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TEXTILE CALCULATIONS

MANUFACTURE AND MECHANISM

BY

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WITH 186 DIAGRAMS

LONDON
SIR ISAAC PITMAN AND SONS, LIMITED
PARKER STREET, KINGSWAY, W.C. 2.
BATH, MELBOURNE, TORONTO, NEW YORK
1920

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PREFACE

One object of this book is to deal with the Arithmetical and Geometrical principles underlying the calculations in textile production, and the calculations in the running and output of textile machinery.

A second object of the book is to elucidate by examples the theory and practice in working out problems for ascertaining quantitative results, costs, and ratios of output, in all classes of yarn and fabric manufacture.

The author has found that the average textile student commits certain formulæ and rules to memory, and applies them mechanically; he is not sufficiently grounded in the theory and science on which these formulæ and rules are based and constructed. It has, therefore, been sought in this treatise to give a full explanation of these, side by side with an exposition of the mathematical problems peculiar to productive routine, and work, in the spinning and weaving industries.

Each variety of calculation, relative to the different branches of these industries, has been treated of, including Woollen and Worsted Spinning, Woollen and Worsted Weaving, Mill Management, Cotton Spinning, Plain and Fancy Cotton Weaving, and Linen and Silk Weaving.

All examples are worked step by step from first principles, the reason for each step being given. From these examples it will be observed that *a knowledge of the various methods of counting yarns and of the various sett systems in use is all that is really required. With a clear understanding of the basic factors on which textile as well as other calculations are formulated, the student perceives the theory as well as the practice comprised in such calculations, and is enabled to work them out with or without reference to tabulated data.*

To this end it has been considered more valuable to give illustrative examples, with fully worked out details, than sets of exercises which may be readily obtained.

Attention may be drawn to the following special points :—

(i) The use of Logarithms and the Slide Rule is encouraged

in the simplification of the complex fractions such as are obtained when finding the weight and cost of warp and weft.

(ii) Simple equations are used in finding the count of a yarn required to twist with one or more yarns to produce a yarn of a given count.

(iii) The choice of scales in constructing graphs to be used as Ready-reckoners for changing the count of a yarn in one system to its equivalent count in some other system, etc. A small diagram is thus made to serve for all numbers of counts (§ 91).

(iv) The diagrams of tappets and the instructions given for drawing these (§ 158).

(v) The use of squared paper in discussing the motion of the "Going-Part" of the loom (§ 159).

The graphs, diagrams, and other illustrations in the book have been specially prepared, and the author has applied the scheme of work they exemplify, in the teaching of Textile Mathematics for some years.

I have to acknowledge my indebtedness to Mr. H. Leach, late Head of the Textile Department of the Holmfirth Technical Institute, for assistance in the definition of technical terms and descriptions of various pieces of mechanism. In this connection also I have received much help from consulting the following works :—

"Woollen and Worsted," by Roberts Beaumont, M.Sc., M.I.Mech.E.

"Tappet and Dobby Looms," by Thomas Roberts.

"Manufacture of Woollen and Worsted Yarns," by J. W. Radcliffe.

"Calculations in Yarns and Fabrics," by Fred Bradbury.

My thanks are also due to the Controller of His Majesty's Stationery Office and to Messrs. Macmillan & Co., Ltd., for permission to include their Mathematical Tables and Useful Constants as an Appendix ; to the City and Guilds of London Institute for permission to make use of Examination questions ; and to the Editor of the Wool Year Book for permission to use particulars relating to various pieces of machinery in the framing of examples.

In addition, I am appreciative of the help and criticisms of Professor Roberts Beaumont, the Editor of this series of Textile Works, and also of the consideration the publishers have given to the issue of the book.

J. H. W.

September, 1920.

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1. *The Various Systems of Counting Yarns.*—The numbering of yarns is necessary to enable spinners and manufacturers to tell at a glance the relative thickness and weight of different yarns. Thus numbers expressing the *counts* have been applied to yarns to denote the thickness and weight of any specific yarn whether of silk, cotton, linen, woollen or worsted. Unfortunately no standard designation has been adopted, the method of counting varying with nearly every class of material.

The terms " counts," " skeins," " grist," " cut," " spindle," etc., are literally the same, for they all represent a certain length of yarn for a specific weight.

The systems of counting yarns may be divided into two classes.

I. Those Systems in which the Length is Variable and the Weight Constant.—As an example consider the counting of various worsted yarns.

The constant weight is 1 lb. Lengths are reckoned in hanks, each hank being 560 yards.

Then if 1 hank or 560 yards weigh 1 lb. the yarn is 1's counts,
if 2 hanks or 1120 yards weigh 1 lb. the yarn is 2's counts,
if 20 hanks or (11,200) yards weigh 1 lb. the yarn is 20's counts.

The term "hank" has a relative importance. The yarn may be on spools or bobbins, or in the form of warp and in balls. What matters is the length which weighs 1 lb., and since it must be a great length for this particular weight, a *unit of* 560 yards is fixed upon, and

The number of these units (560 yards) which weigh 1 lb. is called the "counts" of the yarn.

Thus 30's worsted yarn means
30 hanks (each hank 560 yards long) weigh 1 lb., or
(30×560) yards weigh 1 lb.

II. Those Systems in which the Weight is Variable and the Length Constant.—Thus in the American "Grain System" of counting woollen yarn a fixed length of 20 yards is taken.

The *weight of this length in grains* is the number of counts in the yarn.

Hence if 20 yards weigh 12 grains the counts are 12's grain.
,, if 20 yards ,, 16 ,, ,, ,, ,, 16's grain.

2. *The following tables show the Various Systems in use.*

TABLE I*

Weight Constant—Length Variable

System.	Basis of Counting.	Constant Weight.
Worsted	Number of hanks of 560 yards each	which weigh 1 lb.
Cotton	Number of hanks of 840 yards each	,, ,, 1 lb.
Spun Silk (or Waste Silk)	Number of hanks of 840 yards each	,, ,, 1 lb.
Organzine and Tram (Raw Silk)	Number of yards	,, ,, 1 oz.
Linen	Number of Leas or Cuts of 300 yards each	,, ,, 1 lb.
Yorkshire skeins (woollen)	Number of skeins of 256 yards each	,, ,, 1 lb.
Galashiels (woollen)	Number of Cuts of 300 yards each	,, ,, 24 ozs.

* See *Woollen and Worsted*, by Professor Beaumont; *Calculations in Yarns and Fabrics*, by F. Bradbury.

TABLE I—*cont.*

System.	Basis of Counting.	Constant Weight.
Hawick (woollen)	Number of Cuts of 300 yards each	which weigh 26 ozs.
West of England (woollen)	Number of hanks of 320 yards each	,, ,, 1 lb.
Heavy Woollen District (woollen)	Number of yards	,, ,, 1 oz.
Run (American) (woollen)	Number of runs of 1,600 yards each	,, ,, 1 lb.
Cut (American) (woollen)	Number of hanks of 300 yards each	,, ,, 1 lb.
Stirling and Alloa (woollen)	Number of spindles of 48 cuts each (240 yards per cut)	,, ,, 24 lbs.
Continental Worsted	Number of hanks of 1,000 metres each	,, ,, 1 kilogramme
Continental Linen and Cotton	Number of hanks of 1,000 metres each	,, ,, 500 grammes

A further reduction of some of the foregoing systems of counting may be made. This is done and frequently used in practice thus :

YORKSHIRE SKEINS may be stated as the NUMBER OF YARDS PER DRAM.

Galashiels, the number of cuts of				200	yards	each	which	weigh	1 lb.
West of England	,,	,,	,,	20	,,	,,	,,	,,	1 oz.
Run (American)	,,	,,	,,	100	,,	,,	,,	,,	1 oz.
Stirling & Alloa	,,	,,	,,	480	,,	,,	,,	,,	1 lb.
or	,,	,,	,,	30	,,	,,	,,	,,	1 oz.

TABLE II

WEIGHT VARIABLE—LENGTH CONSTANT

System.	Basis of Counting.	Constant Length.
Dram (Raw Silk)	The number of drams that	1,000 yards weigh
Denier (Raw Silk)	The number of deniers that	{ 476 metres or 520 yards } ,,
Halifax Rural District (woollen)	The number of drams that	80 yards ,,
Grain (American) (woollen)	The number of grains that	20 yards ,,
Aberdeen (woollen)	The number of lbs. that	a spindle weighs (one spindle = 48 cuts of 300 yards each)

TABLE III

SHOWING THE NUMBER OF YARDS THERE ARE IN 1 LB. OF 1's COUNTS OF THE SYSTEMS GIVEN

Worsted	=560 yards	Run (American)	= 1,600 yards
Cotton	=840 ,,	Stirling and Alloa	= 480 ,,
Spun Silk	=840 ,,	Cut (American)	= 300 ,,
Linen	=300 ,,	Dram (Raw Silk)	=256,000 ,,
Yorkshire Skeins	=256 ,,	Halifax and Rural District	= 20,480 ,,
Galashiels	=200 ,,	Aberdeen	= 14,400 ,,
Hawick	$=184\frac{8}{13}$,,	Organzine and Tram (Raw Silk 66's Denier)	= $4{,}202\frac{2}{99}$ yds. per oz.
West of England	=320 ,,		
Heavy Woollen District	=16 ,,		

Mohair, alpaca, and cashmere yarns come under the head of worsted and are *counted on that basis.*

Foreign yarns delivered into this country are usually counted on the worsted although the threads may have been spun upon the woollen principle. Sometimes the yarns may be a mixture of wool and cotton, *e.g.* a yarn may be equal to a 10's worsted and may be composed of 50% wool and 50% cotton.

This yarn would be stated as 1/10's . 50/50.

This signifies that the yarn is single 10's counts with 50% wool and 50% cotton in its composition.

Again, a yarn stated 1/10's . 70/30 would mean that it was single 10's worsted counts, composed of 70% wool and 30% cotton. Also 1/10 . 40/60 signifies that the yarn is composed of 40% wool and 60% cotton.

3. *Single and Composite Yarns.*—(*a*) In single yarns, or yarns that are composed of one thread, the counts may be indicated 30's or we may write 1/30's. This signifies that the yarn is composed of one thread and that there are 30 hanks of this particular yarn in a fixed weight, if belonging to any system in the first class as shown in Table I. If the yarn belongs to any system in the second class as shown in Table II, then 1/30's or 30's indicates a single yarn in which a fixed length weighs 30 units of weight (dram, oz., lb., grain, etc.).

If the yarn is composed of two, three, or more threads of the same sort twisted together, then the number denoting the counts is prefaced by a number indicating the number of threads of which the yarn is composed.

Thus 2/30's means that 2 threads of 1/30's have been twisted together to make a single thread.

3/30's means that 3 threads of 1/30's counts are twisted together to form the resultant thread.

(*b*) *Counts of these composite threads.*

If 1/30's means that 30 hanks of that particular thread has a certain weight (say 1 lb.)

and in a 2/30's two of these threads are twisted together, then 30 hanks of this composite thread evidently weigh 2 lbs. or 15 hanks weigh 1 lb.

Hence a 2/30's thread is evidently equivalent to a 1/15's thread of the same denomination.

Similarly

The true counts of a 3/30's thread would be 1/10's or 10's.

„ „ „ „ 3/60's „ „ „ 1/20's or 20's.

The student must note that the threads twisted are all of the same material and the same counts.

(Later we shall deal with the counts of a twist thread composed of different materials and having different counts.)

Now this holds good for folded yarns of all denominations with the exception of spun silk, and in this system the counts of the folded thread would be written 30's/2 or 30's/3, and these mean that *the compound thread* is equivalent to a 30's but has been formed by twisting two or three threads together.

Hence

in 2/30's worsted there would be 15 hanks in 1 lb.
„ 30/2 spun silk „ „ „ 30 „ „ 1 lb.
„ 3/30's worsted „ „ „ 10 „ „ 1 lb.
„ 30/3 spun silk „ „ „ 30 „ „ 1 lb.

Worsted yarns are sometimes sold by the gross instead of by the lb. This means that the spinner quotes a price for a gross of hanks. Suppose the yarn be 1/36's.

Then since 36 hanks weigh 1 lb.

$$144 \text{ hanks } \text{„ } \frac{144}{36} \text{ lbs. or 4 lbs.}$$

and the price per lb. may thus be deduced from the price per gross of hanks.

4. *Continental Methods of Counting Yarns.*—(1) Worsted Yarns.—This is based upon the hank of 1,000 metres, and the number of such hanks which weigh 1 kilogramme is the number of the counts.

This may be further reduced to

Counts = *Number of metres* that weigh 1 gramme.

(2) Cotton and Linen.—Each of these systems is based upon a hank of 1,000 metres and the number of such hanks which weigh $\frac{1}{2}$ kilogramme (500 grammes) represents the counts.

(3) Silk (Raw).—This is based upon the hank of 400 French ells equivalent to 476 metres or 520 yards, and the weight of this hank in deniers represents the counts. ($533\frac{1}{3}$ deniers = 1 oz.)

The *Legal Silk Count* (approved Paris 1900) is based upon a skein of 450 metres; the weight of such a length in half deci-grammes indicates the counts.

In converting counts of a yarn in an English system to the equivalent counts in the Continental system and *vice versa* we make use of the following equivalents in the British and Metric units of Weights and Measures.

1 metre	= 39·37011 inches = 1·094 yards.
1 yard	= 0·9144 metres.
1 kilogramme	= 2·2046 lbs. = 2·205 lbs.
1 lb.	= 453·593 grammes.
	= 0·4536 kilogramme.
1 gramme	= 15·432 grains and 7000 grains = 1 lb.

5. *Reduction of Counts of one Denomination to its Equivalent in another System.*—The necessity of reducing from one system to another lies in the fact that in the production of some yarns different materials are twisted together. This may be done either for strength or for the purpose of forming effects in yarns that can only be produced by this method of twisting together different materials such as silk, wool, cotton, etc. Also yarn spun in one locality is sent into various districts where the system of counting is different. It is then necessary to transpose the counts of the yarn from the system in which it has been spun to the system of counting in vogue in the district to which the yarn is sent.

6. *Worked Examples on the Various Systems of Counting Yarns.*

Example 1.

What is the weight of 3,360 yards of 15's worsted yarn?

In the worsted system of counting the yarn we have the *hank of* 560 *yards* and the "*counts*" tell us the number of these hanks which weigh 1 lb.

Thus 15's worsted indicates that (15 × 560) yards weigh 1 lb.

∴ If (15 × 560) yards weigh 1 lb.

$$1 \text{ yard weighs } \frac{1}{15 \times 560} \text{lb.}$$

and 3,360 yards weigh $\frac{3360}{15 \times 560}$ or $\frac{2}{5}$ lb.

Example 2.

Suppose 44,800 yards of yarn weigh 5 lbs., what is the count of the yarn in (*a*) worsted, (*b*) Yorkshire skeins, (*c*) Galashiels denominations?

Since 44,800 yards weigh 5 lbs. then $\frac{44,800}{5}$ or 8,960 yards weigh 1 lb.

(*a*) In the worsted system the "*counts*" give the number of hanks of 560 yards which weigh 1 lb., so that in 8,960 yards there are $\frac{8960}{560}$ or 16 hanks (560 yards long), or the yarn is 16's worsted.

(*b*) The Yorkshire skeins system is based on the hank of 256 yards long.

Thus in 8,960 yards of yarn there are $\frac{8960}{256}$ or 35 hanks (256 yards long), or the yarn is 35's Yorkshire skeins.

(*c*) The Galashiels (Gala) system is based on a "*cut*" of 300 yards and the number of these "*cuts*" which weigh 24 ounces represents the counts.

Since 300 yards weigh 24 ozs. then 50 yards weigh 4 ozs. and 200 yards weigh 16 ozs. or 1 lb.

In 8,960 yards of yarn there are $\frac{8960}{200}$ or 44·8 hanks (200 yards long).

$\therefore$ 44·8 hanks of 200 yards weigh 1 lb., or the yarn is 44·8 or $44\frac{4}{5}$'s Gala.

Example 3.

(*a*) What is the weight of 6 gross hanks of 2/48's worsted?

(*b*) If 5 gross hanks of Yorkshire skeins woollen weigh 24 lbs. what would be the counts of the yarn?

(*a*) A 2/48's worsted is equivalent to a 1/24's worsted, that is 24 hanks (560 yards long) weigh 1 lb. or 1 hank weighs $\frac{1}{24}$ lb. and (6 × 144) hanks weigh $\frac{6 \times 144}{24}$ or 36 lbs.

$\therefore$ Weight of 6 gross hanks = 36 lbs.

(*b*) In this case (5 × 144) hanks (256 yards long) weigh 24 lbs. or 1 hank weighs $\frac{24}{5 \times 144}$ lbs.

Now $\frac{24}{5 \times 144} = \frac{1}{30}$ lb. $\therefore$ 30 hanks weigh 1 lb.; or the counts = 30's Y.S.

Example 4.

Give in worsted counts the equivalent of 20 skeins wool, 2/40's cotton, and 13,000 yards Organzine silk.

Here it has to be noted—

1. Worsted counts are based on a hank of 560 yards.
2. Y. skeins wool are based on a hank of 256 yards.
3. Cotton counts are based on a hank of 840 yards.
4. 13,000 Organzine silk means 13,000 yards weigh 1 oz.

(*a*) 20's Y.S. means (20 × 256) yards weigh 1 lb. Since a worsted hank is 560 yards long we wish to know how many hanks each 560 yards long there are in (20 × 256) yards, which is the length of the yarn given as weighing 1 lb.

This is evidently $\frac{20 \times 256}{560}$.

Hence, equivalent worsted counts $= \frac{20 \times 256}{560} = 9\frac{1}{7}$'s worsted.

(*b*) A 2/40's cotton yarn is equivalent to 1/20's cotton, and 20's cotton means (20 × 840) yards weigh 1 lb.

Here again we require the number of worsted hanks (560 yards long) in (20 × 840) yards, and this is $\frac{20 \times 840}{560}$.

$\therefore$ Equivalent worsted counts $= \frac{20 \times 840}{560} = 30$'s.

(*c*) 13,000 Organzine silk means 13,000 yards weigh 1 ounce or (13,000 × 16) yards weigh 1 lb.

$\therefore$ Equivalent worsted counts $= \frac{13{,}000 \times 16}{560} = 371\frac{3}{7}$'s.

Example 5.

Find the equivalent Continental counts of 2/32's worsted yarn.

1. Continental counts for worsted are based on the number of hanks (1,000 metres long) which weigh 1 kilogramme.
2. 1 metre = 39·37011 inches = 1·094 yards to nearest $\frac{1}{1000}$ or 1 yard = 0·9144 metre.
3. 1 kilogramme = 2·205 lbs. or 1 lb. = 453·593 grammes
   = 0·4536 kilogramme.

2/32's worsted is equivalent to 1/16's worsted.

16's worsted means (16 × 560) yards weigh 1 lb.

Since 1 yard = 0·9144 metre and 1 lb. = 0·4536 kilogramme then (16 × 560 × 0·9144) metres weigh 0·4536 kilogramme,

or $\frac{16 \times 560 \times 0{\cdot}9144}{0{\cdot}4536}$ metres weigh 1 kilogramme,

and the number of hanks 1000 metres long in this length of yarn indicates the counts required.

Hence equivalent Continental counts $= \dfrac{16 \times 560 \times 0{\cdot}9144}{0{\cdot}4536 \times 1000}$

$= 18{\cdot}06$ (using log. tables).

Example 5 (a).

To find the constant multiplier which may be used to change *worsted counts to their equivalent Continental worsted counts.*

Suppose the English count is $1/x$'s worsted.

Proceeding as in Example 5, $1/x$'s worsted means that $(x \times 560)$ yards weigh 1 lb.

Since 1 yard $= 0{\cdot}9144$ metre and 1 lb. $= 0{\cdot}4536$ kilogramme we have $(x \times 560 \times 0{\cdot}9144)$ metres weigh 0·4536 kilogramme,

or $\dfrac{x \times 560 \times 0{\cdot}9144}{0{\cdot}4536}$ metres weigh 1 kilogramme,

and the number of hanks 1000 metres long in this length of yarn indicates the counts required.

Hence equivalent Continental counts $= \dfrac{x \times 560 \times 0{\cdot}9144}{0{\cdot}4536 \times 1000}$.

Now if we are converting 16's English worsted then $x = 16$, and we get Continental counts $= \dfrac{16 \times 560 \times 0{\cdot}9144}{0{\cdot}4536 \times 1000}$, which is the same as Example 5.

Again if we are converting 20's worsted then $x = 20$, and Continental counts $= \dfrac{20 \times 560 \times 0{\cdot}9144}{0{\cdot}4536 \times 1000}$.

Note that the English counts are always multiplied by $\dfrac{560 \times 0{\cdot}9144}{0{\cdot}4536 \times 1000}$ or 1·129 (correct to 4 figures).

Then 16's English worsted $= 16 \times 1{\cdot}129$ Continental worsted
$= 18{\cdot}06$ (Example 5),

20's English $= 20 \times 1{\cdot}129 = 22{\cdot}58$'s Continental, and x's English $= (x \times 1{\cdot}129)$ Continental,

or Continental worsted counts = (English worsted counts) $\times$ 1·129.

Example 6.

The counts of a certain cotton yarn are 30's Continental system. Find the equivalent cotton counts in the English system.

Remembering :

1. Continental counts for cotton are based on the number of hanks (1,000 metres long) which weigh $\frac{1}{2}$ kilogramme.

2. English cotton counts based on number of hanks (840 yards) which weigh 1 lb.

3. 1 metre = 1·094 yards to nearest $\frac{1}{1000}$ yard.

4. 1 kilogramme = 2·205 lbs. to nearest $\frac{1}{1000}$ lb.

30's Continental cotton means (30 × 1000) metres weigh 500 grammes or 0·5 kilogramme.

Since 1 metre = 1·094 yards and 1 kilogramme = 2·205 lbs., we have (30 × 1000 × 1·094) yards weigh (0·5 × 2·205) lbs., or (1094 × 30) yards weigh 1·1025 lb.

or $\frac{1094 \times 30}{1{\cdot}1025}$ yards weigh 1 lb., and the number of hanks 840 yards long in this length of yarn indicates the counts required.

$\therefore$ Equivalent English counts $= \frac{1094 \times 30}{1{\cdot}1025 \times 840} = 35{\cdot}42$'s (using log. tables).

Example 6 (a).

To find the constant multiplier which may be used to change *Continental cotton counts to their equivalent English cotton counts.*

Suppose the Continental cotton count is x's.

Proceeding as in Example 6,

x's Continental cotton means that (x × 1000) metres weigh 500 grammes or 0·5 kilogramme.

Since 1 metre = 1·094 yards and 1 kilogramme = 2·205 lbs.,

$\therefore$ (x × 1000 × 1·094) yards weigh (0·5 × 2·205) lbs.,

or (x × 1094) yards weigh 1·1025 lb.

or $\frac{x \times 1094}{1{\cdot}1025}$ yards weigh 1 lb., and the number of hanks 840 yards long in this length of yarn indicates the counts required.

Hence equivalent English cotton counts $= \frac{x \times 1094}{1{\cdot}1025 \times 840}$.

Or, in converting 30's Continental counts, $x = 30$, and equivalent English counts $= \frac{30 \times 1094}{1{\cdot}1025 \times 840}$, which is the result in Example 6.

Similarly in converting 20's Continental counts, $x = 20$, and equivalent English counts $= \frac{20 \times 1094}{1{\cdot}1025 \times 840}$.

It will thus be noticed that the Continental counts are always multiplied by $\frac{1094}{1{\cdot}1025 \times 840}$ or 1·180 (correct to 4 figures).

∴ 30's Continental cotton = 30 × 1·18's English cotton = 35·4's (Example 6),

20's Continental cotton = 20 × 1·18 = 23·6's English cotton,

x's Continental cotton = (x × 1·18) English cotton,

or (English cotton counts) = (Continental cotton counts) × 1·18.

Example 7.

In a certain system of yarn numbering the counts increase as the yarns become thicker, thus a 4-oz. yarn is half the weight of an equal length of an 8-oz. yarn. The 4-oz. yarn is equivalent to a 2/48's worsted. Find the standard length which is the basis of the system and make out a list of equivalents to 10, 12, 14, 16, 18 and 20 *skeins woollen.*

Note that in this method we find the weight of a certain fixed length of each yarn and *number the yarns according to the weight.*

Hitherto the weight has been fixed (*e.g.*, 1 lb., 1 kgm.) and the yarn *numbered according to the number of hanks* of certain lengths in this fixed weight.

We are told that the yarn which is numbered a 4-oz. yarn is equivalent to a 2/48's worsted, and a 2/48's worsted is equivalent to a 1/24's worsted.

24's worsted means (24 × 560) yards weigh 1 lb., or $\frac{24 \times 560}{4}$ yards weigh 4 ounces.

Hence in this particular yarn a fixed length of $\frac{24 \times 560}{4}$ yards must weigh 4 ounces,

or *the Standard Length* = $\frac{24 \times 560}{4}$ = 3,360 yards.

Thus the yarn will be numbered according to the weight of 3,360 yards in each case.

(*b*) 10's Yorkshire skeins means (10 × 256) yards weigh 1 lb.

Hence 1 yard weighs $\frac{16}{10 \times 256}$ ounces,

or 3,360 yards weigh $\frac{16 \times 3360}{10 \times 256}$ ounces,

and $\frac{16 \times 3360}{10 \times 256} = \frac{210}{10}$ = 21 ozs.

Then 10's Yorkshire skeins is equivalent to a 21-oz. yarn.

(*c*) Again 12's Yorkshire skeins means (12×256) yards weigh 1 lb.

Hence 1 yard weighs $\frac{16}{12 \times 256}$ ounces,

or 3,360 yards weigh $\frac{16 \times 3360}{12 \times 256}$ ounces,

and $\frac{16 \times 3360}{12 \times 256} = \frac{210}{12} = 17\frac{1}{2}$ ozs.

$\therefore$ 12's Yorkshire skeins is equivalent to a $17\frac{1}{2}$-oz. yarn.

(*d*) Notice that in finding equivalents for 10's and 12's Yorkshire skeins we have $\frac{21}{10}^{0}$ and $\frac{21}{12}^{0}$ finally.

Then similarly

14's Yorkshire skeins is equivalent to a $\frac{21}{14}^{0}$ or a 15-oz. yarn
16's ,, ,, ,, ,, $\frac{21}{16}^{0}$ or a $13\frac{1}{8}$-oz yarn
18's ,, ,, ,, ,, $\frac{21}{18}^{0}$ or a $11\frac{2}{3}$-oz. yarn
20's ,, ,, ,, ,, $\frac{21}{20}^{0}$ or a $10\frac{1}{2}$-oz. yarn

Hence the equivalents to 10's, 12's, 14's, 16's, 18's, 20's Yorkshire skeins are 21 oz., $17\frac{1}{2}$ oz., 15 oz., $13\frac{1}{8}$ oz., $11\frac{2}{3}$ oz., $10\frac{1}{2}$ oz.

7. *Average Counts.*—It is often useful and necessary to find the average counts of yarns that are used in the production of a certain cloth in which different thicknesses of yarns have been used in the building up of the fabric.

Suppose, for example, it is desired to reproduce a cloth of this description, but in place of yarns of unequal counts we have to use yarns of one count only and at the same time use the same number of ends and picks per inch and get the same weight per yard as in the original cloth. Then if we get the average counts of the various yarns used in the original cloth and use the same sett and picks per inch we should get a cloth the same weight and build as the original, although the effect may not be the same.

As an example:

Suppose a warp is one thread of 16's, three of 60's, and four of 80's worsted. The warp has a certain length and weight. If the warp is to consist of one kind of thread only, keeping the length and weight the same, the counts of this particular thread will be the *Average counts* of the various threads used in the original warp.

Now every separate thread in the warp whatever its counts will be the same length, *i.e.*, the length of the warp. The length may be 1 hank, 20 hanks, 80 hanks, x hanks.

Suppose we use 1 hank of 16's, then we use 3 hanks of 60's and 4 hanks of 80's, and altogether in the warp we use 8 hanks of thread, *or* if we use 80 hanks of 16's, then we use (3×80) hanks of 60's and (4×80) hanks of 80's, and altogether we use (8×80) hanks of thread, and the *number of these hanks* which weigh 1 lb. (supposing all threads the same) indicates the AVERAGE COUNTS.

Thus 80 hanks of 16's weigh 5 lbs.
(3×80) „ „ 60's „ 4 lbs.
(4×80) „ „ 80's „ 4 lbs.
Then weight of (8×80) hanks = 13 lbs.
or „ „ $\frac{8 \times 80}{13}$ „ = 1 lb.

$\therefore$ Average counts of thread in warp $= \frac{8 \times 80}{13} = 49\frac{3}{13}$'s worsted.

8. *Resultant Counts.*—If two or more threads have been twisted or folded together to form a new yarn, then it is necessary to find the number of hanks of this new yarn which weigh a lb. or other standard weight. This number is known as the resultant counts of the threads which have been twisted.

(*a*) If the compound thread is composed of two or more threads of the same material and of the same counts the resultant counts is easily determined, provided also that there is no variation in "take up" of the threads twisted together. *E.g.*:

If we twist three threads of 60's worsted to make a compound thread *of the same length* it is very evident that

60 hanks of the twisted thread will weigh 3 lbs.
or 20 „ „ „ „ „ „ 1 lb.
Then resultant counts = 20's worsted.

This case of a folded yarn has been dealt with before, when we said that

2/60's cotton is equivalent to 1/30's cotton.
3/30's Y. skeins is equivalent to 1/10 Y. skeins, etc.

(*b*) If the threads which are folded or twisted together be of different materials, or of different counts, or if in the twisting the length of the twisted yarn differs from the lengths of the component threads, *the resultant counts* are then rather more difficult to find.

The student must study the examples worked at the end.

(c) Resultant counts may be wanted for any of the following cases :—

1. Twisted yarns in which the compound threads are of the same denomination, and there is no variation in " take up " of the threads twisted together.
2. Twisted yarns composed of threads of various denominations such as silk, cotton, wool, etc., with no variation in " take up."
3. Twisted yarns composed of threads of either the same or different denominations, but in which one or more of the component threads is allowed to run in slacker in the twisting. We thus get variation in " take up " of the threads used in the formation of the compound yarn.

9. *" Take up " in Compound Yarns.*—" Take up " enters into consideration in finding the resultant counts of some yarns, because fancy twist, such as spirals, loop, knop, curls, etc., are made by using different lengths of the component threads. For example, in the production of spiral yarns, one of the threads is held tight, whilst the other is let out of the delivery rollers of the twisting frame at a quicker rate, and is allowed to twine itself around the tighter thread in a cork-screw fashion. If therefore the slacker thread were allowed to be delivered twice as fast as the tight thread, the spiral yarn would " take up " twice the length of the slack thread to that of the tight thread and the " take up " in twisting would be 2 to 1. Thus if we required 10 hanks of the tight thread to produce a certain amount of spiral yarn, then we should require 20 hanks of the slacker thread. If no variation in " take up " were allowed, the effect would be entirely destroyed and we should simply have an ordinary twist yarn.

In " loop " yarns one of the component threads is allowed to run in quicker than the other thread at certain intervals, and made to form a loop.

In " knop " yarn one of the threads forms a small " *knop* " or " *snarl* " caused by letting this thread run in quickly at intervals whilst keeping the other thread tight.

10. *Worked examples on " Average " Counts.*

EXAMPLE 1.

A warp consists of four threads of 80's, two threads of 40's, and one thread of 16's worsted. What is the average count ?

If the warp requires 1 hank of 16's worsted it also requires 2 hanks of 40's and 4 hanks of 80's, *or*

If we use 80 hanks of 16's we must also use (2 × 80) hanks of 40's and (4 × 80) hanks of 80's, and so on in like proportion.

Now 80 hanks of 16's weigh 5 lbs.
(2×80) ,, ,, 40's ,, 4 lbs.
(4×80) ,, ,, 80's ,, 4 lbs.
Hence 560 hanks of yarn weigh 13 lbs.
or $\frac{560}{13}$,, ,, ,, ,, 1 lb.
$\therefore$ Average counts $= \frac{560}{13} = 43\frac{1}{13}$'s.

Note.—The choice of the number of hanks required is optional. We chose 80 in this case since 80 is the L.C.M. of 16's, 40's, and 80's, and this choice gives the weights as integers.
We might have chosen 1 hank or x hanks. (*See* Example 3.)

Example 2.

A warp weighing 60 lbs. is arranged as follows :—
1 thread of 2/40's worsted ; 1 thread of 2/32's worsted ; 1 thread of 22's Yorkshire skeins. Find the average count in skeins and the weight of each yarn in the warp.
(*a*) First change the worsted counts to their equivalent counts in Yorkshire skeins.
Now worsted counts are based on a hank of 560 yards and Yorkshire skeins are based on a hank of 256 yards. Then 2/40's or 1/20's worsted means that (20×560) yards weigh 1 lb., and number of hanks (256 yards) in (20×560) yards $= \frac{20 \times 560}{256}$.

$\therefore$ Equivalent Yorkshire skeins $= \frac{20 \times 560}{256} = 43\frac{3}{4}$'s.

Similarly, taking 2/32's worsted or 1/16's.,

Equivalent Yorkshire skeins $= \frac{16 \times 560}{256} = 35$'s.

Now the length of each thread used in making the warp is the same.
Suppose there are 175 hanks of each required.
Then 175 hanks of $43\frac{3}{4}$'s Yorkshire skeins weigh 4 lbs.
175 ,, ,, 35's ,, ,, ,, 5 lbs.
175 ,, ,, 22's ,, ,, ,, $7\frac{2}{2}\frac{1}{2}$ lbs.
and in the whole warp we have
525 hanks of thread weighing $16\frac{2}{2}\frac{1}{2}$ lbs.
and number of hanks per lb. $= \frac{525}{16\frac{2}{2}\frac{1}{2}} = 30{\cdot}96$ (using logs.).
$\therefore$ Average counts $= 30{\cdot}96$'s $= 31$'s Yorkshire skeins (approx.).

(*b*) *To find the weight of each thread used in the warp.*
From the above we see that :—
In $16\frac{2}{2}\frac{1}{2}$ lbs. of thread used there are 4 lbs. of $43\frac{3}{4}$'s Yorkshire skeins or 2/40's worsted.

In 1 lb. of thread used there are $\frac{4}{16\frac{2}{2}\frac{1}{2}}$ lbs. of $43\frac{3}{4}$'s Yorkshire skeins or 2/40's worsted.

In 60 lbs. of thread used there are $\frac{4 \times 60}{16\frac{2}{2}\frac{1}{2}}$ lbs. of $43\frac{3}{4}$'s Yorkshire skeins or 2/40's worsted.

$\therefore$ Weight of 2/40's worsted $= \frac{4 \times 60}{16\frac{2}{2}\frac{1}{2}} = 14{\cdot}16$ lbs.

Again

In $16\frac{2}{2}\frac{1}{2}$ lbs. of thread used there are 5 lbs. of 2/32's worsted.

,, 1 lb. ,, ,, ,, ,, $\frac{5}{16\frac{2}{2}\frac{1}{2}}$ lbs. ,, ,,

,, 60 lbs. ,, ,, ,, ,, $\frac{5 \times 60}{16\frac{2}{2}\frac{1}{2}}$ lbs. ,, ,,

$\therefore$ Weight of 2/32's worsted $= \frac{5 \times 60}{16\frac{2}{2}\frac{1}{2}} = 17{\cdot}69$ lbs.

Also

In $16\frac{2}{2}\frac{1}{2}$ lbs. of thread used there are $7\frac{2}{2}\frac{1}{2}$ lbs. of 22's Y. skeins.

,, 1 lb. ,, ,, ,, $\frac{7\frac{2}{2}\frac{1}{2}}{16}$ lbs. ,, ,,

,, 60 lbs. ,, ,, ,, $\frac{7\frac{2}{2}\frac{1}{2} \times 60}{16\frac{2}{2}\frac{1}{2}}$ lbs. ,, ,,

$\therefore$ Weight of 22's Yorkshire skeins $= \frac{7\frac{2}{2}\frac{1}{2} \times 60}{16\frac{2}{2}\frac{1}{2}} = \frac{175 \times 60}{373}$

$= 28{\cdot}15$ lbs.

Total weight $= 14{\cdot}16 + 17{\cdot}69 + 28{\cdot}15 = 60$ lbs.

Example 3.

A cloth is woven with one pick of 24's mohair and one pick of 16's Yorkshire skeins woollen. What are the average counts in skeins ?

Here it is necessary to find the equivalent of 24's mohair in the Yorkshire skeins denomination.

Mohair comes under the head of worsted and is based on a hank of 560 yards.

Yorkshire skeins counts are based on a hank of 256 yards.

Then 24's mohair $= \frac{24 \times 560}{256} = 52\frac{1}{2}$'s Yorkshire skeins.

The same length of each thread will be used. Suppose it is x hanks.

Then since 16 hanks of 16's Yorkshire skeins weigh 1 lb.
or 1 hank ,, 16's ,, ,, weighs $\frac{1}{16}$ lb.

$\therefore$ x hanks of 16's weigh $\frac{x}{16}$ lbs.

and x ,, ,, $52\frac{1}{2}$'s ,, $\frac{x}{52\frac{1}{2}} = \frac{2x}{105}$ lbs.

$\therefore$ $2x$ hanks of thread altogether weigh $\left(\frac{x}{16} + \frac{2x}{105}\right)$ lbs. or $\frac{137x}{1680}$ lbs., and number of hanks weighing 1 lb. $= 2x \div \frac{137x}{1680}$

or average counts $= \frac{2x \times 1680}{137x} = \frac{3360}{137} = 24{\cdot}53$'s.

11.—*Worked examples on Resultant Counts of Twisted Yarns.*

EXAMPLE 1.

Find the resultant counts of a yarn made by twisting 1/30's and 1/60's worsted.

Assuming there is no "take up" during the twisting, then the length of each yarn used will be the same.

For purposes of calculation we may choose to twist any number of hanks of each (*e.g.*, 1, 2, 30, 60, or x).

(*a*) Suppose we twist 1 hank (256 yards) of each thread, then we shall have 1 hank (256 yards) of twist yarn.

Now 60 hanks of 60's weigh 1 lb.
or 1 hank of 60's weighs $\frac{1}{60}$ lb.
and 1 hank of 30's ,, $\frac{1}{30}$ lb.

$\therefore$ 1 hank of twist yarn weighs $(\frac{1}{60} + \frac{1}{30}) = \frac{1}{20}$ lb.

Then 20 hanks of twist yarn weigh 1 lb. But the number of hanks (256 yards) which weigh 1 lb. indicates the Counts (Worsted Denomination).

$\therefore$ Counts of the twist yarn = 20's worsted.

(*b*) The working will not involve fractions if we choose to twist 60 hanks of each thread.

Thus 60 hanks of 60's yarn weigh 1 lb.
60 ,, ,, 30's ,, ,, 2 lbs.
$\therefore$ 60 hanks of twisted yarn weigh 3 lbs.
or 20 ,, ,, ,, ,, 1 lb.
Hence Counts of the twist yarn = 20's worsted.

EXAMPLE 2.

Give the size of a 4-fold thread of 10's, 15's, 20's and 30's Yorkshire skeins.

Assuming there is no take up, if we twist 30 hanks of each of the four threads we shall have 30 hanks of twist yarn.

Now 30 hanks of 30's Yorkshire skeins weigh 1 lb.
30 ,, ,, 20's ,, ,, ,, $1\frac{1}{2}$ lb.
30 ,, ,, 15's ,, ,, ,, 2 lbs.
30 ,, ,, 10's ,, ,, ,, 3 lbs.

Then 30 hanks of twist yarn weigh $7\frac{1}{2}$ lbs.

or $\frac{30}{7\frac{1}{2}}$,, ,, ,, ,, ,, 1 lb.

Hence Counts of the twist yarn $= \frac{30}{7\frac{1}{2}} = 4$'s Yorkshire skeins.

Example 3.

Find the worsted counts of a folded yarn made of 2/40's cotton and 1/32's mohair.

Note.—Mohair is worsted.

Here it is necessary to find the equivalent worsted counts of the cotton thread.

Now
2/40's or 1/20's cotton means that (20 × 840) yards weigh 1 lb. and worsted counts are based on a hank of 560 yards.

$\therefore$ 2/40's or 1/20's cotton $= \frac{20 \times 840}{560} = 30$'s worsted.

Then 32 hanks of 32's worsted weigh 1 lb.
and 32 ,, ,, 30's ,, ,, $\frac{32}{30}$ or $1\frac{1}{15}$ lb.

Assuming no "take up" there will be 32 hanks of folded yarn and $\therefore$ 32 hanks of twisted yarn weigh $2\frac{1}{15}$ lb.

or $\frac{32}{2\frac{1}{15}}$,, ,, ,, ,, 1 lb

$\therefore$ Counts of the twist yarn $= \frac{32}{2\frac{1}{15}} = 15\frac{15}{31}$'s worsted.

Example 4.

Give the resulting counts in skeins of 2/50's cotton and $12\frac{1}{2}$ skeins woollen, also the counts if 100 yards of each yield 90 yards of twist.

(*a*) First change the cotton counts to their equivalent in the skeins woollen denomination.

Now 2/50's or 1/25's cotton means (25 × 840) yards weigh 1lb.
and Yorkshire skeins counts are based on a hank of 256 yards.

$\therefore$ 2/50's or 1/25's cotton $= \frac{25 \times 840}{256} = 82\frac{1}{32}$'s Y. skeins.

Then 100 hanks of $82\frac{1}{2}$'s Yorkshire skeins weigh $\frac{100}{82\frac{1}{2}}$ or $\frac{128}{105}$ lbs.

and 100 hanks of $12\frac{1}{2}$'s ,, ,, ,, $\frac{100}{12\frac{1}{2}}$ or 8 lbs.

If there is no "take up" then

100 hanks of twist yarn weigh $9\frac{23}{105}$ lbs.

or $\frac{100}{9\frac{23}{105}}$,, ,, ,, ,, ,, 1 lb.

$\therefore$ Counts of twist yarn $= \frac{100}{9\frac{23}{105}} = 10\frac{25}{42}$ Yorkshire skeins.

(*b*) If 100 yards of each thread when twisted yield 90 yards of twist,

then (2×100) yards of each yield (2×90) yards of twist

and (256×100) ,, ,, ,, ,, (256×90) ,, ,, ,,

or 100 hanks (Yorkshire skeins) of each yield 90 hanks of twist.

From the solution in (*a*) we see that on twisting 100 hanks of 2/50's cotton ($82\frac{1}{2}$'s Yorkshire skeins) and 100 hanks of $12\frac{1}{2}$'s Yorkshire skeins we have

90 hanks of twist yarn weigh $9\frac{23}{105}$ lbs.

or $\frac{90}{9\frac{23}{105}}$,, ,, ,, ,, ,, 1 lb.

$\therefore$ Counts of twist yarn $= \frac{90}{9\frac{23}{105}} = 9\frac{369}{483} = 9\frac{3}{4}$'s Y. skeins.

Example 5.—Involving "Take up."

A spiral yarn is made by twisting a 2/60's Botany with a 1/8's Botany. If 4 inches of the former are required to 3 inches of the latter, what is the number of hanks per lb. of the twist thread ?

When 4″ of 2/60's are twisted with 3″ of 1/8's we have 3″ of twist

,, 4 yards of 2/60's ,, ,, 3 yards of 1/8's ,, 3 yards ,,

,, 4 hanks of 2/60's ,, ,, 3 hanks of 1/8's ,, 3 hanks ,,

Now 4 hanks of 2/60's or 1/30's weigh $\frac{4}{30}$ or $\frac{2}{15}$ lb.

3 ,, ,, 1/8's ,, $\frac{3}{8}$ lb.

$\therefore$ 3 hanks of twist weigh $(\frac{2}{15} + \frac{3}{8})$ or $\frac{61}{120}$ lb.

Hence $\frac{3}{\frac{61}{120}}$,, ,, ,, ,, 1 lb.

$\therefore$ Counts of twist yarn $= \frac{3}{\frac{61}{120}} = \frac{3 \times 120}{61} = 5\frac{55}{61} =$ 6's Botany.

Example 6.—Involving "Take up."

A fancy loop yarn is composed of two threads of 1/40's white cotton, two threads of 1/40's red cotton, and two threads of 2/100's black cotton. The relative lengths of material used are 8″, 9″ and 4″ respectively. Find the resultant counts if the last thread is straight.

Here two threads of 1/40's are equivalent to one thread of 1/20's
" " 2/100's " " " " " 1/25's
and if we twist 8 hanks of white cotton with 9 hanks of red cotton and 4 hanks of black cotton, we shall have 4 hanks of twist thread.

Now 8 hanks of 1/20's white cotton weigh $\frac{2}{5}$ lb.
9 " 1/20's red " " $\frac{9}{20}$ lb.
4 " 1/25's black " " $\frac{4}{25}$ lb.

$\therefore$ 4 hanks of twist weigh $(\frac{2}{5} + \frac{9}{20} + \frac{4}{25})$ or $\frac{101}{100}$ lb.

or $(4 \div \frac{101}{100})$ hanks weigh 1 lb.

$\therefore$ Count of the twist yarn $= 4 \div \frac{101}{100} = 3\frac{97}{101}$'s cotton.

Example 7.—Involving "Take up."

Find the resulting worsted counts of a 3-fold fancy yarn made as follows :—

1 thread of 8's mohair taking up 60%
1 " 1/30's worsted " " 6%
1 " 1/30's cotton " " $1\frac{1}{2}$%

It must be noted in this problem :

1. Mohair is counted in the same system as worsted.

2. $1/30\text{'s cotton} = \dfrac{30 \times 840}{560} = 45\text{'s worsted.}$

3. In producing 100 hanks of folded yarn we shall require 160 hanks of mohair, 106 hanks of 1/30 worsted, $101\frac{1}{2}$ hanks of 1/30's cotton.

Now 160 hanks of 8's mohair weigh 20 lbs.
106 " 1/30's worsted " $3\frac{8}{15}$ lbs.
$101\frac{1}{2}$ " 45's worsted (30's cotton) weigh $2\frac{23}{50}$ lbs.

Hence 100 worsted hanks of twist weigh $25\frac{71}{90}$ lbs.

or $(100 \div 25\frac{71}{90})$ hanks " " 1 lb.

$\therefore$ Counts of the folded yarn $= 100 \div 25\frac{71}{90}$

$$= \frac{100 \times 90}{2321} = 3{\cdot}877 = 3\tfrac{3}{4}\text{'s (approx.).}$$

EXAMPLE 8.—INVOLVING YARNS IN TABLE II.

Find the resultant counts of a yarn made by twisting 2 threads of 6 dram (raw silk) with 1 thread of 5 dram (raw silk) and 2 threads of 4 dram (raw silk).

Here 6 dram (raw silk) means that 1000 yards weigh 6 drams
5 dram „ „ „ „ 1000 „ „ 5 „
4 dram „ „ „ „ 1000 „ „ 4 „

If each thread be supposed to be 1000 yards long and there is no take up we shall have 1000 yards of twist yarn.

Taking 2 threads of 6 dram silk

(2 × 1000) yards of 6 dram silk weigh 12 drams
also 1000 „ 5 „ „ „ 5 „
and (2 × 1000) „ 4 „ „ „ 8 „

∴ 1000 yards of twist yarn weigh 25 drams, and the number of drams which a length of 1000 yards weighs is the number of the counts.

∴ Resultant counts = 25 dram (raw silk).

EXAMPLE 9.

To obtain the resulting counts of any *two threads* twisted together we may use the following formula :—

"a counts multiplied by b counts ; this product divided by a counts plus b counts = c resulting counts."

Give the reasoning to show how the formula is arrived at.

Suppose the count of the first thread is a's (any denomination, cotton, worsted, etc.)
„ „ „ „ „ second thread „ b's (same denomination)
„ „ „ „ „ folded thread „ c's „ „

Whatever kind of material we suppose for the first thread, we must count the second thread in the same system.

Then a's count means a hanks of material chosen weigh 1 lb.

or 1 hank „ „ weighs $\frac{1}{a}$ lb.

Similarly 1 hank of b's count weighs $\frac{1}{b}$ lb.

If there is no "take up" the length of each thread and the length of the twist thread will be the same.

Let us twist 1 hank of each thread.

Then 1 hank of a's count weighs $\frac{1}{a}$ lb.

1 „ b's „ „ $\frac{1}{b}$ „

and 1 hank of folded thread weighs $\left(\frac{1}{a}+\frac{1}{b}\right)$ lb.

or $\left\{1 \div \left(\frac{1}{a}+\frac{1}{b}\right)\right\}$ hanks of folded thread weigh 1 lb.

$$\therefore \text{ Resultant counts } (c) = 1 \div \left(\frac{1}{a}+\frac{1}{b}\right)$$
$$= 1 \div \left(\frac{a+b}{ab}\right)$$
$$= \frac{ab}{a+b}.$$

12.—*Relative Weights and Cost of Twist Yarns.*

Example 1.

A twist thread is composed of 2/48's and 1/40's cotton. Find the resultant counts and the weight of each thread in 100 lbs. of twist.

A 2/48's cotton is equivalent to a 1/24's cotton. Since there is no " take up " the lengths of each thread will be the same.

We may choose any number of hanks to be twisted 1, 24, 40, 80, or x hanks just as we please. Let us take 120 hanks (120 is L.C.M. of 24 and 40) and the weights of each will be integers.

(a) Then 120 hanks of 24's cotton weigh 5 lbs.
and 120 ,, 40's ,, ,, 3 lbs.
Hence 120 hanks of twist ,, 8 lbs.
or $\frac{120}{8}$,, ,, ,, 1 lb.

$$\therefore \textit{ Resultant counts} = \frac{120}{8} = 15\text{'s cotton.}$$

Also from above we see that

8 lbs. of twist thread contains 5 lbs. of 2/48's.
or 1 lb. ,, ,, ,, $\frac{5}{8}$ lb. ,,
$\therefore$ 100 lbs. ,, ,, ,, $\frac{5 \times 100}{8}$,,

Hence *Weight of* 2/48*'s cotton* $= \frac{5 \times 100}{8} = \frac{500}{8} = 62\frac{1}{2}$ lbs.
and *Weight of* 1/40*'s cotton* $= 100 - 62\frac{1}{2} = 37\frac{1}{2}$ lbs.

Example 2.

A twist thread is composed of 1/20's worsted @ 3*s*. per lb. 40 skeins woollen @ 2*s*. 3*d*. per lb. and 60/2's spun silk at 12*s*.

per lb. Find the resultant counts and the cost per lb. of the twist thread.

First find the equivalent worsted counts of the woollen and the silk threads.

40 skeins woollen means (40 × 256) yards weigh 1 lb.
Worsted counts based on a hank of 560 yards.

$$\therefore \quad \text{Equivalent worsted counts} = \frac{40 \times 256}{560} = 18\tfrac{2}{7}\text{'s.}$$

Again 60/2 spun silk means (60 × 840) yards weigh 1 lb.

$$\text{Hence Equivalent worsted counts} = \frac{60 \times 840}{560} = 90\text{'s.}$$

(*a*) Suppose we twist 180 hanks of each thread.
Then 180 hanks of 20's worsted weigh 9 lbs.

$$180 \quad \text{,,} \quad 18\tfrac{2}{7}\text{'s} \quad \text{,,} \quad \text{or 40 skeins ,,} \quad \frac{180}{18\tfrac{2}{7}} \text{ lbs.}$$

$$= 9\tfrac{7}{32} \text{ lbs.}$$

180 ,, 90's ,, or 60/2 silk weigh 2 lbs.
Hence 180 hanks of twist weigh $20\tfrac{7}{32}$ lbs.

$$\therefore \quad \text{Resultant counts} = \frac{180}{20\tfrac{7}{32}} = \frac{180 \times 32}{667} = 8{\cdot}636.$$

(*b*) *To find the cost of* 1 *lb. of twist yarn.*

In finding counts above we twisted 9 lbs. of 1/20's worsted with $9\tfrac{7}{32}$ lbs. of 40's skeins and 2 lbs. of 60/2 silk and obtained $20\tfrac{7}{32}$ lbs. of twist.

Now 9 lbs. of 1/20's worsted @ 3*s.* per lb. cost 27*s.*
$9\tfrac{7}{32}$ lbs. of 40 skeins @ 2*s.* 3*d.* ,, ,, $22s.\ 1\tfrac{25}{32}d.$
2 lbs. of 60/2 spun silk @ 12*s.* ,, ,, 24*s.*

$\therefore$ $20\tfrac{7}{32}$ lbs. of twist cost $73/1\tfrac{25}{32}$ or $877\tfrac{25}{32}$ pence.

Then

$$\text{Cost of 1 lb. of twist yarn} = \frac{877\tfrac{25}{32}}{20\tfrac{7}{32}} \text{ pence}$$

$$= \frac{28089}{32} \times \frac{32}{667} = 3/6\tfrac{1}{9}.$$

Example 3.

A compound twist thread contains one thread of 2/80's worsted at 3*s.* per lb., one of 60/2 spun silk at 16*s.*, and one thread of 2/60's worsted at 2*s.* 10*d.* per lb. Find the counts, price per lb., and quantities of each thread in 100 lbs. of twist.

First find the worsted counts equivalent to 60/2 spun silk.
Now
60/2 spun silk means that (60 × 840) yards weigh 1 lb.
and worsted counts are based on a hank of 560 yards.

Hence

$$\text{Equivalent worsted counts} = \frac{60 \times 840}{560} = 90\text{'s.}$$

Suppose we twist 360 hanks of each thread; assuming there is no "take up," then

360 hanks of 2/80's or 1/40's worsted weigh 9 lbs.
360 ,, 90's worsted or 60/2 silk ,, 4 lbs.
360 ,, 2/60's or 1/30's worsted ,, 12 lbs.

Then

360 hanks of twist yarn weigh 25 lbs.

and

(*a*) Counts of twist yarn $= \frac{360}{25} = 14\frac{2}{5}$'s worsted.

(*b*) To find the price per lb. of the Twist Yarn.

Now the

9 lbs. of 2/80's worsted @ 3*s.* per lb. cost 27*s.*
4 lbs. of 60/2 spun silk @ 16*s.* ,, ,, 64*s.*
12 lbs. of 2/60's worsted @ 2*s.* 10*d.* ,, ,, 34*s.*

∴ Cost of 25 lbs. of twist = 125 shillings

and Cost of 1 lb. of twist = 5*s.*

(*c*) To find Weight of each Yarn in 100 lbs. of Twist.

In making 25 lbs of twist we use

9 lbs. of 2/80's worsted; 4 lbs. of 60/2 spun silk; 12 lbs. of 2/60's worsted

Hence

In making 100 lbs. of twist we shall require

36 lbs. of 2/80's worsted; 16 lbs. of 60/2 spun silk; 48 lbs. of 2/60's worsted.

Example 4.

A compound yarn is made to the following particulars, viz., 2 threads of 2/40's worsted at 3*s.* per lb., 115 yards of each in 100 yards of the compound yarn, and 1 thread of 1/12's mohair at 2*s.* per lb., with 250 yards in 100 yards of the compound yarn. Find the resultant counts and the price per lb., allowing 4*d.* per lb. for waste and cost of twisting.

Notice.—1st. Mohair is counted on the worsted system.

2nd. To produce 100 hanks of twist we require 115 hanks of 2/40's worsted twice and 250 hanks of 1/12's mohair.

Since 115 hanks of 2/40's worsted weigh $5\frac{3}{4}$ lbs.
115 ,, ,, ,, ,, $5\frac{3}{4}$ lbs.
250 ,, 1/12's mohair ,, $20\frac{5}{6}$ lbs.

Then 100 hanks of twist yarn weigh $32\frac{1}{3}$ lbs.

(*a*) Hence Counts of the twist yarn $= \frac{100}{32\frac{1}{3}} = \frac{300}{97} = 3\frac{9}{97}$'s worsted.

(*b*) *To find cost of* 1 *lb. of twist yarn.*

To get $32\frac{1}{3}$ lbs. of twist we use $11\frac{1}{2}$ lbs. of 2/40's worsted and $20\frac{5}{6}$ lbs. of mohair.

Now $11\frac{1}{2}$ lbs. of 2/40's worsted @ 3*s.* cost 34*s.* 6*d.*
and $20\frac{5}{6}$ lbs. of 1/12's mohair @ 2*s.* cost 41*s.* 8*d.*
Then $32\frac{1}{3}$ lbs. of twist yarn cost 76*s.* 2*d.*
and

$$\text{Cost of twist per lb.} = \frac{76/2}{32\frac{1}{3}} = \frac{228{\cdot}5}{97} \text{ shillings}$$
$$= 2s.\ 4\tfrac{1}{4}d. \text{ to nearest farthing.}$$

Allowing 4*d.* per lb. for waste and cost of twisting,
Cost of twist yarn = 2*s.* $8\frac{1}{4}d.$ per lb. to nearest farthing.

Example 5.

A fancy loop yarn is made as follows: 1 ground thread of 1/40's cotton taking up 2% in twisting at 1*s.* 5*d.* per lb.; 1 loop thread of 1/16's mohair taking up 60% in twisting at 3*s.* 2*d.* per lb.; 1 binder thread of 1/40's cotton taking up 5% in twisting at 1*s.* 5*d.* per lb.

What are (*a*) the resulting worsted counts, (*b*) the price per lb., and (*c*) the weight of each thread in 200 lbs. of twist yarn?

Here it must be noted

1st. Mohair counts are based on the worsted system.

2nd. To obtain 100 hanks of twist yarn we shall require 102 hanks of ground thread, 160 hanks of loop thread, and 105 hanks of binder thread.

3rd. We must find the equivalent worsted counts of the 1/40's cotton.

Now

40's cotton means (40 × 840) yards weigh 1 lb.
and worsted counts are based on a hank of 560 yards.

Then Equivalent worsted counts $= \frac{40 \times 840}{560} = 60$'s.

102 hanks of 60's worsted or 1/40's cotton weigh $1\frac{51}{60}$ lbs.
160 ,, 16's mohair ,, 10 lbs.
105 ,, 60's worsted or 1/40's cotton ,, $1\frac{3}{4}$ lbs.
Hence 100 hanks of twist yarn weigh $13\frac{9}{20}$ lbs.
and

(*a*) Resultant counts $= \frac{100}{13\frac{9}{20}} = \frac{2000}{269} = 7{\cdot}43$ to nearest $\frac{1}{100}$.

(*b*) *To find cost of* 1 *lb. of twist.*

To make $13\frac{9}{20}$ lbs. of twist we used $1\frac{21}{80}$ lbs. of ground thread, 10 lbs. of loop thread and $1\frac{3}{4}$ lbs. of the binder thread.

The cost of $3\frac{9}{20}$ lbs. of 1/40's cotton @ 1*s.* 5*d.* per lb. = 4*s.* $10\frac{1}{2}\frac{3}{0}d.$

The cost of 10 lbs. of 1/16's mohair @ 3*s.* 2*d.* per lb. = 31*s.* 8*d.*

$\therefore$ Cost of $13\frac{9}{20}$ lbs. of twist = 36*s.* $6\frac{1}{2}\frac{3}{0}d.$

and Cost of 1 lb. of twist yarn $= \dfrac{36s.\ 6\frac{1}{2}\frac{3}{0}d.}{13\frac{9}{20}} = 2s.\ 8\frac{3}{4}d.$ nearly.

(*c*) *To find weight of each yarn in* 200 *lbs. of twist.*

From (*a*) it is seen that

In $13\frac{9}{20}$ lbs. of Twist there are $3\frac{9}{20}$ lbs. of 1/40's cotton.

„ 1 lb. „ „ „ „ $\dfrac{3\frac{9}{20}}{13\frac{9}{20}}$ „ „ „

„ 200 lbs. „ „ „ „ $\dfrac{3\frac{9}{20} \times 200}{13\frac{9}{20}}$ „ „ „

$\therefore$ *Weight of* 1/40*'s cotton* $= \dfrac{3\frac{9}{20} \times 200}{13\frac{9}{20}} = 51{\cdot}3$ lbs.

and *Weight of* 1/16*'s mohair* $= \dfrac{10 \times 200}{13\frac{9}{20}}$ lbs.

(Method i)

$= (200 - 51{\cdot}3)$ lbs.

(Method ii) shorter

$= 148{\cdot}7$ lbs.

The student having studied the foregoing examples carefully will have little difficulty in working similar exercises. He should take each part of the question separately and having first found the counts of the twisted yarn he will notice that he gets data to enable him to find the cost per lb. and the proportion by weight of the component threads in a certain weight of twist.

13.—*The Testing of Yarns for Counts.*

When yarn is received from the spinner, the manufacturer usually makes a series of tests of the material delivered, to ascertain if the counts are correct. Also during the spinning operations it is necessary that the material should be weighed to ensure correct results in these processes.

The testing may be accomplished in two ways.

(*a*) With small quantities of yarn, it is usual to weigh a short length against some standard weight as explained below.

(*b*) When possible the length of a hank is reeled and weighed, and the counts of the material thus determined with greater accuracy.

(*a*) *Method I.—When short lengths are weighed against standard weights.*

(A) *Considering worsted yarn.*

The number of hanks (560 yards) which weigh 1 lb. gives the counts. Let us alter the unit of length to 1 yard.

Now 1 lb = 7000 grains,

and in 1's counts we have 560 yards weigh 1 lb.

or „ 1's „ „ „ 1 yard weighs $\frac{1}{560}$ lb.

Now $\frac{1}{560}$ lb. $= \frac{7000}{560}$ grains $= 12{\cdot}5$ grains.

Then by 1's worsted we mean 1 yard weighs 12·5 grains.

Again

By 20's counts worsted we mean (20 × 560) yards weigh 1 lb. or by 20's counts worsted we mean 20 yards weigh $\frac{1}{560}$ lb. or 12·5 grains.

Having in this way changed from a standard length of 560 yards to a length of 1 yard and from a standard weight of 1 lb. to a standard of 12·5 grains, we may say that—

The worsted counts of a yarn is the *number of hanks each* 560 *yards long which weigh* 1 *lb.* OR *the number of yards which weigh* 12·5 *grains.*

Thus if 2, 3, 4, $16\frac{1}{2}$, or 30 yards of yarn weigh 12·5 grains the counts are 2's, 3's, 4's, $16\frac{1}{2}$'s, or 30's worsted respectively.

Example 1.

Suppose 3 threads of a worsted yarn are obtainable each measuring 12 inches. These are weighed as accurately as possible on a delicate balance and the weight is 0·25 grain.

Then When weight is 0·25 grain the length = 1 yard.

„ „ 1 „ „ $= \frac{1}{0{\cdot}25}$ „

„ „ 12·5 grains „ $= \frac{12{\cdot}5}{0{\cdot}25}$ „

And $\frac{12{\cdot}5}{0{\cdot}25} = \frac{2{\cdot}5}{0{\cdot}05} = \frac{0{\cdot}5}{0{\cdot}01} = 50$ yards.

Hence 50 yards weigh 12·5 grains,

or the *yarn is* 50*'s worsted counts.*

Example 2.

$2\frac{1}{2}$ yards of yarn are found to weigh 1·5 grains.

What are its counts in the worsted denomination ?

As before we must find the number of yards which weigh 12·5 grains.

When weight is 1·5 grains the length = 2·5 yards.

" " 1 " " = $\frac{2·5}{1·5}$ "

" " 12·5 grains " = $\frac{2·5 \times 12·5}{1·5}$

= $20\frac{5}{6}$ yards.

Then $20\frac{5}{6}$ yards weigh 12·5 grains.

or the yarn is $20\frac{5}{6}$'s worsted counts.

(B) *Again considering woollen* (West of England).

1's West of England means that 320 yards weigh 1 lb. or 7000 grains.

∴ 1's West of England means that 1 yard weighs $\frac{1}{320}$ lb. or $\frac{7000}{320}$ grains.

$$\text{Now } \frac{7000}{320} \text{ grains} = \frac{700}{32} = \frac{175}{8} = 21·875 \text{ grains.}$$

and Hence the *number of yards which weigh* 21·875 *grains* is the count of the yarn in the West of England system.

In a similar manner we may find the *standard weight* for any system always taking the *standard length to be* 1 *yard*. So that for any particular system THE COUNTS OF THE YARN WILL BE THE NUMBER OF YARDS WHICH BALANCE THE STANDARD WEIGHT FOR THAT SYSTEM.

The student should now work out for himself the values of the standard weights for other systems.

We give a list of a few of the most common.

SYSTEM.	STANDARD WEIGHT.
Worsted	12·5 grains
Cotton	$8\frac{1}{3}$ or 8·3 grains approx.
Woollen (cut Gala)	35·0 grains
Spun silk	$8\frac{1}{3}$ or 8·3 grains approx.
Linen	$23\frac{1}{3}$ or 23·3 " "
Hawick	37·91 " "
Woollen (American run)	$4\frac{3}{8}$ or 4·375 "
" (American cut)	$23\frac{1}{3}$ or 23·3 " "
" (Yorkshire skeins)	$27\frac{11}{32}$ or 27·34 " "

Note.—For Yorkshire skeins *the dram* is also taken as the standard weight and *the number of yards that weigh* 1 *dram* indicates the counts.

(*b*) *Method II.—When greater lengths of yarn are available.*

When possible, a length of yarn equal to that of the standard hank for the particular system under consideration is reeled off and weighed. This method gives greater accuracy.

The weight of 1 hank will be some fraction of the constant weight—generally 1 lb. (*See* Table I, page 2.)

Now since *the number of hanks which weigh* 1 *lb.* = counts.
If 1 hank weighs $\frac{1}{32}$ lb.
Then 32 hanks weigh 1 lb. and the counts = 32's.
Again If 1 hank weighs $\frac{1}{20}$ lb.
Then 20 hanks weigh 1 lb. and the counts = 20's.

Example 3.

If 10 yards of woollen (Yorkshire skeins) weigh 8·29 grains find the counts of the yarn.

In 1's Y. skeins 256 yards weigh 1 lb. or 7000 grains
or ,, 1's ,, ,, 1 yard weighs $\frac{1}{256}$,, or $\frac{7000}{256}$,,

Then the *number of yards which weigh* $\frac{7000}{256}$ *grains is the count.*

When the weight is 8·29 grains the length = 10 yards.
,, 1 ,, 1 grain ,, $= \frac{10}{8{\cdot}29}$,,
,, ,, $\frac{7000}{256}$ grains ,, $= \frac{10 \times 7000}{256 \times 8{\cdot}29}$ yds.

and $\frac{10 \times 7000}{256 \times 8{\cdot}29} = \left[\begin{array}{l}\textit{Roughly} \\ \frac{10 \times 7000}{2000} = 35 \text{ yards.}\end{array}\right]$
$= 32{\cdot}98$ (using logs.).
∴ Counts of yarn = 32·98 = 33's Yorkshire skeins.

Note.—The student will notice that we might have used the preceding list of standard weights and stated at once that 1 yard of 1's Yorkshire skeins weighs $27\frac{1}{32}$ grains. This is very convenient when the list is at hand for use, but for examination purposes it is sufficient to memorise Tables I and II, and this will be accomplished by the frequent use of them in working numerous examples.

Example 4.

On testing a yarn a manufacturer finds that 30 yards of worsted weigh 12·5 grains, but the yarn ordered was 32's. What length of yarn does the manufacturer lose per lb. ?

1's worsted means that 560 yards weigh 1 lb. or 7000 grains
or 1's ,, ,, ,, 1 yard weighs $\frac{1}{560}$ lb. or 12·5 ,,

Then
if 30 yards weigh 12·5 grains
the counts of the yarn sent = 30's worsted.
The yarn ordered was 32's.

Now

In 32's worsted there are (32×560) yards per lb.

,, 30's ,, ,, ,, (30×560) ,, ,,

$\therefore$ Loss = 2 hanks of 560 yards = 1120 yards per lb.

Example 5.

A hank of cotton weighs $\frac{1}{18}$ lb. When reeled the length is found to be only 820 yards. What are the actual counts of the yarn ?

Here we have 820 yards weigh $\frac{1}{18}$ lb.

or 18×820 ,, ,, 1 lb.

But cotton counts are based on a hank of 840 yards, and the number of these hanks in (18×820) yards $= \frac{18 \times 820}{840}$.

$\therefore$ Actual counts $= \frac{18 \times 820}{840} = 17\frac{4}{7}$'s cotton.

CHAPTER II

"Sett" Systems

14.—Explanation of terms. 15.—Chief "Sett" Systems in use. Table IV. 16.—Table V showing the number of ends per inch in a 1's sett of the various systems. 17.—Given the "Sett" System to find the number of ends per inch. (Worked examples.) 18.—Given the number of ends per inch to find the "sett" in various systems. (Worked examples.) 19.—Given the "sett" in any one system, to find its "equivalent sett" in any other system. (Worked examples.) 20.—Miscellaneous examples worked. 21.—Calculation of the Mails on each shaft in a set of Gears or Healds. 22.—Weight and Cost of Warp and Weft. i. Calculations involving warp yarns of one colour and counts. ii. Calculations involving weight of weft only. iii. Calculations involving warp and weft. iv. Calculations involving weight of warp and weft in fancy woven fabrics. v. Miscellaneous examples.

14.—*Explanation of Terms.*

By the technical term "sett" we mean the number of warp threads that are placed side by side in a certain chosen distance during the process of preparing the warp for the loom and in the subsequent process of weaving.

As in the case of the counting of yarns, there is no standard method of indicating the sett, each locality having its own system and terms. Hence the frequent occurrence of such technical terms as Porty, Portie, Porter, Beer, Knott, or Bunch. These are used in different localities, and are all arbitrary in name and value.

Each term signifies a certain number of ends, the number of ends varying according to the locality.

To spread the threads evenly across the width of the warp, equal distances must contain the same number of threads for any particular warp. Different localities have fixed upon different distances for units, such as 36 inches, 90 inches, 9 inches, etc., and *the number of bunches of threads* (Porties or Beers, etc., as the case may be) *which are placed in the unit of distance constitutes the sett.*

Consider the Bradford System.—This is based upon a "*group*

of 40 threads"—this group is termed a "sett." The *unit of distance* is 36 inches and the "*sett*" is *the number of these groups which are placed in a width of 36 inches.*

Thus 40 threads in 36 inches constitutes 1's "sett" Bradford System,

80 threads (2 groups of 40) in 36 inches constitutes 2's "sett" Bradford System,

2000 threads (50 "setts" of 40) in 36 inches constitutes 50's "sett" Bradford System,

or 54's "sett" Bradford indicates that 54 *groups of 40 threads* ("setts") occupy a width of 36 inches in the warp. That is (54 × 40) or 2160 threads lay side by side in every 36 inches width of warp.

Again in the *Leeds System* :—

The *unit of distance* is 9 inches. A *group of 38 threads* is chosen and termed a Portie, and 12's "sett" Leeds indicates that 12 porties (groups of 38 threads) occupy a width in the warp of 9 inches ; that is :—

(12 × 38) or 456 threads are spread over a width of 9 inches.

Then *in general terms*

If we choose a GROUP OF *a* THREADS (call this group a Porter, Beer, or Bunch or any other suitable name), and a UNIT OF DISTANCE of *x* inches.

If we put *a* threads in a width of *x* inches the "sett" is 1's.

„ „ $2a$ „ „ „ x „ „ „ 2's.

„ „ $34a$ „ „ „ x „ „ „ 34's.

„ „ pa „ „ „ x „ „ „ p's.

Since no standard method of indicating the "sett" has been adopted, it is advisable for students and those employed in the manufacture of woven fabrics to become conversant with the system adopted in their own particular locality, and also those systems which are in most general use, so as to be able to transfer from one system of setting to its equivalent in any other system.

15. The following are the *chief "sett" systems in use* :—

TABLE IV *

SYSTEM.	METHOD OF COUNTING.
Huddersfield and U.S.A. .	By the *number of splits per inch.*
Bradford . .	By *the number of* setts of 40 ends each in 36 inches.
Dewsbury .	„ „ beers „ 38 „ „ 90 „
Leeds . .	„ „ porties „ 38 „ „ 9 „

* See *Woollen and Worsted*, by Professor Beaumont ; *Calculations in Yarns and Fabrics*, by F. Bradbury.

TABLE IV—(*cont.*)

System.	Method of Counting.
Scotch Tweed .	By *the number of* porters of 40 ends each in 37 inches
Holmfirth .	„ „ porties „ 40 „ „ 48 „
Stalybridge (Colne Valley)	„ „ porties „ 40 „ „ 12 „
Bolton . .	„ „ beers „ 40 „ „ 24¼ „
Blackburn .	„ „ beers „ 40 „ „ 45 „
Manchester .	„ „ splits in a width of 36 „
Stockport. .	„ „ splits „ „ „ 2 „
Glasgow . .	„ „ splits „ „ „ 37 „
Belfast (Linen)	„ „ splits „ „ „ 40 „
Silk . . .	„ „ splits in the full width of the warp.
Metric . .	By the *number of threads* in 10 centimetres.

In all the above systems, with the exception of the Huddersfield System, it is understood that there are TWO ENDS IN EACH SPLIT OF THE REED OR SLEY.

In the Huddersfield System, the number of ends through each split is usually stated, *e.g.*,

15's reed 3's means that there are 15 splits per inch and 3 ends are passed through each split, therefore we should have (15 × 3) or 45 ends per inch.

Again we may have a "sett" stated as 16's reed 4's. This implies that the reed has 16 splits per inch and that 4 threads are passed through each split. Consequently there are 64 threads per inch.

Sometimes more than two ends are put through a split, *e.g.*, 3 or 4 or 6. This would mean that the sett would be finer and there would be more ends per inch in the proportions of 3 to 2, 4 to 2 and 6 to 2 respectively.

Thus a 54's sett Bradford means that,

54 groups of 40 threads occupy a width of 36 inches.

$$\therefore \text{ Number of ends per inch } = \frac{54 \times 40}{36} = 60.$$

and „ splits „ (two ends per split) = 30.

The expression 54 6 sett Bradford implies that the reed is made on the 54's sett Bradford basis and has therefore 30 splits per inch, but 6 threads pass through a split instead of the ordinary 2.

Thus the number of ends per inch = 30 × 6 = 180 and the number of groups of 40 threads in 36 inches will be trebled.

$$\text{Hence the true sett of the warp } = \frac{54 \times 6}{2} = 162\text{'s}.$$

Similarly 54/3 sett implies that 3 threads pass through a

split instead of 2. Thus the number of ends per inch will be increased in the ratio of 3 : 2 and

$$\text{The true sett of the warp} = \frac{54 \times 3}{2} = 81\text{'s}.$$

Note.—(i) This practice of increasing the number of ends per split beyond the usually understood number 2 is done to save expense, for, as we have seen, one particular reed may be made to serve for more than one sett.

(ii) The terms "split" or "dent" mean the spaces in the reed or sley.

TABLE V

SHOWING THE NUMBER OF ENDS PER INCH IN A 1's SETT OF THE VARIOUS SYSTEMS

Huddersfield	1 × ends in a split.	
Bradford	40/36 = $1\frac{1}{9}$	ends per inch.
Dewsbury	38/90 = 19/45	,, ,,
Leeds	38/9 = $4\frac{2}{9}$	,, ,,
Scotch Tweed	40/37 = $1\frac{3}{37}$	,, ,,
Holmfirth	40/48 = 5/6	,, ,,
Stalybridge	40/12 = $3\frac{1}{3}$	,, ,,
Bolton	40/24$\frac{1}{4}$ = $1\frac{63}{97}$	,, ,,
Blackburn	40/45 = 8/9	,, ,,
Manchester	2/36 = 1/18	,, ,,
Stockport	2/2 = 1	,, ,,
Glasgow	2/37 = 2/37	,, ,,
Belfast (Linen)	2/40 = 1/20	,, ,,

This table is given for reference only and must not be committed to memory.

The student must make himself familiar with the table in the beginning of the chapter, which gives the methods of counting the setts in the various systems.

A careful study of the worked examples which follow will it is hoped enable him to attack any problems of a similar character.

17. *Given the "Sett" System to find the number of ends per inch.*

EXAMPLE 1.

Reduce 63's sett Bradford, 22/6 Scotch tweed and 15's reed 5's Huddersfield to ends per inch.

(*a*) 1's sett Bradford means 1 group of 40 threads ("sett") spread over a width of 36 inches

and 63's sett Bradford means 63 groups of 40 threads (63 "setts") spread over the same width of 36 inches.

Then

In 36 inches there are 63×40 threads.

In 1 inch there are $\frac{63 \times 40}{36} = 70$ threads.

$\therefore$ Number of ends per inch $= 70$.

(*b*) The Scotch tweed system is based upon the number of porters (groups of 40 threads) spread over a width of 37 inches, OR on the number of groups of 20 splits, 2 *ends in a split*, in the unit distance.

Now 22/6 indicates that 6 ends are threaded through each split. So that if the usual 2 ends were placed in each split, the actual sett is 66/2, for we should require 3 splits in place of every one.

Then 66's sett Scotch tweed means 66 groups of 40 threads (66 porters) spread over a width of 37 inches.

Thus in 37 inches there are (66×40) threads.

„ 1 inch „ $\frac{66 \times 40}{37}$ or $71\frac{3}{37}$ threads.

$\therefore$ Number of ends per inch $= 71\frac{3}{37}$.

(*c*) 15's reed 5's Huddersfield System.

This implies that there are 15 splits per inch and that 5 threads are passed through each split.

Consequently Number of ends per inch $= 15 \times 5 = 75$.

EXAMPLE 2.

Find (*a*) the number of ends per inch, (*b*) the number of splits per inch in a 1300 reed Glasgow, 900's sett Manchester, and a 63's sett Bolton.

(*a*) In the Glasgow Cotton System the "sett" is the number of splits, 2 ends in a split, in a width of 37 inches.

Then 1300's sett Glasgow means 1300 splits, 2 ends per split, in a width of 37 inches.

Hence In 37 inches there are (1300×2) threads.

„ 1 inch „ $\frac{1300 \times 2}{37}$ or $70\frac{10}{37}$ threads.

$\therefore$ *Number of ends per inch* $= 70\frac{10}{37}$.

Also since 2 threads pass through each split,

Number of *splits* per inch $= 70\frac{10}{37} \div 2 = 35\frac{5}{37}$.

(*b*) 900's *sett Manchester*.

In the Manchester system the "sett" is the number of splits, 2 threads per split, in a width of 36 inches.

Thus 900's sett Manchester means 900 splits, 2 ends in a split, in a width of 36 inches.

Hence

In 36 inches there are (900×2) threads and 900 splits.

„ 1 inch „ $\frac{900 \times 2}{36}$ „ and $\frac{900}{36}$ „

$\therefore$ *Number of threads* per inch $= \frac{900 \times 2}{36} = 50$

and *Number of splits* per inch $= \frac{900}{36} = 25.$

(*c*) *63's sett Bolton.*

The Bolton system is based on the number of beers of 40 ends each, in a unit distance of $24\frac{1}{4}$ inches.

Then 63's sett Bolton means (63×40) ends in a width of $24\frac{1}{4}$ inches

and *Number of ends* per inch $= \frac{63 \times 40}{24\frac{1}{4}} = 103\frac{8}{97}$.

And since 2 threads are passed through each split,

Number of splits per inch $= 103\frac{8}{97} \div 2 = 51\frac{83}{97}$.

18. *Given the Number of ends per inch to find the "sett" in various systems.*

Example 3.

Given 120 threads per inch, find the sett (*a*) Bradford, (*b*) Huddersfield (4 in a dent), (*c*) Dewsbury, and (*d*) Scotch Tweed.

(*a*) The "sett" Bradford is the number of "setts" (group of 40 threads) in a width of 36 inches.

Since In 1 inch there are 120 threads

$\therefore$ in 36 inches there are (120×36) threads.

It is necessary to find the number of groups (40 ends in a group).

$\therefore$ Sett $= \frac{120 \times 36}{40} = 3 \times 36 = 108$'s Bradford.

(*b*) For the Huddersfield sett we require the number of splits per inch having 4 threads per split.

Since there are 120 threads per inch and 4 threads in a split,

$\therefore$ Number of splits per inch $= \frac{120}{4} = 30.$

$\therefore$ Sett required $= 30$'s reed 4's Huddersfield.

(*c*) In the *Dewsbury System.*

The "sett" is the number of beers (38 ends each) in a standard width of 90 inches.

If there are 120 threads per inch,

then there are $\frac{120}{38}$ beers „ „

And in 90 inches there are $\frac{120}{38} \times 90$ beers.

$\therefore$ Sett required $= \frac{120 \times 90}{38} = 284\frac{4}{19}$'s Dewsbury.

(*d*) *Scotch Tweed System.*

The "sett" is the number of porters (40 ends each) in a width of 37 inches.

If there are 120 threads per inch,

then there are $\frac{120}{40}$ porters per inch.

And in 37 inches there are $\left(\frac{120}{40} \times 37\right)$ porters.

$\therefore$ Sett required $= \frac{120 \times 37}{40} = 111$'s Scotch tweed.

19. *Given the "Sett" in any one system, to find its "equivalent sett" in any other system.*

Calculations of this type are best made in two steps.

(i) Find the number of ends per inch in the given sett as in §17.

(ii) Express this number of ends per inch in terms of the required sett as in §18.

Study the following examples.

Example 4.

Find the equivalent of an 80's sett Bradford in *the Blackburn System.*

1*st Step.—Reduce 80's sett Bradford to ends per inch.*

80's sett Bradford means (80×40) ends in a width of 36 inches.

$\therefore$ Number of ends per inch $= \frac{80 \times 40}{36}$ (i).

[*Note.*—Don't work this step out.]

2*nd Step.—Change this number of ends per inch* (i) *to "setts" Blackburn.*

In the Blackburn System,

The "sett" is the number of beers (40 ends each) in a width of 45 inches.

Since there are $\frac{80 \times 40}{36}$ threads per inch (i),

then „ „ $\frac{80 \times 40}{36 \times 40}$ beers „ „

and in 45 inches there are $\frac{80 \times 40 \times 45}{36 \times 40}$ beers (40 threads each).

$\therefore$ Sett required $= \frac{80 \times 40 \times 45}{36 \times 40} = 100$'s sett Blackburn.

Hence 80's sett Bradford is equivalent to 100's sett Blackburn.

Example 5.

Find the equivalent of 24/3 Scotch tweed in (*a*) Leeds and (*b*) *Huddersfield Systems.*

24/3 implies that 3 ends pass through a split instead of the usual 2 ends.

Then whereas 6 ends now occupy 2 splits (3 ends per split),

For the standard form 6 ends should occupy 3 splits (2 ends per split).

Then the number of splits is increased in the ratio of 3 to 2,

or $24/3 = \frac{24 \times 3}{2} = 36/2$ sett Scotch tweed.

(*a*) *1st Step.—Reduce 36's sett Scotch tweed to ends per inch.*

36's sett Scotch tweed means (36×40) ends in a width of 37 inches.

$\therefore$ Number of ends per inch $= \frac{36 \times 40}{37}$ (i).

[*Note.*—Leave this in fractional form.]

2nd Step.—Change number of ends per inch (i) *to "sett" Leeds.*

In the Leeds System,

The "sett" is the number of porties (38 ends each) in a width of 9 inches.

Since there are $\frac{36 \times 40}{37}$ ends per inch (i),

Then ,, ,, $\frac{36 \times 40}{37 \times 38}$ porties ,, ,,

and in 9 inches there are $\frac{36 \times 40 \times 9}{37 \times 38}$ porties (38 ends each).

$\therefore$ Sett required $= \frac{36 \times 20 \times 9}{37 \times 19} = 9{\cdot}217$ (using logs.)

$= 9{\cdot}2$ sett Leeds.

Hence 9·2's sett Leeds is equivalent to 24/3 Scotch tweed.

(*b*) The Huddersfield system is based on the number of splits per inch with 2 threads per split.

The number of ends per inch $= \frac{36 \times 40}{37}$ (i).

If 2 threads pass through a split,

Then number of splits per inch $= \frac{36 \times 40}{37 \times 2} = 19\frac{1}{2}$ (approx.),

Hence

24/3 Scotch tweed is equivalent to $19\frac{1}{2}$'s reed 2's Huddersfield.

20.—*Miscellaneous Examples.*

EXAMPLE 6.

A worsted warp which contains 2346 threads has to be woven in a 64/2 sett Bradford. What will be the width in the loom ?

In the Bradford System,

The "sett" is the number of setts (40 ends in each) in a width of 36 inches and 2 ends pass through each split.

Then

64/2 sett Bradford means (64×40) ends in a width of 36 inches.

Thus there are (64×40) ends in 36 inches

or 1 end ,, $\frac{36}{64 \times 40}$,,

and 2346 ends in $\frac{36 \times 2346}{64 \times 40}$ inches.

$\therefore$ Width in loom $= \frac{36 \times 2346}{64 \times 40} = 32{\cdot}99 = 33$ inches.

EXAMPLE 7.

A woollen warp contains 76 porties Dewsbury. It has to be sleyed in a 95 sett Dewsbury. What is the width in the reed ?

A portie or beer in the Dewsbury system is a group of 38 threads, and 76 porties Dewsbury means (76×38) ends in a width of 90 inches. This number of ends must be sleyed in a 95 sett Dewsbury. That is at the rate of (95×38) ends in a width of 90 inches.

Then if (95×38) ends are sleyed in 90 inches

1 end is ,, ,, $\frac{90}{95 \times 38}$ inches

and (76×38) ends are sleyed in $\frac{90 \times 76 \times 38}{95 \times 38}$ or 72 ins.

$\therefore$ Width in reed = 72 inches.

EXAMPLE 8.

A crammed stripe is made as follows :—

Reed, 50 "sett" Bradford ; cotton ground, 1 thread in a split ; silk warp, 3 in a split.

Plan. Cotton | 60 | | 5 | | 5 | | } 115 threads in
Silk .. | | 15 | | 15 | | 15 | } each repeat.

Find (*a*) the number of threads of each material, and (*b*) the total number of ends in the warp required to produce a piece 54 inches wide in reed.

A 50 " sett " Bradford means (50 × 40) ends in width of 36 inches.

$$\therefore \text{ Number of ends per inch} = \frac{50 \times 40}{36}$$

and since 2 ends are sleyed in a split,

$$\therefore \text{ Number of splits per inch in reed} = \frac{50 \times 40}{36 \times 2} = \frac{250}{9}.$$

Then in 54 inches of reed

$$\text{Number of splits} = \frac{250}{9} \times 54 = 250 \times 6 = 1500.$$

Now IN ONE REPEAT OF PATTERN there are
70 threads of cotton which will require 70 splits
45 ,, silk (3 in split) ,, ,, ,, 15 splits.
Then *for one repeat of pattern* we require 85 splits.

and NUMBER OF REPEATS in 54 inches $= \frac{1500}{85}$.

Now in every repeat there are
70 threads of cotton and 45 threads of silk.
Hence

(*a*) Number of cotton threads $= 70 \times \frac{1500}{85} = 1235$ (to nearest unit).

Number of silk threads $= \frac{45 \times 1500}{85} = 794$ (to nearest unit).

(*b*) ∴ Total number of threads = 1235 + 794 = 2029.

(*c*) Suppose we have also to find the *Average sett* Bradford of the warp.

We have
Total number of threads = 2029 and width in reed = 54 inches.

Then in 1 inch there are $\frac{2029}{54}$ threads

and ,, 1 inch ,, ,, $\frac{2029}{54 \times 40}$ " setts " of 40 threads

and ,, 36 inches ,, ,, $\frac{2029 \times 36}{54 \times 40}$ " setts " ,, ,,

$$\therefore \text{ Average sett} = \frac{2029 \times 36}{54 \times 40} = \frac{2029}{60} = 33{\cdot}8 \text{ Bradford.}$$

Example 9.

A figured pattern, of which one repeat occupies 5 inches, is made as follows :—The first inch contains 5 ends in each dent, the remaining 4 inches have only 2 ends in each dent ; the width of the fabric in the reed is 30 inches, and the total number of ends is 2400.

Find (1) the average sett of the healds, (2) the sley, (3) the sett of the finer portion of the stripe, and (4) the coarser portion, all in the Bradford system.

(1) Total number of threads = 2400
Width in reed = 30 inches

$\therefore$ Number of ends per inch $= \frac{2400}{30} = 80$

The "sett" Bradford is the number of "setts" (40 ends in each) in a width of 36 inches.

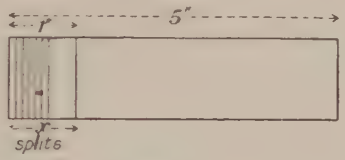


Fig. 1.

Now in 1 inch there are 80 ends or 2 "setts" (of 40 ends) and in 36 inches there are (80 × 36) ends or 72 "setts."

Hence Average sett of healds = 72's Bradford.

(2) Suppose in the sley or reed we have x splits or dents per inch (Fig. 1).

In first inch we have 5 threads per dent
„ next 4 inches „ 2 „ „ „ } for one repeat.

Hence we have $(5x + 8x)$ or $13x$ threads in 5 inches.

Then in 30 inches (width in reed) there are $78x$ threads.

But total number of threads in 30 inches = 2400.

Then $78x = 2400$

or $x = \frac{2400}{78} = \frac{400}{13}$.

$\therefore$ Number of splits per inch $= \frac{400}{13}$.

In the standard Bradford system two threads pass through each split.

Hence *for the coarser portion and the sley*,

we must have $\frac{400}{13} \times 2$ *or* $\frac{800}{13}$ ends per inch.

Thus in 1 inch there are $\frac{800}{13}$ ends

or „ 1 „ „ „ $\frac{800}{13 \times 40}$ "setts" (40 ends each)

and „ 36 inches „ $\frac{800 \times 36}{13 \times 40}$ „ „ „

(2) and (4).

$\therefore$ "Sett" of Sley and Coarser portion $= \frac{800 \times 36}{13 \times 40} = 55\frac{5}{13}$'s Bradford.

(3) The finer portion is sleyed 5 threads in a dent.

Now Number of splits per inch $= \frac{400}{13}$.

$\therefore$ „ threads „ $= \frac{400 \times 5}{13}$.

and Number of setts per inch $= \frac{400 \times 5}{13 \times 40}$

or „ „ in 36 inches $= \frac{400 \times 5 \times 36}{13 \times 40}$.

$\therefore$ "*Sett*" *of finer portion* $= \frac{400 \times 5 \times 36}{13 \times 40} = 138\frac{6}{13}$'s Bradford.

21. *Calculation of the mails on each shaft in a set of gears or healds.* Sets of healds may be of two types, namely, *wire healds* or *knitted healds*. Whatever type we use it is necessary that we should be able to calculate the number of mails per inch required on each shaft in the set, so as to be able to place the required number of mails on the shafts when wire healds are used or to give proper instructions to the heald-knitter when knitted healds are used.

I. When the ends are drawn through the healds *straight-gate*, or in consecutive order, it is easy to estimate the number of mails per inch required on each shaft.

Example.

A 12 shaft design has to be woven on a cloth containing 72 ends per inch; what number of mails per inch would be required on each shaft?

As stated, 12 shafts are required to weave this cloth, and since there must be 72 ends per inch in the gears,

Then *Number of mails per inch on each shaft* $= \frac{72}{12} = 6.$

Also If the width of the warp is 40 inches,

Then *Number of mails on each shaft* $= 6 \times 40 = 240.$

II. If the warp threads are required to be drawn through the healds in a special order for the purpose of getting effects by means of drafting, then the number of mails per inch may vary on each shaft; consequently it is necessary to know how many mails are required on each shaft so as to be able to build up a correct set of gears.

EXAMPLE.

The ends are drawn through the gears in the following order:—

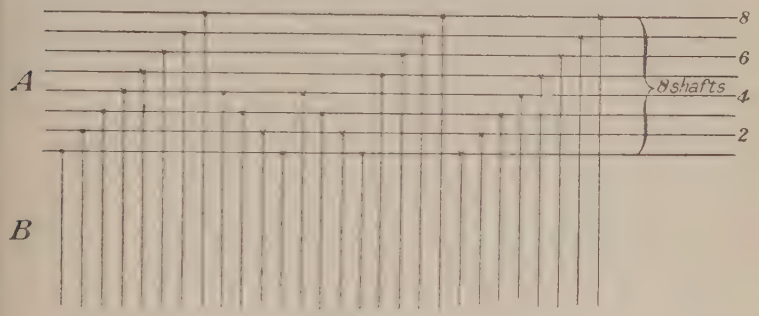


FIG. 2.

The horizontal lines (A) represent the warp shafts.

„ vertical „ (B) „ „ warp threads, and the crosses (×) represent the method or order of drawing the ends through the mails on the shaft.

It is desired to weave a cloth with 84 ends per inch, draft as above.

Now, the diagram shows that there are 28 ends in one repeat of the pattern.

Since there are 84 ends per inch in the warp,

Then *Number of repeats of the pattern in one inch* $= \frac{84}{28} = 3.$

From the diagram we also see that

In one repeat of the pattern

Shaft number	1	2	3	4	5	6	7	8
Number of threads through the mails	4	4	4	4	3	3	3	3

And since there are 3 repeats of the pattern in one inch,
Shafts Nos. 1–2–3–4 will require (4 × 3) or 12 mails per inch.
„ Nos. 5–6–7–8 „ „ (3 × 3) or 9 „ „ „
We have therefore
4 shafts with 12 mails per inch and
4 „ „ 9 „ „ „
making a total of (48 + 36) or 84 mails per inch.

III. Another type of gears used is one in which there are more mails per inch in one part of a shaft than in another; for example, when the production of a stripe is desired, the stripe part being crammed or set much finer than the body or ground of the cloth.

To produce these to the best advantage, using knitted healds, it is essential to have the healds knitted to suit the design. If specially knitted healds are not used, there is a great danger of friction on the threads caused by the mail cords being pulled to one side, and at the same time the warp threads are constantly being chafed during weaving owing to the threads not being straight.

In the case of wire healds the mails slide into the desired position, and are kept in the correct position by the tightness of the warp threads.

Example 1.

(B)

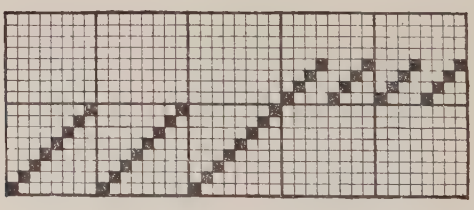


Fig. 3.

In the knitting of healds for draft (B) to suit the design, it is desirable that we have 80 threads per inch in the gears; what would be the rate and order of knitting for this set of healds?

The rate of knitting is usually based on 4 shafts.

Then if more than 4 shafts are used for a certain part of the draft it is obvious that the rate of knitting will be less.

For example, if we want 80 threads per inch and we are only using 4 shafts (this being the base of the knitting) we shall

have 20 mails per inch on each shaft. But if we use 8 shafts we shall only require half this number or (10) on a shaft to produce 80 ends per inch in the gears.

Thus considering draft (B),

For the 1st eight shafts,

If x be the rate of knitting,

$$\frac{x}{80} = \frac{4}{8}.$$

Multiply by 80 and

$$x = \frac{4 \times 80}{8} = 40\text{'s rate.}$$

For the back four shafts,

Rate of knitting = 80's rate.

Then order of knitting would be.

For first 8 shafts—Knit 3 and miss 2.

,, last 4 ,, —Knit 4 and miss 6.

EXAMPLE 2.

(C)

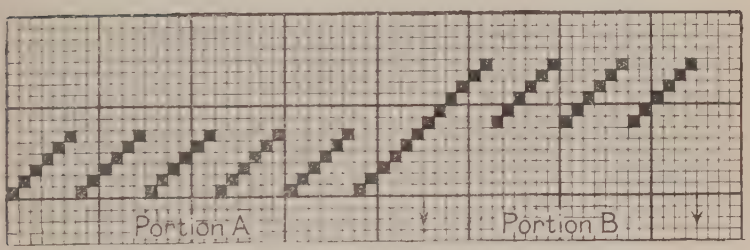


FIG. 4.

What would be the rate and order of knitting healds suitable for draft (C). The spacing in the portion marked A must be equal to 90 threads per inch and in the portion marked B equal to 60 threads per inch.

Considering portion A—6 shafts are required.

If rate of knitting = x,

Then $$\frac{x}{90} = \frac{4}{6}.$$

Multiply both sides by 90 and

$$x = \frac{90 \times 4}{6} = 60\text{'s rate.}$$

Again for portion B—6 shafts are required.

If rate of knitting $= x$,

$$\frac{x}{60} = \frac{4}{6}.$$

Multiply both sides by 60 and

$$x = \frac{4 \times 60}{6} = 40\text{'s rate.}$$

Then order of knitting would be

For first 6 shafts—knit 6, miss 6—rate 60.

„ last 6 „ —miss 4, knit 4—rate 40.

Note.—The width of portion A in the gears is exactly the same as that of portion B due to portion A being crammed more than portion B in the ratio of 6 to 4. Then in the sleying of the warp for draft (C), the portion A will be crammed in the reed, or in other words more ends will be passed through each dent in the reed than there will be for portion B.

Thus for portion A we may sley 3 per dent

and „ „ B „ „ 2 „

In this way we get portions A and B to be the same width in the gears and sley although portion A has more threads in a given width than portion B in ratio of 6 to 4 or 3 to 2.

For the 36 threads in portion A sleyed 3 per dent will require 12 dents and the 24 threads in portion B sleyed 2 per dent will require 12 dents. Also width of each portion in reed $= \frac{2}{5}$ inch.

From this it will be seen that specially knitted healds are the best to use for a draft of this type, for in this way we avoid the pulling of the heald cords and the chafing of the warp threads.

22. *Weight and Cost of Warp and Weft.*

To calculate the weight and cost of the warp and weft used in making a specified cloth the student must be quite familiar with (*a*) the various systems of counting yarns and (*b*) the various systems of setting the warp threads.

He must also note :—

(1) The length of each warp thread is the same as the given length of warp.

(2) There are *two methods of calculating the weight of the weft.*

(*a*) To obtain the total number of picks in the piece we take the length of the piece out of the loom (sometimes called the grey-length), and having obtained the weight of the weft, add on say 5% of this weight to allow for waste in weaving.

(*b*) Another method is to calculate the total number of picks using the length of the warp and let this compensate the waste.

A perusal of the following examples will perhaps make these points clear.

I. *Calculations involving Warp Yarns of one colour and counts.*

Example 1.

Calculate the weight of a warp made of 2/36's worsted, warped 80 threads per inch, 63 inches in reed and 160 yards long.

Width of warp = 63 inches.
Number of threads per inch = 80.

Then

Number of threads in warp = 80×63.
Each warp thread is 160 yards long.

$\therefore$ Length of thread in warp = $80 \times 63 \times 160$ yards.

Now 2/36's worsted means 18 hanks each 560 yards long weigh 1 lb.

$$\therefore \quad \text{Weight of warp} = \frac{80 \times 63 \times 160}{18 \times 560} = 80 \text{ lbs.}$$

Example 2.

How many pieces could be warped from 136 lbs. of 36's cut Gala, 40 ends per inch, 68 inches wide, 60 yards each ?

(*a*) Let us first find the weight of yarn in one warp.

Width of warp = 68 inches ; number of ends per inch = 40. Total ends in the warp = (40×68) ; length of each warp = 60 yards. $\therefore$ Length of thread in warp = $40 \times 68 \times 60$ yards.

Since 36's cut Gala means (36×200) yards weigh 1 lb.,

$$\text{Then} \quad \text{Weight of warp} = \frac{40 \times 68 \times 60}{36 \times 200} = \frac{68}{3} \text{ lbs.}$$

Total weight of yarn = 136 lbs.

and each warp requires $\frac{68}{3}$ lbs.

$$\text{Then} \quad \text{Number of warps} = 136 \div \frac{68}{3} = 6.$$

Example 3.

What count in Yorkshire skeins is required to warp 64 yards, 2800 ends, weighing 35 lbs. ?

Each warp thread is 64 yards long and there are 2800 threads.

Then Total length of yarn used = 64×2800 yards.

Since the Yorkshire skeins hank is 256 yards long,

$\therefore$ Number of Yorkshire skeins hanks in warp $= \dfrac{64 \times 2800}{256}$

and these weigh 35 lbs.

Now the number of hanks which weigh 1 lb. indicates the counts.

$\therefore$ Counts of yarn $= \dfrac{64 \times 2800}{256 \times 35} = 20$'s Yorkshire skeins.

Example 4.

A warp of 2/40's worsted contains 2000 ends and weighs 50 lbs. What is the length ?

Suppose the length $= x$ yards.

Then the total length of yarn $= 2000\,x$ yards. (A)

2/40's worsted means that 20 hanks each 560 yards long weigh 1 lb.

Then in 1 lb. of yarn we have (20×560) yards

and ,, 50 lbs. ,, ,, ,, $(20 \times 560 \times 50)$ yards

or Length of yarn in warp $= (20 \times 560 \times 50)$ yards. (B)

Equating (A) and (B) we have

$$2000\,x = 20 \times 560 \times 50.$$

Divide both sides by 2000 and

$$x = \frac{20 \times 560 \times 50}{2000} = 280.$$

$\therefore$ Length of warp $= 280$ yards.

Example 5.

What weight of warp is there in a piece 48 yards long, 60 inches wide, 160 ends per inch finished ; made of 2/60's worsted. Shrinkage one-seventh.

Total number of warp threads $= 160 \times 60$.

To find the length of each thread in the warp itself.

The piece is 48 yards long finished, and there has been a shrinkage of $\frac{1}{7}$ *the original length.*

Then $\frac{6}{7}$ of original length $= 48$ yards

or $\frac{1}{7}$,, ,, $= 8$,,

$\therefore$ Original length of warp $= 56$ yards.

Hence Length of thread in the warp $= (56 \times 160 \times 60)$ yards

Since the yarn is 2/60's, worsted, (30×560) yards weigh 1 lb.

$\therefore$ Weight of warp $= \dfrac{56 \times 160 \times 60}{30 \times 560} = 32$ lbs.

II. *Calculations involving Weight of Weft only.*

EXAMPLE 6.

How many lbs. of 2/48's worsted are required to weft a piece 60 yards long, 66 inches wide, 72 picks per inch ?

Now Each pick of weft is 66 inches or $\frac{66}{36}$ yards long.

Number of picks per inch $= 72$

" " " yard $= 72 \times 36$

" " in 60 yards $= 72 \times 36 \times 60$.

$\therefore$ Total length of weft $= \frac{66}{36} \times \frac{72 \times 36 \times 60}{1}$ yards

$= 66 \times 72 \times 60$ yards.

Now 2/48's worsted means that (24×560) yards weigh 1 lb.

$\therefore$ Weight of weft $= \frac{66 \times 72 \times 60}{24 \times 560} = 21\frac{3}{14}$ lbs.

NOTE.—In obtaining the weight of weft we may shorten the working if we show that—

$$\left\{\begin{matrix}\text{The number}\\ \text{of picks per}\\ \text{inch}\end{matrix}\right\} \times \left\{\begin{matrix}\text{The number of}\\ \text{inches in the}\\ \text{width}\end{matrix}\right\} = \left\{\begin{matrix}\text{The number}\\ \text{of yards of}\\ \text{weft in one}\\ \text{yard of}\\ \text{cloth.}\end{matrix}\right.$$

Thus in the last example,

Number of picks per inch $= 72$

" " " yard $= 72 \times 36$.

Length of each pick $= 66$ inches $= \frac{66}{36}$ yards.

$\therefore$ Length of weft in 1 yard of cloth $= 72 \times 36 \times \frac{66}{36}$ yards

$= (72 \times 66)$ yards.

Generalising,

If the number of picks per inch $= a$,

then " " " " " yard $= a \times 36$.

If Width of piece $= b$ inches,

then Length of each pick $= b$ inches

$= \frac{b}{36}$ yards.

and Length of weft in 1 yard of cloth $= a \times 36 \times \frac{b}{36}$ yards

$= (a \times b)$ yards

$$= \left\{\begin{matrix}\text{Number of}\\ \text{picks per}\\ \text{inch}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Width of}\\ \text{piece in}\\ \text{inches}\end{matrix}\right\}$$

Example 7.

What length of cloth can be woven from 30 lbs. of 35-skein Yorkshire, 70 inches wide, 60 picks ?

In a 35's Yorkshire skeins yarn (35×256) yards weigh 1 lb. and the weight of the yarn is 30 lbs.

Hence Length of yarn at our disposal $= (35 \times 256 \times 30)$ yards.

Also $\left.\begin{matrix}\text{Length of weft}\\ \text{in 1 yard of}\\ \text{cloth}\end{matrix}\right\} = \left\{\begin{matrix}\text{The number}\\ \text{of picks per}\\ \text{inch}\end{matrix}\right\} \times \left\{\begin{matrix}\text{The number of}\\ \text{inches in the}\\ \text{width}\end{matrix}\right\}$

$= (60 \times 70)$ yards.

Then Length of Cloth woven $= \dfrac{35 \times 256 \times 30}{60 \times 70} = 64$ yards.

Example 8.

The speed of a loom is 200 picks per minute, and it is weaving cloth with 17 picks per $\frac{1}{4}$ inch ; assuming that it is running 85 per cent. of a working week of 55 hours, what length of cloth would be woven ? State what weight of 40's cotton weft would be used (irrespective of waste) if the width of warp in the reed was $38\frac{1}{2}$ inches.

(*a*) The working week $= 55$ hours $= (55 \times 60)$ minutes.

$$85\% = \frac{85}{100} = \frac{17}{20}.$$

Then Time during which the loom is running

$$= \frac{17}{20} \times 55 \times 60 \text{ minutes}$$

and Total number of picks made during the week

$$= \frac{17 \times 55 \times 60 \times 200}{20} = (17 \times 55 \times 3 \times 200).$$

Now every 68 picks means 1 inch of cloth woven.
and every (68×36) picks means 1 yard ,, ,,

$$\therefore \text{ Length of cloth woven} = \frac{17 \times 55 \times 3 \times 200}{68 \times 36} = 229\tfrac{1}{6} \text{ yds.}$$

(*b*) *To find the weight of the weft.*

Length of weft in 1 yard of cloth $= 68 \times 38\frac{1}{2}$ yards.

Then Total length of weft $= 68 \times 38\frac{1}{2} \times 229\frac{1}{6}$ yds.

But 40's cotton means that (40×840) yards weigh 1 lb.

$$\therefore \text{ Weight of weft} = \frac{68 \times 38\frac{1}{2} \times 229\frac{1}{6}}{40 \times 840} \text{ lbs.}$$

$$= \frac{68 \times 77 \times 1375}{40 \times 2 \times 6 \times 840} = 17{\cdot}86 \text{ lbs.}$$

III. *Calculations involving Warp and Weft.*

Example 9.

A piece is woven from 65 yards of warp, 2/48's Botany, 54 sett Bradford, 64 inch reed, 56 picks per inch 1/24's Botany. Find the weight of warp and weft required. The length of the warp to be taken to cover the waste of weft in weaving.

(*a*) *To find weight of warp.*

54 Sett Bradford means (54×40) ends in a width of 36 inches

or $\frac{54 \times 40}{36}$ ends per inch.

Since reed width = 64 inches,

Then Number of ends in the warp $= \frac{54 \times 40 \times 64}{36}$.

Each end is 65 yards long.

Then Length of warp threads $= \frac{54 \times 40 \times 64 \times 65}{36}$ yards.

2/48's Botany means that (24×560) yards weigh 1 lb.

$\therefore$ Weight of warp $= \frac{54 \times 40 \times 64 \times 65}{36 \times 24 \times 560} = 18\frac{4}{7}$ lbs.

(*b*) *To find weight of weft.*

$$\text{Length of weft in 1 yard of cloth} = \left\{\begin{matrix}\text{Number of}\\ \text{picks per}\\ \text{inch}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Number of}\\ \text{inches in}\\ \text{the width}\end{matrix}\right\}$$

$$= 56 \times 64 \text{ yards.}$$

We are instructed to take the length of the cloth as the length of the warp (65 yards) and let this compensate the loss of weft due to weaving.

Hence Total length of weft $= 56 \times 64 \times 65$ yards.

Since 1/24's Botany means (24×560) yards weigh 1 lb.,

$$\text{Weight of weft} = \frac{56 \times 64 \times 65}{24 \times 560} = 17\frac{1}{3} \text{ lbs.}$$

Example 10.

Find the cost of the materials used in the construction of a cloth with the following details :—

Warp, 2/28's worsted at 2*s.* 6*d.* per lb.; weft, 2/12's worsted at 2*s.* per lb.; 17 reed 4's; 34 inches wide; 38 picks per inch; warp 50 yards; cloth finished 42 yards; allow 5% for waste of weft.

(*a*) *To find weight and cost of warp.*

17 reed 4's means 17 splits per inch with 4 ends through a split or 68 ends per inch.

Width of warp = 34 inches.

Then Number of ends in warp = 68×34.

Since each end is 50 yards long,

Total length of warp threads = $68 \times 34 \times 50$ yards.

2/28's worsted means that (14×560) yards weigh 1 lb.

$\therefore$ Weight of yarn in the warp = $\frac{68 \times 34 \times 50}{14 \times 560} = 14\frac{7}{8}\frac{3}{8}$ lbs.

Cost of 1 lb. of warp thread = $2s.\ 6d. = £\frac{1}{8}$.

$\therefore$ Cost of material in the warp = $£(\frac{1}{8} \times 14\frac{7}{8}\frac{3}{8}) = £1{\cdot}843$

$= £1\ 16\ 10\frac{1}{4}$.

(*b*) *To find weight and cost of the weft.*

Length of weft in 1 yard of the cloth = 38×34 yards.

Length of cloth finished = 42 yards.

Hence Total length of weft = $38 \times 34 \times 42$ yds.

Now in a 2/12's worsted there are (6×560) yards weigh 1 lb.

Then Weight of weft in the cloth = $\frac{38 \times 34 \times 42}{6 \times 560} = 16{\cdot}15$ lbs.

Allowing 5% for waste in weaving, we have

Weight of weft yarn used = $16{\cdot}15 + (\frac{1}{2}\frac{1}{0} \times 16{\cdot}15)$

$= 16{\cdot}15 + 0{\cdot}8075 = 16{\cdot}9575$ lbs.

Cost of 1 lb. of weft thread = $2s. = £\frac{1}{1}\frac{1}{0}$.

$\therefore$ Cost of material in the weft = $£16{\cdot}9575 \times \frac{1}{1}\frac{1}{0}$.

$= £1{\cdot}696 = £1\ 13\ 11$.

Total cost of warp and weft = £1 16 $10\frac{1}{4}$ + £1 13 11.

= £3 10 9 to the nearest penny.

IV. *Weight of Warp and Weft in Fancy Woven Fabrics.*

If there are several colours or different counts of warp and weft introduced, as in the production of fancy woven goods, it is necessary to ascertain the weight and cost of each sort or colour. A study of the following examples will show the method of determining these factors.

EXAMPLE 11.

Calculate the weight and cost of each material used in a cloth made as follows :—

Warp Pattern.

Medium grey	; 2/28's Worsted @	2*s.* 10*d.*	per lb.	84–822–8	6–
Dark grey	; 2/28's	,,	,,	,,	8–88–228	–
Black	; 2/28's	,,	2*s.* 8*d.*	,,	8–88––8	8–
Black & white	; 4/84's	,,	3*s.* 10*d.*	,,	22––44–	––
Bright blue	; 40/2 spun silk at		15*s.*	,,	––––––––	4–

Weft.—Black 25's Yorkshire skeins at 2*s.* 6*d.* per lb.
Loom particulars : 80 ends and 48 picks per inch ; 68 inches vide ; 60 yards warped ; 58 yards woven ; allow 5% for loss n weight of weft due to waste in weaving.

From the warping plan we see that

Total number of threads *in one repeat* of the pattern } = 144.

Now Total number of threads in the warp } = 80 × 68 = 5440.

Then Number of repeats in the full width of the warp } = $\frac{5440}{144}$ = 37 and 112 threads over.

To find the number of threads of each colour in the remaining portion of the repeat, we mark off this portion (112 threads) as shown and afterwards total up the number of threads of each colour in this portion of repeat of the pattern.

[*N.B.*—The table is read downwards. Thus the threads occur in this order: 8 medium grey, 8 dark grey, 8 black, 2 black and white, 4 medium grey, 2 black and white, 8 dark grey, 8 black, 8 medium grey, and so on.]

(*a*) *Medium Grey.* In each repeat of the pattern there are 38 ends, and in the remaining portion there are 32 ends.

∴ Number of ends of medium grey in full width of warp = (38 × 37) + 32 = 1406 + 32 = 1438.

Each end is 60 yards long.

∴ Length of medium grey = 60 × 1438 yards.

Since 2/28's worsted means (14 × 560) yards weigh 1 lb.,

$$\text{Weight of medium grey} = \frac{60 \times 1438}{14 \times 560} = 11 \text{ lbs.}$$

Cost of medium grey = 2/10 × 11 = 31*s.* 2*d.*

(*b*) *Dark Grey.* In each repeat of the pattern there are 42 threads, and in the remaining portion there are 36 threads.

Then Number of ends of dark grey in full width of warp = (42 × 37) + 36 = 1554 + 36 = 1590.

As before, Length of dark grey = 60 × 1590 yards

and Weight of dark grey = $\frac{60 \times 1590}{14 \times 560} = 12\frac{1}{6}$ lbs. (approx.)

Cost of dark grey = 2/10 × $12\frac{1}{6}$ = 34*s.* 6*d.* to nearest penny.

(*c*) *Black worsted.*

Number of ends in each repeat = 48
" " remainder = 32.

Then Number of ends of black in full width of warp } = (48 × 37) + 32
= 1776 + 32 = 1808

s shown in (*a*) and (*b*).

$$\text{Weight of black} = \frac{60 \times 1808}{14 \times 560} = 13{\cdot}84 \text{ lbs. (using logs.).}$$

Cost of black $= 2/8 \times 13{\cdot}84 = 36/11$ to nearest penny.

(*d*) *Black and white worsted.*

Number of ends in each repeat $= 12$
,, ,, remainder $= 12$.

Then Number of ends of black and white in full width of warp $= (12 \times 37) + 12$
$= 12 \times 38 = 456$.

4/84's worsted means that (21×560) yards weigh 1 lb.

$$\therefore \quad \text{Weight of black and white} = \frac{60 \times 456}{21 \times 560} = 2{\cdot}327 \text{ lbs.}$$

Cost of black and white $= 3/10 \times 2{\cdot}327 = 8s.\ 11d.$

(*e*) *Blue silk.*

Number of ends in each repeat $= 4$
,, ,, remainder $= 0$.

Then Number of ends of blue silk in full width of warp $= 4 \times 37 = 148$.

40/2 spun silk means (40×840) yards weigh 1 lb.

$$\text{Hence} \quad \text{Weight of silk} = \frac{60 \times 148}{40 \times 840} = \frac{37}{140} \text{ lb.}$$

$$\text{Cost of silk} = 15s. \times \frac{37}{140} = 3s.\ 11\tfrac{1}{2}d. \text{ to nearest farthing.}$$

(*f*) *Weight and cost of weft.*

Length of weft in 1 yard of the cloth $= 48 \times 68$ yards
Length of cloth woven $= 58$ yards.

$\therefore$ Total length of weft in the cloth $= 48 \times 68 \times 58$ yards.

Now in 25's Yorkshire skeins we have (25×256) yards weigh 1 lb.

$$\text{Then Weight of weft in the cloth} = \frac{48 \times 68 \times 58}{25 \times 256} = 29{\cdot}58 \text{ lbs.}$$

Add 5% or $\frac{1}{20}$ of this weight for waste in weaving and

Weight of weft yarn used $= 29{\cdot}58 + (\frac{1}{20} \times 29{\cdot}58)$
$= 29{\cdot}58 + 1{\cdot}479 = 31{\cdot}059$ lbs.

$$\text{Cost of weft} = 2/6 \times 31{\cdot}059 = £\frac{31{\cdot}059}{8}$$

$= £3{\cdot}882 = £3\ 17s.\ 8d.$ to nearest penny.

Example 12.

What length of warp with 4800 ends can be made using 20 lbs. of cotton and warped as follows ?

Cotton 2/20 . .	1	1
Organzine Silk 3500 . .	0	2
Worsted 2/48 . .	2	0

Here we have 6 ends in each repeat of the pattern, and 4800 ends in the full width of the warp.

Then Number of repeats of pattern $= \frac{4800}{6} = 800.$

In each repeat there are 2 ends of cotton.

Hence Number of ends of cotton in full width of warp $= 800 \times 2 = 1600.$

Now in a 2/20's cotton we have (10×840) yards weigh 1 lb.

Then in 20 lbs. of 2/20's cotton we have $(10 \times 840 \times 20)$ yards.

But there are 1600 ends.

$\therefore$ Length of each end and warp $= \frac{10 \times 840 \times 20}{1600} = 105$ yards.

V. *Miscellaneous Examples.*

Example 13.

What would be the cost in wages for preparing the warps to make 150 pieces of cloth each requiring 84 yards of yarn, 1800 ends each piece and 24's cotton counts ? Rates of payment : winding 2*s.* 2*d.* per 100 lbs. ; warping $1\frac{3}{4}d.$ per 450 ends per 1000 yards ; sizing, $4\frac{1}{8}d.$ per 1000 ends per 1000 yards ; twisting, $4\frac{1}{4}d.$ per 1000 ends, assume 15 warps for the 150 pieces.

Length of yarn in each piece $= 84 \times 1800$ yards.

" " in 150 pieces $= 84 \times 1800 \times 150$ yards. (A)

Now in 24's cotton we have (24×840) yards weigh 1 lb.

Hence

$$\text{Weight of yarn required} = \frac{84 \times 1800 \times 150}{24 \times 840} \text{ lbs.} \quad \text{(B)}$$

$$= \frac{4500}{4} \text{ lbs.} = (11\tfrac{1}{4} \times 100) \text{ lbs.}$$

(*a*) Payment for winding is at the rate of 2/2 per 100 lbs.

$\therefore$ Wages for *winding* $= 2/2 \times 11\frac{1}{4} =$ £1 4*s.* $4\frac{1}{2}d.$

(*b*) Rate of payment for warping is $1\frac{3}{4}d.$ per 450 ends per 1000 yards, which is $1\frac{3}{4}d.$ for (450×1000) yards of yarn.

Take the total length of yarn in (A) and

$$\text{Wages for } \textit{warping} = \frac{84 \times 1800 \times 150}{450 \times 1000} \times \frac{7}{4} \text{ pence}$$
$$= 88{\cdot}2 \text{ pence} = 7s.\ 4\tfrac{1}{4}d.$$

(*c*) Payment for sizing is at the rate of $4\frac{1}{8}d.$ per 1000 ends per 1000 yards, which is $4\frac{1}{8}d.$ for (1000 × 1000) yards of yarn.

$$\text{Wages for } \textit{sizing} = \frac{84 \times 1800 \times 150}{1000 \times 1000} \times \frac{33}{8} \text{ pence}$$
$$= 93{\cdot}56 \text{ pence} = 7s.\ 9\tfrac{1}{2}d.$$

(*d*) Total number of ends to twist = 1800 × 14. Since there are 15 warps,

$$\therefore \quad \text{Wages for } \textit{twisting} = \frac{1800 \times 14}{1000} \times \frac{17}{4} \text{ pence}$$
$$= 107 \text{ pence} = 8s.\ 11d.$$

$$\therefore \text{ Total wages} = £1\ 4s.\ 4\tfrac{1}{2}d. + 7s.\ 4\tfrac{1}{4}d. + 7s.\ 9\tfrac{1}{2}d. + 8s.\ 11d.$$
$$= £2\ 8s.\ 5\tfrac{1}{4}d.$$

Example 14.

Calculate the percentage weight of each material in the following warp: 36 ends 24 skeins; 1 end 20/2 spun silk, 2 ends 24 skeins, 2 ends 2/60's worsted twisted with 2/40's cotton, 2 ends 24 skeins, 1 end 20/2 spun silk.

(*a*) Reduce each kind of material to its equivalent cotton counts.

$$\text{24's Yorkshire skeins} = \frac{24 \times 256}{840} = \frac{512\text{'s}}{70} \text{ cotton},$$

$$\text{20/2 spun silk} = 1/20\text{'s cotton},$$

$$\text{2/60's worsted} = \frac{30 \times 560}{840} = 1/20\text{'s cotton},$$

and 2/60's worsted twisted with 2/40's cotton = 1/10's cotton.

(*b*) Suppose length of warp is 840 yards or 1 cotton hank.

In the warp there are 40 ends of 24's skeins; 2 ends of 20/2 spun silk; 2 ends of twisted yarn.

$$\therefore \text{ Weight of 40 hanks of } \frac{512\text{'s}}{70} \text{ cotton} = \frac{40 \times 70}{512} = 5{\cdot}469 \text{ lbs.}$$

Weight of 2 hanks of 1/20's cotton = 0·1 lb.
,, 2 hanks of 1/10's cotton = 0·2 lb.
Total weight of warp = 5·769 lbs.

(*c*) Percentage of 24's skeins $= \frac{5{\cdot}469 \times 100}{5{\cdot}769} = 94{\cdot}8\%$

Percentage of 20/2 spun silk $= \frac{0{\cdot}1 \times 100}{5{\cdot}769} = 1{\cdot}73\%$

Percentage of twisted yarn $= \frac{0{\cdot}2 \times 100}{5{\cdot}769} = 3{\cdot}47\%$

Example 15.

How many warp cuts (70 yards) can be got on to a beam of a 2/36 worsted warp, 54 ends per inch, 97 porties (40 threads), assuming that the depth of the flanges or plates will permit of $4\frac{1}{8}$ lbs. of warp per inch width being beamed ?

Total number of ends in warp $= 97 \times 40$.

Length of yarn in one warp cut $= 97 \times 40 \times 70$ yards (A)

$$\text{Width in reed} = \frac{97 \times 40}{54} \text{ inches} \quad . \quad . \quad . \quad . \quad \text{(B)}$$

In a 2/36 worsted we have (18×560) yards weigh 1 lb. Then from (A),

$$\text{Weight of yarn in one warp cut} = \frac{97 \times 40 \times 70}{18 \times 560} = \frac{485}{18} \text{ lbs.}$$

From (B),

$$\text{Weight of yarn which can be put on beam} = \frac{97 \times 40}{54} \times 4\tfrac{1}{8} \text{ lbs.}$$

Then

$$\text{Number of warp cuts} = \left\{ \frac{97 \times 40 \times 33}{54 \times 8} \right\} \div \frac{485}{18} = 11.$$

[*Note.*—Do not work out at each step, but wait till the final step.]

CHAPTER III

Simple Equations

23.—Introduction to Simple Equations. 24.—Axioms used in solving Equations. 25.—Examples to illustrate the use of the above Axioms. 26.—To find the counts of a thread which when twisted with any known thread will produce one of a given count. Worked examples.

23. *Introduction to Simple Equations.*

Suppose the following problem presents itself :—

"If 11,200 yards of yarn weigh 1 lb., what would be the counts in the woollen (Yorkshire skeins) denomination ?"

Let us try to obtain the required result, in a manner different from that adopted in Chapter I.

Suppose the unknown counts be represented by the symbol x.

Since the Yorkshire skein is 256 yards and

1's counts (Y.S.) means 256 yards weigh 1 lb.

2's ,, ,, ,, (2×256) yards weigh 1 lb., etc.

Then x's ,, ,, ,, $256x$ yards weigh 1 lb.

Since the yarn we are considering weighs 1 lb., then it must be $256x$ yards long. But we are told it is 11,200 yards long, and we express the fact that these two lengths are the same thus :—

$$256x = 11{,}200 \quad . \quad . \quad . \quad . \quad . \quad \text{(A)}$$

This is called AN EQUATION and is *a statement that two quantities* (i) 256 times an unknown quantity x (ii) 11,200 *are equal.*

Again : Suppose we require a tank to hold 500 gallons of water. What height must it be if its length and breadth are to be 5 ft. and 4 ft. respectively ?

Let us represent the unknown height by the symbol h (feet). Then the capacity of the tank will be $(5 \times 4 \times h)$ or $20h$ cubic feet.

But the capacity of the tank is 500 gallons or 80·2 cubic feet (since 1 gallon = 0·1604 cubic feet), and since we are considering

the same tank in both cases, these capacities must be the same, and we express this fact by the equation

$$20h = 80{\cdot}2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

Equations therefore contain one or more *unknown* quantities [x in equation (A); h in equation (B)], and our task is to SOLVE the equations, *i.e.* to find out what numerical values must be assigned to these unknown quantities in order that the equation may be satisfied.

Considering equation (A) $256x = 11{,}200$, we have 256 times the number represented by x is equal to 11,200; then, x must represent a number which is 256 times less than 11,200, and this we know is found by dividing 11,200 by 256.

Hence we have $$x = \frac{11200}{256} = 43{\cdot}75 \quad (C)$$

This equation (C) $x = \frac{11200}{256}$ we have obtained from (A) by dividing each side of equation (A) by 256, and it is evident that whatever value of x satisfies the statement of equation (A) will satisfy the statement of equation (C).

24. *Axioms used in Solving Equations.*

All simple equations may be SOLVED by making use of one or more of the following axioms :—

1. If equals be added to equals the sums are equal.
2. If equals be taken from equals, the remainders are equal.
3. If equals be multiplied by equals, the products are equal.
4. If equals be divided by equals, the quotients are equal.

N.B.—It will not be fair to divide by 0, for though $6 \times 0 = 7 \times 0$ it is not true that $6 = 7$.

25. *The following examples will illustrate the use of the above axioms.*

EXAMPLE 1.

Solve $$\frac{x}{5} = 23.$$

Multiply both sides by 5 and $x = 115$ (Axiom 3).

To check or verify this result—substitute 115 for x in $\frac{x}{5} = 23$

and we get $\frac{115}{5} = 23$.

$\therefore \quad \frac{x}{5} = 23$ when $x = 115$.

Example 2.

Solve the equation

$$2x - 5 = 19.$$

Adding 5 to both sides we have

$$2x - 5 + 5 = 19 + 5.$$

or $$2x = 24 \text{ (Axiom 1).}$$

Dividing both sides by 2 we have $x = 12$ (Axiom 4).

Verification.

Substitute 12 for x in the original equation and

$$2x - 5 = 24 - 5 = 19.$$

$\therefore$ The solution $x = 12$ satisfies the equation.

Example 3.

Solve $$7x - 12 = 2x + 8.$$

Here our object is to get the terms containing the unknown quantity (x) to one side, usually the left-hand side, and the numerical terms to the other side.

Add 12 to and subtract $2x$ from both sides and

$$7x - 12 + 12 - 2x = 2x + 8 + 12 - 2x$$

or $$7x - 2x = 8 + 12 \text{ (Axioms 1 and 2)}$$

or $$5x = 20.$$

Divide both sides by 5 and we have $x = 4$ (Axiom 4).

Verification. Substitute 4 for x in left-hand side of equation and $7x - 12 = 28 - 12 = 16.$

Substitute 4 for x in right-hand side

and $$2x + 8 = 8 + 8 = 16.$$

Hence $7x - 12 = 2x + 8$ when $x = 4$.

Example 4.

Solve the equation

$$\frac{1{\cdot}2}{x} = 0{\cdot}36.$$

Now it is clear that if two fractions are equal, then the equality is not destroyed if each fraction be inverted thus :

if $$\frac{3}{5} = \frac{36}{60} \text{ then } \frac{5}{3} = \frac{60}{36}$$

and if $$\frac{x}{y} = \frac{12}{5} \text{ then } \frac{y}{x} = \frac{5}{12}.$$

Then if $$\frac{1{\cdot}2}{x} = \frac{0{\cdot}36}{1}$$

Inverting $$\frac{x}{1{\cdot}2} = \frac{1}{0{\cdot}36}.$$

Multiplying both sides by 1·2 we have

$$x = \frac{1{\cdot}2}{0{\cdot}36} = \frac{120}{36} = 3\tfrac{1}{3}.$$

VERIFICATION.

Substitute $3\frac{1}{3}$ or $\frac{10}{3}$ for x in the equation,

and $$\frac{1{\cdot}2}{x} = \frac{1{\cdot}2}{3\frac{1}{3}} = \frac{1{\cdot}2 \times 3}{10} = 0{\cdot}36.$$

$\therefore$ $x = 3\frac{1}{3}$ satisfies the equation.

EXAMPLE 5.

What value of q makes one half of $3{\cdot}5q - 2{\cdot}5$ equal to two-thirds of $2{\cdot}1q + 5{\cdot}4$?

If $$\frac{3{\cdot}5q - 2{\cdot}5}{2} = \frac{2(2{\cdot}1q + 5{\cdot}4)}{3}$$

Multiply both sides by 6 (L.C.M. of the Denominators) and

$$3(3{\cdot}5q - 2{\cdot}5) = 4(2{\cdot}1q + 5{\cdot}4).$$

Removing the brackets we have

$$10{\cdot}5q - 7{\cdot}5 = 8{\cdot}4q + 21{\cdot}6.$$

Subtract $8{\cdot}4q$ from and add $7{\cdot}5$ to both sides,

and $$10{\cdot}5q - 8{\cdot}4q = 21{\cdot}6 + 7{\cdot}5$$

or $$2{\cdot}1q = 29{\cdot}1.$$

Divide both sides by 2·1 and

$$q = \frac{29{\cdot}1}{2{\cdot}1} = 13{\cdot}86$$

correct to two places of decimals.

EXAMPLE 6.

A piece of sheet zinc in the form of a rectangle measuring 12 inches by 9 inches is found to weigh 1·36 lbs. What is its thickness? Zinc weighs 0·252 lb. per cubic inch.

Suppose the thickness is x inches.

Then volume of the zinc $= 12 \times 9 \times x = 108x$ cubic inches.

Since 1 cubic inch of zinc weighs 0·252 lb.,

Then Weight of the sheet $= 108x \times 0{\cdot}252$ lbs.

But the weight is given as 1·36 lbs.

Hence $108 \times x \times 0{\cdot}252 = 1{\cdot}36.$

Divide both sides by $108 \times 0{\cdot}252$ and

$$x = \frac{1{\cdot}36}{108 \times 0{\cdot}252} = 0{\cdot}05$$ correct to the nearest $\frac{1}{100}$.

$\therefore$ Thickness of sheet $= 0{\cdot}05$ inch to nearest $\frac{1}{100}$".

Note.—On examining the equations solved in the foregoing examples, the student will notice that the highest power of the unknown quantity

(x, h, q) is the first. Equations of this type are termed *Simple Equations* or equations of the FIRST DEGREE.

When the unknown quantity is raised to the second power, *e.g.*

$$x^2 + 5x = 24$$
$$p^2 - 3p = 10$$

the equation is called a QUADRATIC EQUATION or an equation of the SECOND DEGREE.

Similarly $x^3 - 6x^2 + 11x = 6$ is an equation of the THIRD DEGREE.

26. *To find the Count of a Thread, which when twisted with any known thread will produce one of a given count.*—Sometimes a spinner or manufacturer requires that the counts of a thread resulting from the twisting of several threads (whether of the same or different materials) should be a given count; for instance it may be required to twist with a thread of 20's worsted another thread of worsted of such counts that the two together will give a thread of 8's worsted. Or it may be that it is required to twist together worsted and cotton, worsted and silk, or woollen and cotton, or any two materials; then it becomes necessary to find what thread along with the given one will produce a twist thread equal to the count required.

Problems of this character are best worked by obtaining a simple equation from the conditions given and solving this equation.

The following examples should be carefully studied.

EXAMPLE 1.

It is required to twist with a thread of 20's worsted another worsted thread, which together shall make one equal to 8's worsted.

Supposing there is no shrinkage or "take up" during the twisting operation, the lengths of each thread used and the twisted thread will all be equal.

Remembering that 20's counts worsted means that 20 hanks of 560 yards weigh 1 lb.,

Suppose counts of unknown thread $= x$'s worsted and suppose length of each thread and that of twist thread is 20 hanks.

Then 20 hanks of 20's weigh 1 lb.

20 hanks of x's ,, $\frac{20}{x}$ lbs. {Be quite clear about this step}

$\therefore$ 20 hanks of twist thread weigh $\left(1 + \frac{20}{x}\right)$ lbs.

But the twist thread = 8's worsted.

Then 20 hanks of twist thread weigh $\frac{20}{8}$ lbs.

Hence $$1 + \frac{20}{x} = \frac{20}{8}.$$

Subtracting 1 from each side we have

$$\frac{20}{x} = \frac{20}{8} - 1 = \frac{12}{8}.$$

Inverting each side

$$\frac{x}{20} = \frac{8}{12}.$$

Multiply each side by 20 and

$$x = \frac{8 \times 20}{12} = \frac{40}{3} = 13\tfrac{1}{3}.$$

∴ Required thread must be $13\frac{1}{3}$'s worsted.

EXAMPLE 2.

What counts of silk should be doubled with 60's worsted to yield 36's worsted ?

In this case we proceed as if the unknown thread was a worsted thread and then find its equivalent counts in the silk denomination.

Suppose counts of required thread = x's worsted, and assuming there is no "take up."

Then 60 hanks of 60's worsted weigh 1 lb.
60 hanks of x's ,, ,, $\frac{60}{x}$ lbs.

∴ 60 hanks of twist thread weigh $\left(1 + \frac{60}{x}\right)$ lbs.

But since the twist thread is 36's worsted.

60 hanks of twist thread weigh $\frac{60}{36}$ lbs.

Hence $$1 + \frac{60}{x} = \frac{60}{36}.$$

Subtract 1 from each side and

$$\frac{60}{x} = \frac{60}{36} - 1 = \frac{24}{36} = \frac{2}{3}.$$

Inverting each side

$$\frac{x}{60} = \frac{3}{2}.$$

Multiply by 60 and

$$x = \frac{3 \times 60}{2} = 90.$$

∴ Required thread = 90's worsted counts.

Now silk counts are based on a hank of 840 yards.

Hence equivalent silk counts $= \frac{90 \times 560}{840} = 60$'s.

Example 3.

A twist thread is composed of one thread 30's worsted, one of 40's cotton, and a third thread of silk. Find the counts of the silk thread assuming that the compound thread is equal to 18's worsted.

40's cotton is equivalent to $\frac{40 \times 840}{560}$ or 60's worsted.

Suppose counts of unknown thread $= x$'s worsted.

Assuming there is no "take up,"

60 hanks of 60's worsted weigh 1 lb.
60 ,, 30's ,, ,, 2 lbs.
60 ,, x's ,, ,, $\frac{60}{x}$ lbs.

Then 60 hanks of twist thread weigh $\left(3 + \frac{60}{x}\right)$ lbs.

But since the compound thread is equal to 18's worsted,

60 hanks of twist thread weigh $\frac{60}{18}$ lbs.

Hence $$3 + \frac{60}{x} = \frac{60}{18}.$$

Subtract 3 from each side and

$$\frac{60}{x} = \frac{60}{18} - 3 = \tfrac{1}{3}.$$

Inverting each side

$$\frac{x}{60} = 3.$$

$$\therefore \quad x = 180$$

And required thread is equal to 180's worsted.

Now counts in spun silk are based on a hank of 840 yards.

$\therefore$ 180's worsted is equivalent to $\frac{180 \times 560}{840}$ or 120's spun silk.

Example 4.—When the Lengths Vary.

A loop yarn has a resultant count of 4's cotton. It is made up of two threads of 2/28's black cotton, and a grey thread in

worsted. There are two yards of this thread used to each yard of cotton. Find the counts of the worsted.

Now 4's cotton is equivalent to $\frac{4 \times 840}{560}$ or 6's worsted.

Also 2 threads of 2/28's cotton is equivalent to 1 thread of 28's cotton and 7's cotton is equivalent to $\frac{7 \times 840}{560}$ or $10\frac{1}{2}$'s worsted.

Suppose counts of grey thread = x's worsted,

And since 2 yards of x's are twisted with 1 yard of black cotton,

2 hanks of x's worsted weigh $\frac{2}{x}$ lb.

1 hank of $10\frac{1}{2}$'s ,, weighs $\frac{2}{21}$ lb.

$\therefore$ 1 hank of twist thread weighs $\left(\frac{2}{x} + \frac{2}{21}\right)$ lb.

But since twist thread is equivalent to 6's worsted, 1 hank of twist weighs $\frac{1}{6}$ lb.

$$\therefore \quad \frac{2}{x} + \frac{2}{21} = \frac{1}{6}$$

$$\text{or} \quad \frac{2}{x} = \frac{1}{6} - \frac{2}{21} = \frac{3}{42}.$$

Inverting both sides,

$$\frac{x}{2} = \frac{42}{3}$$

$$\text{or} \quad x = \frac{84}{3} = 28.$$

$\therefore$ Required thread must be 28's worsted

Example 5.

Find the count and price of a spun silk yarn to twist to a 2/48's worsted yarn at 2*s*. 10*d*. per lb. and make a yarn equal to 2/36's worsted at 5*s*. Allow 3*d*. per lb. for twisting, etc.

Relative lengths :—Twist, 100 ; worsted, $102\frac{1}{2}$; silk, 110.

(a) *To find the counts of the Spun Silk Yarn.*—Work as if the yarn were worsted and then find its equivalent counts in the spun silk system.

Suppose the counts are x's worsted, and 110 hanks of x's twisted with $102\frac{1}{2}$ hanks of 2/48's produce 100 hanks of twist thread 2/36's.

Now 110 hanks of x's worsted weigh $\frac{110}{x}$ lbs.

$102\frac{1}{2}$,, 2/48's ,, ,, $\frac{102\frac{1}{2}}{24} = \frac{205}{48}$ lbs.

$\therefore$ 100 ,, twist yarn ,, $\left(\frac{110}{x} + \frac{205}{48}\right)$ lbs.

But 100 hanks of twist 2/36's weigh $\frac{100}{18}$ lbs.

$\therefore$ $$\frac{110}{x} + \frac{205}{48} = \frac{100}{18}$$

or $$\frac{110}{x} = \frac{50}{9} - \frac{205}{48} = \frac{185}{144}.$$

Inverting both fractions, $\frac{x}{110} = \frac{144}{185}$.

Multiply both sides of the equation by 110 and

$$x = \frac{144 \times 110}{185} = 85\tfrac{1}{2}.$$

Then Worsted counts $= 85\frac{1}{2}$'s

and Equivalent spun silk counts $= \frac{85\frac{1}{2} \times 560}{840} = \frac{171}{3} = 57$'s.

(b) *To find the cost per lb. of the spun silk thread.*

From (*a*) we see :—

In $\frac{100}{18}$ lbs. of twist yarn we have $\frac{205}{48}$ lbs. of 2/48's worsted and $\frac{110}{x}$ lbs. of spun silk.

Now Cost of $\frac{100}{18}$ lbs. of twist at 4/9 (deduct 3*d*. from 5*s*. for twisting) $= \frac{100 \times 57}{18}$ pence

$= 316{\cdot}7$ pence.

Cost of $\frac{205}{48}$ lbs. of 2/48's worsted at 2/10 $= \frac{205 \times 34}{48}$.

$= 145{\cdot}2$ pence.

$\therefore$ Cost of $\frac{110}{x}$ lbs. of spun silk $= 316{\cdot}7 - 145{\cdot}2$

$= 171{\cdot}5$ pence.

Since $x = 85\frac{1}{2}$,

Cost of 1 lb. of spun silk $= 171{\cdot}5 \div \frac{110}{x} = \frac{171{\cdot}5 \times x}{110}$

$= 171{\cdot}5 \times \frac{85{\cdot}5}{110} = 133{\cdot}3$ pence

$= 11s.\ 1d.$ to the nearest penny.

Example 6.

Suppose a cloth is to be made, the warp of which shall weigh 40 lbs., the face cloth to be 30 skeins woollen, and the back to be 10 skeins, both Yorkshire counts. There are to be two threads of face to one of back, the width of the piece to be 64 inches in the reed, and the length of warp 50 yards. How many ends per inch of face and back respectively will be required?

Suppose there are x ends per inch back warp.

Then ,, $2x$,, ,, ,, front ,,

Width of warp in reed = 64 inches.

Then Total number of ends of back warp $= 64x$.

Each end is 50 yards long.

Then Total length of back warp $= 64x \times 50$ yards.

In a 10's Y.S. we have (10×256) yards weigh 1lb.

$$\therefore \quad \text{Weight of back warp} = \frac{64 \times x \times 50}{10 \times 256} = \frac{5x}{4} \text{ lbs.}$$

$$\text{Similarly weight of front warp} = \frac{64 \times 2x \times 50}{30 \times 256} = \frac{5x}{6} \text{ lbs.}$$

Hence $$\frac{5x}{4} + \frac{5x}{6} = 40.$$

Multiply every term on both sides by 12 and

$$15x + 10x = 480$$

or $$25x = 480.$$

$$\therefore \quad x = 19{\cdot}2.$$

Then there are 19·2 ends per inch back warp

and ,, 38·4 ,, ,, front ,,

CHAPTER IV

Indices and Logarithms

27.—Positive Indices. 28.—Laws of Indices for Positive Integers. 29.—(*a*) Square Root, (*b*) Cube Root by factors. 30.—Fractional and Negative Indices. 31.—To find a meaning for $a^{\frac{1}{2}}$. 32.—To find a meaning for a^0. 33.—To find a meaning for a^{-2}. 34.—Introductory to Logarithms. 35.—To construct a Table giving the Values of certain powers of 10. 36.—Graph of $y = 10^x$ and its Use. 37.—Relation between Indices and Logarithms. 38.—Definition of Logarithm. 39.—Fundamental Laws of Logarithms. 40.—Common Logarithms—Characteristic and Mantissa. 41.—The Mantissæ of the Logs. of Numbers, formed with the same significant digits, are the same. 42.—Manipulation of Logarithms. 43.—Use of Tables of Logarithms. 44.—Antilogarithms. 45.—Calculations using a Table of Logarithms.

27. *Positive Indices.*—We are already acquainted with the fact that the number of units of area in a rectangle is obtained by multiplying the number of units in the length by the number of units in the breadth. Thus if length be a units and breadth b units,

Then Area $= (a \times b)$ or (ab) units of area.

Now if the breadth of the rectangle equals the length, the figure becomes a square, each side being a units long, and

Area of square $= (a \times a)$ units of area.

The product $(a \times a)$ may be abbreviated to aa and further to a^2.

This a^2 is called "the second power of a" or "a to the second power" or more commonly "a squared."

Again in the case of a rectangular prism if

Length $= a$ linear units
Breadth $= b$,, ,,
Height $= c$,, ,,

then volume $= (a \times b \times c)$ units of volume.

If the length, breadth and height be the same (a units), the rectangular prism becomes a cube and

Volume of cube $= (a \times a \times a)$ units of volume.

The product $(a \times a \times a)$ may be written $(a.a.a)$ or a^3. ^{3} is called " the third power of a " or " a to the third power " r " the cube of a."

The conception of a power is extended to cases which have o immediate application to Geometry ; thus :

$.x.x.x$ is called " the fourth power of x " and is denoted by x^4.

$c.c.c.c$ is called " the fifth power of c " and is denoted by c^5.

We may also say that

2^4 is the *product* of 4 factors, each factor being 2.

x^7 ,, ,, 7 factors, ,, ,, ,, x.

And in general $x^n = x.x.x.x \ldots$ to n factors, or x^n is the roduct of n factors, each factor being x.

In a^3 the number 3 is called the " *index* " (plural " *indices* ") the power a^3.

The student must carefully distinguish between $4x$ and x^4. Thus $4x = x + x + x + x$, which is the *sum* of four terms, ach term being x, whilst as stated before $x^4 = x.x.x.x$, which a *product*.

8. *Laws of Indices for Positive Integers.*

(*a*) Considering $3^4 = 81$ we say 3 is the quantity which is ised to the 4th power, 4 is the index of the power, and 3^4 r 81 is the fourth power of 3.

Now $a^3 = a.a.a$ and $a^5 = a.a.a.a.a$.

Then $a^3 \times a^5 = (a.a.a) \times (a.a.a.a.a)$

= the product of 8 factors each factor being a

$= a^8$.

We notice that the index 8 in the product a^8 is the sum f 5 and 3, the indices of a^3 and a^5.

Also $x^2 \times x^4 = (x.x) \times (xxxx) = x^{(2+4)} = x^6$.

In general $x^m \times x^n = (x.x.x \ldots$ to m factors$) \times (x.x.x \ldots$ to n factors$)$

$= x.x.x.x \ldots$ to $(m + n)$ factors

$= x^{(m+n)}$.

Notice we are multiplying powers of the same quantity, ius : $x^2 \times x^4$; $a^3 \times a^5$; $c^5 \times c^8$; etc.

If the quantities are different we have

$$a^3 \times b^2 = a.a.a.b.b = a^3b^2.$$

The above leads up to the following law :—

" In multiplying together powers of the *same quantity* add ie indices."

(*b*) Consider $x^8 \div x^5$.

$$\frac{x^8}{x^5} = \frac{x.x.x.x.x.x.x.x}{x.x.x.x.x} = x^3.$$

Here 5 factors of the numerator (x^8) cancel with the 5 factors

of the denominator (x^5) and in the numerator ($8-5$) or 3 factors remain.

Thus $\dfrac{x^8}{x^5} = x^{(8-5)} = x^3.$

Similarly :— $x^7 \div x^3 = x^{(7-3)} = x^4.$
$a^{12} \div a^9 = a^{(12-9)} = a^3.$
$c^7 \div c^5 = c^{(7-5)} = c^2.$

Generalising $\dfrac{x^m}{x^n} = \dfrac{x.x.x.x \ldots \text{to } m \text{ factors}}{x.x.x \ldots \text{to } n \text{ factors}}$
$= x^{(m-n)}$ where m is greater than n.

As in multiplication we can only deal with " powers of the same quantity " in this way.

Thus $a^{12} \div a^9 = a^{(12-9)} = a^3$, *but* $a^{12} \div b^9 = \dfrac{a^{12}}{b^9}$, since there are no common factors in the numerator and denominator of the fraction.

The " Index Law for Division " may now be stated :—

" In dividing any power of a quantity by another power of the same quantity, subtract the index of the divisor from that of the dividend."

29. *Square Root and Cube Root by Factors.*—(*a*) We have defined a^2 as the " second power of a " or the " square of a " and a^3 as the " third power of a " or the " cube of a."

The quantity a which is the repeated factor of (a^2) is spoken of as the " square root " of a^2.

Using the symbol $\sqrt{\ }$ for the words *the square root of* we have $\sqrt{a^2} = a.$

Since $3^2 = 3.3 = 9$ then $\sqrt{9} = \sqrt{3.3} = 3.$
$13^2 = 13.13 = 169$,, $\sqrt{169} = \sqrt{13.13} = 13.$
$(\frac{3}{5})^2 = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$,, $\sqrt{\frac{9}{25}} = \sqrt{\frac{3}{5} \times \frac{3}{5}} = \frac{3}{5}.$

A perfect square is " the product of two factors each factor being the same " ; then being given a square number, if we can express it as the product of *two groups of prime factors* each group being the same, one of these two groups is the square root.

Example 1.

$$\sqrt{25} = \sqrt{5 \times 5} = 5.$$

Example 2.

$$\sqrt{3600} = \sqrt{2.2.2.2.3.3.5.5} = \sqrt{(2.2.3.5)(2.2.3.5)} = 2.2.3.5 = 60$$

EXAMPLE 3.

$$\sqrt{99225} = \sqrt{5.5.3969} = \sqrt{5.5.3.3.441}$$
$$= \sqrt{5.5.3.3.3.49} = \sqrt{5.5.3.3.3.7.7}$$
$$= \sqrt{(5.3.3.7)(5.3.3.7)} = 315.$$

EXAMPLE 4.

If the length of a side of a square be l units of length and rea of square be A units of area, then $A = l \times l = l^2$, and $= \sqrt{A}$.

EXAMPLE 5.

If radius of a circle be r units of length and area of circle be A units of area, we have $A = \pi r^2$, and $r^2 = \frac{A}{\pi}$.

$$\therefore \qquad r = \sqrt{r^2} = \sqrt{\frac{A}{\pi}}.$$

EXAMPLE 6.

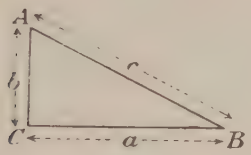


FIG. 5.

If ABC be a right-angled triangle having ACB a right angle, we have already learnt that the area of the square described on AB is equal to the sum of the areas of the squares described on AC and CB.

If CB be a units long; $CA = b$ units; $AB = c$ units,

Then $c^2 = a^2 + b^2$ or $c = \sqrt{a^2 + b^2}$; also $a^2 = c^2 - b^2$ or $a = \sqrt{c^2 - b^2}$, and $b^2 = c^2 - a^2$ or $b = \sqrt{c^2 - a^2}$.

(*b*) Again we have $4^3 = 4.4.4 = 64$ where 64 is the "CUBE" of 4 or "the third power of" 4, and 4 is the "CUBE ROOT" of 64.

Using the symbol $\sqrt[3]{\ }$ for the words "the cube root of" we have $\sqrt[3]{64} = \sqrt[3]{4.4.4} = 4$, and $\sqrt[3]{a^3} = \sqrt[3]{a.a.a} = a$.

Seeing that a *cube* is the product of three factors, each factor being the same, then if we require the *cube root* of a number

we must seek for three factors, or groups of prime factors, of the given number, each factor or group being the same.

Thus :—

Example 7.

$$\sqrt[3]{3375} = \sqrt[3]{5.675} = \sqrt[3]{5.5.135}$$
$$= \sqrt[3]{5.5.5.27} = \sqrt[3]{5^3.3^3}$$
$$= \sqrt[3]{(5.3)(5.3)(5.3)} = 5.3 = 15.$$

Example 8.

$$\sqrt[3]{13824} = \sqrt[3]{2.2.2.1728} = \sqrt[3]{2^3.2^3.216}$$
$$= \sqrt[3]{2^3.2^3.6^3} = \sqrt[3]{(2.2.6)(2.2.6)(2.2.6)}$$
$$= 2.2.6 = 24.$$

Example 9.

$$\sqrt[3]{3\tfrac{3}{8}} = \sqrt[3]{\tfrac{27}{8}} = \sqrt[3]{\frac{3^3}{2^3}} = \sqrt[3]{\tfrac{3}{2} \times \tfrac{3}{2} \times \tfrac{3}{2}} = \frac{3}{2}.$$

Example 10.

$$\sqrt[3]{0{\cdot}000125} = \sqrt[3]{\frac{125}{10^6}} = \sqrt[3]{\frac{5.5.5}{10^2.10^2.10^2}}$$
$$= \frac{5}{10^2} = 0{\cdot}05$$

Example 11.

If one edge of a cube be l units of length and the volume of the cube be V units of volume, then $V = l \times l \times l = l^3$, and since $l = \sqrt[3]{l^3}$ $\therefore$ $l = \sqrt[3]{V}$.

30. *Fractional and Negative Indices.*—It has already been seen that

$a^3 = a.a.a$; $a^5 = a.a.a.a.a$; $a^n = a.a.a.a$ . . . to n factors.

$a^m \times a^n = a^{(m+n)}$.

$a^m \div a^n = a^{(m-n)}$ if m be greater than n.

$(a^m)^n = a^{mn}$.

These results depend upon the definition that a^n is the product of n factors, each factor being a ; and this definition evidently requires that n should be a positive whole number.

The question now arises, what meaning can be given to

expressions such as $2^{\frac{1}{2}}$, $10^{0.75}$, $a^{1.5}$, $x^{0.25}$, 10^{-3}, etc., or to a^n when n is a fraction or negative?

Such expressions often occur in our calculations and we must be able to interpret them.

Now it is important that all indices, whether positive, negative, integral or fractional, should be subject to the same laws. We, therefore, when trying to find meanings for expressions such as $a^{\frac{1}{2}}$, a^{-2}, $a^{-\frac{3}{4}}$, *assume* that they conform to the laws established for positive, whole number indices, viz.: "In multiplying together powers of the same quantity, add the indices; in dividing, subtract the index of the divisor from that of the dividend."

In other words we assume that $a^m \times a^n = a^{(m+n)}$; $a^m \div a^n = a^{(m-n)}$ for ALL VALUES of m and n.

31. *To find a meaning for* $a^{\frac{1}{2}}$. — Assuming then that $a^m \times a^n = a^{(m+n)}$ for all values of m and n

$$2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2^{(\frac{1}{2}+\frac{1}{2})} = 2^1 = 2.$$

Thus 2 which is the product of two factors each being $2^{\frac{1}{2}}$ is the square of $2^{\frac{1}{2}}$, and $2^{\frac{1}{2}} = \sqrt{2}$.

Again $5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5^{(\frac{1}{3}+\frac{1}{3}+\frac{1}{3})} = 5^1 = 5$, and 5 being the product of three factors each $5^{\frac{1}{3}}$ is the cube of $5^{\frac{1}{3}}$, or

$$5^{\frac{1}{3}} = \sqrt[3]{5}.$$

Similarly $3^{\frac{1}{5}} \times 3^{\frac{1}{5}} \times 3^{\frac{1}{5}} \times 3^{\frac{1}{5}} \times 3^{\frac{1}{5}} = 3^{(\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5})} = 3^1 = 3^3$, or

$$3^{\frac{3}{5}} = \sqrt[5]{3^3},$$

and in general $a^{\frac{1}{n}} = \sqrt[n]{a}$; $a^{\frac{m}{n}} = \sqrt[n]{a^m}$.

32. *To find a meaning for* a^0. — Since $a^m \div a^n = a^{(m-n)}$ for all values of m and n, $a^3 \div a^3 = a^{(3-3)} = a^0$.

But $$a^3 \div a^3 = \frac{a.a.a}{a.a.a} = 1.$$

Hence $$a^0 = 1.$$

Note that $2^0 = 1$; $10^0 = 1$; $x^0 = 1$ and $a^0 = 1$ for all values of a.

33. *To find a meaning for* a^{-2}. — Since $a^m \div a^n = a^{(m-n)}$.

$$a^3 \div a^5 = a^{(3-5)} = a^{-2}.$$

But $$\frac{a^3}{a^5} = \frac{a.a.a}{a.a.a.a.a} = \frac{1}{a^2}.$$

$$\therefore \quad a^{-2} = \frac{1}{a^2}$$

and in general $$a^{-n} = \frac{1}{a^n}$$

As examples we may take

$$10^{-1} = \frac{1}{10} = 0{\cdot}1$$

$$10^{-2} = \frac{1}{10^2} = 0{\cdot}01$$

$$100^{-0{\cdot}5} = \frac{1}{100^{0{\cdot}5}} = \frac{1}{100^{\frac{1}{2}}} = \frac{1}{\sqrt{100}} = 0{\cdot}1$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{27^2}} = \frac{1}{\sqrt[3]{3^3.3^3}} = \frac{1}{\sqrt[3]{(3.3)(3.3)(3.3)}} = \frac{1}{9}$$

$$3^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 3^{\frac{1}{2}} \times 3^{\frac{2}{2}} = 3^{\frac{3}{2}} = 3.$$

34. *Introductory to Logarithms.*—Let us consider the following table showing the various "*powers* of the *root* 2."

$2^0 = 1$	$2^6 = 64$	$2^{12} = 4096$
$2^1 = 2$	$2^7 = 128$	$2^{13} = 8192$
$2^2 = 4$	$2^8 = 256$	$2^{14} = 16384$
$2^3 = 8$	$2^9 = 512$	$2^{15} = 32768$
$2^4 = 16$	$2^{10} = 1024$	etc.
$2^5 = 32$	$2^{11} = 2048$	

Now suppose we have to multiply 512 by 32.

From the table we find $512 = 2^9$ and $32 = 2^5$.

Then $512 \times 32 = 2^9 \times 2^5 = 2^{14}$ (by indices), and from the table $2^{14} = 16384$.

$\therefore \qquad 512 \times 32 = 16384.$

Verify this by actual multiplication.

Again, suppose we wish to divide 8192 by 64.

From the table we have $8192 = 2^{13}$ and $64 = 2^6$.

Then $\dfrac{8192}{64} = \dfrac{2^{13}}{2^6} = 2^7$ (by indices).

From the table $2^7 = 128$.

Hence $\dfrac{8192}{64} = 128$. Verify this by actual division.

The student should in this manner work with the aid of the table, checking his results by actual multiplication or division,

512×16 ; 1024×32 ; $4096 \div 128$; $32768 \div 512$.

He will in this way notice that, in place of multiplying 512 by 32, he finds what power of 2 each number is, adds the two indices together, and looks up 2^{14}, which has been calculated for him in the table. Similarly a division involves merely the subtraction of two indices.

But he also notices that he can only multiply and divide numbers in the table.

Try 124×36.

The table does not give these numbers as powers of 2, but $124 = 2^x$ and $36 = 2^y$, and we were given x and y, then as before $124 \times 36 = 2^x \times 2^y = 2^{(x+y)}$, and we could at once write down the answer.

5. Now let us construct a table giving the values of certain POWERS *of* 10.

$10^0 = 1$

$10^{0.5} = 10^{\frac{1}{2}} = \sqrt{10} = 3{\cdot}162$

$10^{0.25} = 10^{\frac{1}{4}} = \sqrt[4]{10} = \sqrt{3{\cdot}162} = 1{\cdot}779$

$10^{0.125} = 10^{\frac{1}{8}} = \sqrt[8]{10} = \sqrt{\sqrt[4]{10}} = 1{\cdot}333$

$10^{0.375} = 10^{\frac{3}{8}} = 10^{\frac{1}{4}} \times 10^{\frac{1}{8}} = 1{\cdot}779 \times 1{\cdot}333 =$

$10^{0.625} = 10^{\frac{5}{8}} = 10^{\frac{1}{2}} \times 10^{\frac{1}{8}} = 3{\cdot}162 \times 1{\cdot}333 =$

$10^{0.75} = 10^{\frac{3}{4}} = 10^{\frac{1}{2}} \times 10^{\frac{1}{4}} = 3{\cdot}162 \times 1{\cdot}779 =$

$10^{0.875} = 10^{\frac{7}{8}} = 10^{\frac{3}{4}} \times 10^{\frac{1}{8}} =$

$10^1 = 10$

$10^{-1} = \frac{1}{10} = 0{\cdot}1$

$10^{-0.5} = \frac{1}{10^{\frac{1}{2}}} = \frac{1}{\sqrt{10}} = \frac{1}{3{\cdot}162} =$

$10^{-0.25} = \frac{1}{10^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{10}} = \frac{1}{1{\cdot}779} =$

and so on.

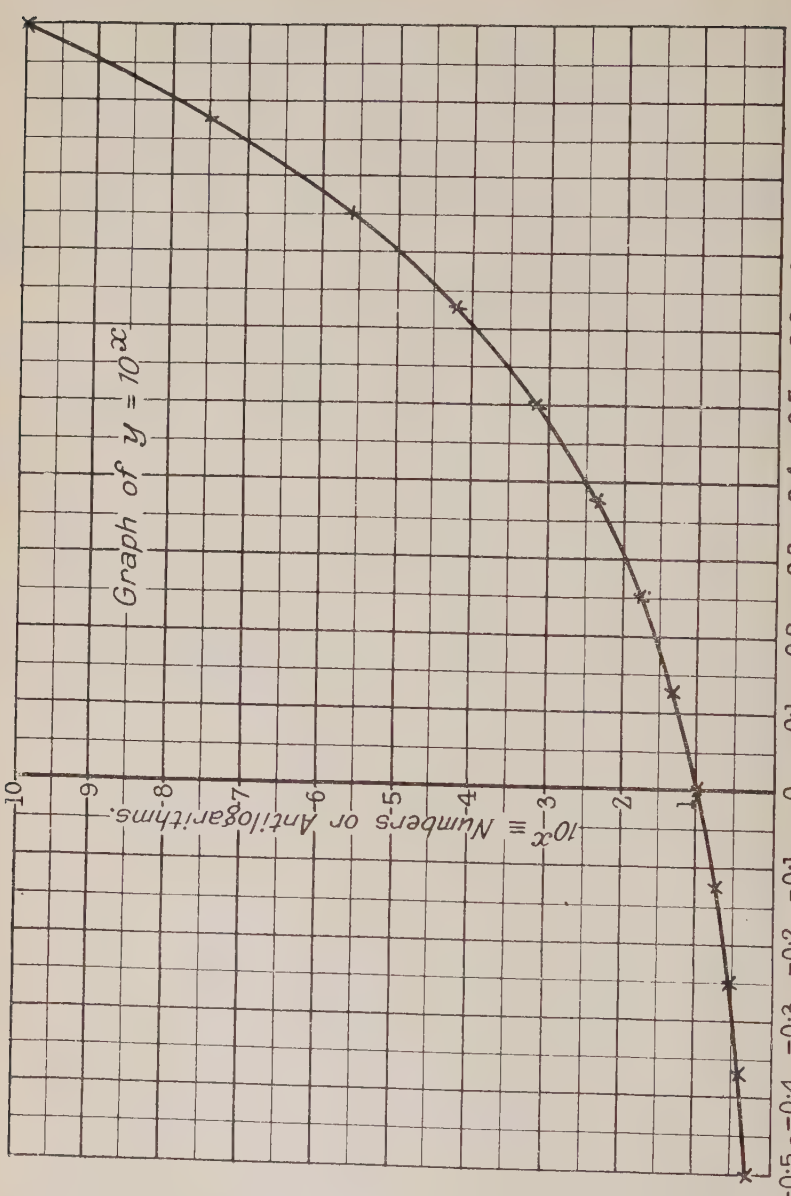
The student should fill in the blanks obtaining the results to four significant figures using contracted methods of multiplication and division.

When this is completed we may collect the results in a table as follows.

$y = 10^x$.

Values for x.	Corresponding Values for 10^x.	Values for x.	Corresponding Values for 10^x.
0	$10^0 = 1$	$-0{\cdot}125$	$10^{-0{\cdot}125} = 0{\cdot}75$
$0{\cdot}125$	$10^{0{\cdot}125} = 1{\cdot}33$	$-0{\cdot}25$	$10^{-0{\cdot}25} = 0{\cdot}56$
$0{\cdot}25$	$10^{0{\cdot}25} = 1{\cdot}78$	$-0{\cdot}375$	$10^{-0{\cdot}375} = 0{\cdot}42$
$0{\cdot}375$	$10^{0{\cdot}375} = 2{\cdot}37$	$-0{\cdot}5$	$10^{-0{\cdot}5} = 0{\cdot}32$
$0{\cdot}5$	$10^{0{\cdot}5} = 3{\cdot}16$	$-0{\cdot}625$	$10^{-0{\cdot}625} = 0{\cdot}24$
$0{\cdot}625$	$10^{0{\cdot}625} = 4{\cdot}22$	$-0{\cdot}75$	$10^{-0{\cdot}75} = 0{\cdot}18$
$0{\cdot}75$	$10^{0{\cdot}75} = 5{\cdot}62$	$-0{\cdot}875$	$10^{-0{\cdot}875} = 0{\cdot}13$
$0{\cdot}875$	$10^{0{\cdot}875} = 7{\cdot}50$	-1	$10^{-1} = 0{\cdot}1$
1	$10^1 = 10$		

36. *Graph of* $y = 10^x$ *and its Use.* Now draw the graph $y = 10^x$ from $x = -1$ to $x = 1$ (Fig. 6), taking 10 cms. as unit for x and 1 cm. as unit for y. Along x axis write


Graph of $y = 10^x$
10^x = Numbers or Antilogarithms.
10
9
8
7
6
5
4
3
2
1
0
-0·5 -0·4 -0·3 -0·2 -0·1 0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
Values for x = Indices or Logarithms.

Indices" and along the y axis "Numbers." Keep the graph and let us see how we can make use of it.

Example 1.

When $10^x = 5$ we see that $x = 0{\cdot}7$.
When $10^x = 1{\cdot}4$,, ,, ,, $x = 0{\cdot}15$.
Thus we get $10^{0{\cdot}7} = 5$ and $10^{0{\cdot}15} = 1{\cdot}4$.
Now $5 \times 1{\cdot}4 = 10^{0{\cdot}7} \times 10^{0{\cdot}15}$
$= 10^{(0{\cdot}7 + 0{\cdot}15)}$
$= 10^{0{\cdot}85}$ (by Indices).
From the graph, if $x = 0{\cdot}85$ then $10^x = 7$, and $5 \times 1{\cdot}4 = 7$.

Example 2.

Again using graph :—
When $10^x = 9$ we see that $x = 0{\cdot}954$.
,, $10^x = 4$,, ,, ,, $x = 0{\cdot}6$.
Then we may say $10^{0{\cdot}954} = 9$ and $10^{0{\cdot}6} = 4$, and

$$9 \div 4 = \frac{10^{0{\cdot}954}}{10^{0{\cdot}6}} = 10^{(0{\cdot}954 - 0{\cdot}6)}$$

$= 10^{0{\cdot}354}$ (by Indices).
Now from graph, if $x = 0{\cdot}354$ $10^x = 2{\cdot}25$, then $10^{0{\cdot}354} = 2{\cdot}25$ and $9 \div 4 = 2{\cdot}25$.

Example 3.

When $10^x = 2{\cdot}9$ $x = 0{\cdot}46$. then $10^{0{\cdot}46} = 2{\cdot}9$ and
$(2{\cdot}9)^2 = 2{\cdot}9 \times 2{\cdot}9 = 10^{0{\cdot}46} \times 10^{0{\cdot}46} = 10^{0{\cdot}92}$.
From graph, if $x = 0{\cdot}92$ then $10^x = 8{\cdot}35$, then $10^{0{\cdot}92} = 8{\cdot}35$ and $(2{\cdot}9)^2 = 8{\cdot}35$.
Compare this result with that obtained by multiplication

Example 4.

When $10^x = 9$ $x = 0{\cdot}954$, then we say $10^{0{\cdot}954} = 9$.
Thus $\sqrt{9} = 9^{\frac{1}{2}} = (10^{0{\cdot}954})^{\frac{1}{2}}$

$= 10^{\frac{0{\cdot}954}{2}}$

$= 10^{0{\cdot}477}$.

and from graph when $x = 0{\cdot}477$ $10^x = 3$.
Hence $10^{0{\cdot}477} = 3$ or $\sqrt{9} = 3$.
Now use the graph to work the following.

1. Express the following as powers of 10 :—
1, 1·5, 2, 2·5, 4, 6, 0·15, 0·4, 3·5, 2, 0·2 [*e.g.* $6{\cdot}5 = 10^{0{\cdot}814}$].

2. Find the values of $10^{0\cdot3}$, $10^{0\cdot4}$, $10^{0\cdot7}$.
   (*a*) By multiplication find value of $10^{0\cdot3} \times 10^{0\cdot4}$.
   (*b*) Compare result with $10^{(0\cdot3+0\cdot4)}$, *i.e.* $10^{0\cdot7}$.
3. Find *a*, *b*, and *c* in the following cases :—
   i. $1\cdot5 \times 4 = 10^a \times 10^b = 10^{(a+b)}$. Find $10^{(a+b)}$ from graph.
   $6 = 10^c$. Compare 10^c with $10^{(a+b)}$.
   ii. $\frac{0\cdot4}{4} = 10^a \div 10^b = 10^{(a-b)}$. Find $10^{(a-b)}$.
   $0\cdot1 = 10^c$. Compare 10^c with $10^{(a-b)}$.
4. Copy the following and fill up the blanks using the graph where necessary :—

$$2\cdot8 \times 3\cdot3 = 10 \cdot\cdot\cdot \times 10 \cdot\cdot\cdot$$
$$= 10 \cdot\cdot\cdot + \cdot\cdot\cdot$$
$$= 10 \cdot\cdot\cdot$$
$$=$$

5. Following example of question 4, perform the following multiplications using your graph. Check each answer by actual multiplication.
   $4\cdot5 \times 2$; $3\cdot7 \times 1\cdot5$; $(1\cdot5)^2$; 2^3 ; $0\cdot4 \times 10$; $0\cdot6 \times 6$.
6. Copy the following and fill up the blanks.

$$2\cdot55 \div 1\cdot7 = 10 \cdot\cdot\cdot \div 10 \cdot\cdot\cdot$$
$$= 10 \cdot\cdot\cdot - \cdot\cdot\cdot$$
$$= 10 \cdot\cdot\cdot$$
$$=$$

Check by actual division.
7. In the same way use the graph to simplify $9\cdot6 \div 4$; $5\cdot2 \div 1\cdot3$ $3\cdot75 \times 5\cdot9 \div 2\cdot7$; $8\cdot7 \times 6\cdot3 \div 7\cdot5$.
8. Remembering that $10^3 \times 10^2 = 10^{(3+2)}$
$$10^{0\cdot75} \times 10^{0\cdot13} = 10^{0\cdot88}$$
Try to express as powers of 10
15, 150, 35, 7·85, 78·5, 785, 7850, 48·5, 485, 0·2, 20

37. *Relation between Indices and Logarithms.*—Now in the above examples we have used a graph to determine the *indices* corresponding to *given numbers* and *vice versa*. By drawing larger and more accurate graphs it would be possible to determine the indices to more significant figures, but it is easier to use tables.

Then suppose, having chosen a *base* or *root*, say 10, we obtain a *Table of Indices* corresponding to all numbers between 1 and 9·999 ; then we have seen that the index corresponding to the *product* of two given numbers is the sum of the indices corresponding to the two numbers respectively. In other words the process of *Multiplication* of numbers is *replaced by* the process of *adding indices*. Similarly the process of *Division* is *replaced by* the process of *subtracting indices*, whilst the process of finding *square root*, *cube root*, *etc.*, *is replaced by dividing an index by* 2, 3, *etc.*

The Meaning of a Logarithm. Let us go back to the statement $3^4 = 81$.

In the index notation we said 4 is the index of the power

which 3 is raised to produce 81 ; 81 is the fourth power of 3 ; whilst 3 is the fourth root of 81.

We proceed now to give these particular numbers fresh names.

4 is called *the Logarithm of* 81 *to the base* 3.

3 is the base of the logarithm.

81 is the *Antilogarithm of* 4 *to the base* 3.

Again $10^3 = 1000$.

When dealing with indices we said 3 is the index of the power to which 10 is raised to produce 1000, whilst in logarithmic notation we say 3 *is the logarithm of* 1000 *to the base* 10, and this is written more shortly $\log_{10}1000 = 3$.

Hence the following statements are different in form but equivalent to one another.

(i) $10^3 = 1000$; (ii) $\sqrt[3]{1000} = 10$; (iii) $\log_{10}1000 = 3$.

Similarly we have

(i) $3^4 = 81$; (ii) $\sqrt[4]{81} = 3$; (iii) $\log_3 81 = 4$.

Now return to your graph of $y = 10^x$.

Example 1.

When $x = 0{\cdot}2$ $y = 1{\cdot}6$ (approx.), then $10^{0{\cdot}2} = 1{\cdot}6$, and we say $\log_{10}1{\cdot}6 = 0{\cdot}2$.

Example 2.

When $y = 9{\cdot}1$ $x = 0{\cdot}96$ (approx.), then $10^{0{\cdot}96} = 9{\cdot}1$ or $\log_{10}9{\cdot}1 = 0{\cdot}96$, and in general if $10^x = y$ we say $\log_{10}y = x$.

8. *Definition of Logarithm.* — The *logarithm* of a number to a given base is the *index* of the power to which the base must be raised to produce the number. *Or*, if a number N can be expressed in the form a^x, the index x is called the *logarithm* of the *number* N to the *base* a.

More shortly if $a^x = N$, then $\log_a N = x$.

Any number might be taken as base, and the logarithms of all positive numbers to any given base are known as a *System of Logarithms.*

In all practical calculations the system in use has 10 for its base. The advantage of this system will appear later.

Logarithms to the base 10 are known as *Common Logarithms.*

Example 1.

Find the value of $\log_2 16$.

We have to find the index of the power to which 2 must be raised to produce 16, or

If x be the required index or log,

$$2^x = 16.$$

But $16 = 2^4$.

$\therefore$ $2^x = 2^4$

and $\therefore$ $x = 4$, whence $\log_2 16 = 4$.

EXAMPLE 2.

Find $\log_{10} 0{\cdot}001$.

If we suppose x be the required logarithm, then we must have $10^x = 0{\cdot}001$.

But $0{\cdot}001 = \dfrac{1}{1000} = \dfrac{1}{10^3} = 10^{-3}$ (by indices).

Then $10^x = 10^{-3}$

and $x = -3$.

$\therefore$ $\log_{10} 0{\cdot}001 = -3$.

EXAMPLE 3.

If $a^n = P$ find the value of n when $a = 0{\cdot}3$. $P = 0{\cdot}027$.

Here $(0{\cdot}3)^n = 0{\cdot}027$.

Now $27 = 3^3$

and $0{\cdot}027 = (0{\cdot}3)^3$.

Hence $(0{\cdot}3)^n = (0{\cdot}3)^3$

and $n = 3$.

$\therefore$ $3 = \log_{0{\cdot}3} 0{\cdot}027$.

EXAMPLE 4.

What is the value of $\log_a P$ when $a = 216$ $P = 6$?

Suppose x is the required logarithm.

Then $216^x = 6$.

Now $6 = \sqrt[3]{216} = 216^{\frac{1}{3}}$ (Indices).

$\therefore$ $216^x = 216^{\frac{1}{3}}$

and $\therefore$ $x = \frac{1}{3}$. Hence $\log_{216} 6 = \frac{1}{3}$.

39. *Fundamental Laws of Logarithms.* — Consider the table of values for x and 10^x for the graph $y = 10^x$ on page 75.

In logarithmic notation we call the values for x LOGARITHMS, and the corresponding values for y or 10^x are NUMBERS or *Antilogarithms.*

Thus we get—

$\log_{10} 1 = 0$
$\log_{10} 1{\cdot}33 = 0{\cdot}125$
$\log_{10} 1{\cdot}78 = 0{\cdot}25$
$\log_{10} 2{\cdot}37 = 0{\cdot}375$
$\log_{10} 3{\cdot}16 = 0{\cdot}5$
$\log_{10} 4{\cdot}22 = 0{\cdot}625$
$\log_{10} 5{\cdot}62 = 0{\cdot}75$
$\log_{10} 7{\cdot}50 = 0{\cdot}875$
$\log_{10} 10 = 1{\cdot}000$.

Using the graph (Fig. 6) we have :

EXAMPLE 1.

If $y=5$ or $1{\cdot}4$

$\text{Log}_{10}5 = 0{\cdot}7 \qquad \log_{10}1{\cdot}4 = 0{\cdot}15.$

Now $\text{Log}_{10}5 + \log_{10}1{\cdot}4 = 0{\cdot}7 + 0{\cdot}15 = 0{\cdot}85,$

ıd when x or the log. is $0{\cdot}85$, the corresponding value for
or the number whose logarithm is $0{\cdot}85$ is found to be 7.
Now 7 is the product of 5 and $1{\cdot}4$.

Hence $\text{Log}_{10}(5 \times 1{\cdot}4) = \log_{10}5 + \log_{10}1{\cdot}4$ . . . . . I.

EXAMPLE 2.

If $y = 9 \quad x = 0{\cdot}954$ (from graph).
If $y = 4 \quad x = 0{\cdot}6$,, ,,

hen $\text{Log}_{10}9 = 0{\cdot}954$ and $\log_{10}4 = 0{\cdot}6$.

Now $9 \div 4 = 2{\cdot}25,$

nd if $y = 2{\cdot}25 \qquad x = 0{\cdot}354$ (from graph),

ıen $\log_{10}2{\cdot}25 = 0{\cdot}354 = 0{\cdot}954 - 0{\cdot}6.$

Hence $\log_{10}\frac{9}{4} = \log_{10}9 - \log_{10}4.$ . . . . . II.

EXAMPLE 3.

If $y = 2{\cdot}9 \quad x = 0{\cdot}46,$

hen $\log_{10}2{\cdot}9 = 0{\cdot}46.$

Now $(2{\cdot}9)^2 = 2{\cdot}9 \times 2{\cdot}9.$

Hence $\log_{10}(2{\cdot}9)^2 = \log_{10}2{\cdot}9 + \log_{10}2{\cdot}9$ (Example 1)
$= 0{\cdot}46 + 0{\cdot}46 = 0{\cdot}92.$

From graph, when $x = 0{\cdot}92 \quad y = 8{\cdot}35,$

nd $(2{\cdot}9)^2 = 8{\cdot}35$ approx. by multiplication.

Then $\log_{10}(2{\cdot}9)^2 = 2\log_{10}2{\cdot}9$ . . . . . III.

EXAMPLE 4.

When $y = 9 \quad x = 0{\cdot}954$ from graph.

'hen $\log_{10}9 = 0{\cdot}954.$

Now $\sqrt{9} = 3,$

nd when $y = 3 \quad x = 0{\cdot}477$ from graph.

$\therefore \quad \log_{10}3 = 0{\cdot}477$ which we notice
$= \frac{1}{2}(0{\cdot}954) = \frac{1}{2}\log_{10}9.$

Then $\log_{10}\sqrt{9} = \frac{1}{2}\log_{10}9$ . . . . . IV.

This result we may also obtain from Example 3, thus

$\sqrt{9} = 9^{\frac{1}{2}}$ (by indices)

nd $\log_{10}9^{\frac{1}{2}} = \frac{1}{2}\log_{10}9$ (by Example 3).

Hence $\log_{10}\sqrt{9} = \frac{1}{2}\log_{10}9.$

Collecting the results I, II, III, and IV, we have :

I. The *Logarithm of a product* of any number of factors is the *sum of the logarithms* of the several factors.

II. The *Logarithm of a quotient* is obtained by *subtracting* the logarithm of the divisor from the logarithm of the dividend.

III. The *Logarithm of any power of a number* is obtained by *multiplying* the logarithm of the number *by the index of the power*.

IV. The *Logarithm of any root of a number* is obtained by *dividing* the logarithm of the number *by the index of the root*,

and *these are true whatever be the base of the logarithms.*

Thus

$$\left.\begin{aligned}\log_a xy &= \log_a x + \log_a y\\ \log_a xyz &= \log_a x + \log_a y + \log_a z\end{aligned}\right\} \quad \text{I.}$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y \quad \text{II.}$$

$$\left.\begin{aligned}\log_a x^3 &= 3\log_a x\\ \log_a x^p &= p \times \log_a x\end{aligned}\right\} \quad \text{III.}$$

$$\left.\begin{aligned}\log_a \sqrt{x} &= \tfrac{1}{2}\log_a x\\ \log_a \sqrt[4]{x} &= \tfrac{1}{4}\log_a x\\ \log_a \sqrt[p]{x} &= \tfrac{1}{p}\log_a x\end{aligned}\right\} \quad \text{IV.}$$

40. *Common Logarithms—Characteristic and Mantissa.*—For practical purposes 10 is chosen as the base of the system of logarithms. In writing down such logarithms the base is omitted, so that $log_{10}8$ is written *log* 8.

Now

$10^0 = 1$	and	log 1	$= 0$
$10^1 = 10$	,,	log 10	$= 1$
$10^2 = 100$	,,	log 100	$= 2$
$10^3 = 1000$	,,	log 1000	$= 3$
$10^{-1} = \frac{1}{10} = 0{\cdot}1$	,,	log 0·1	$= -1$
$10^{-2} = \frac{1}{100} = 0{\cdot}01$	,,	log 0·01	$= -2$
$10^{-3} = \frac{1}{1000} = 0{\cdot}001$	,,	log 0·001	$= -3$

Thus the logarithm of a number between

1	and 10, *i.e.* with 1 digit,	$= 0 +$ a fraction.
10	and 100, *i.e.* with 2 digits,	$= 1 +$ a fraction.
100	and 1000, *i.e.* with 3 digits,	$= 2 +$ a fraction.
0·1	and 1	$= -1 +$ a fraction.
0·01	and 0·1	$= -2 +$ a fraction.
0·001	and 0·01	$= -3 +$ a fraction.

As examples we find from the table of logarithms at the end of the book (how these are got will be shown later):—

Log 3·564	$= 0 + 0{\cdot}5519$	$= 0{\cdot}5519.$
Log 26·37	$= 1 + 0{\cdot}4211$	$= 1{\cdot}4211.$
Log 438·2	$= 2 + 0{\cdot}6417$	$= 2{\cdot}6417.$
Log 1876	$= 3 + 0{\cdot}2732$	$= 3{\cdot}2732.$
Log 0·6725	$= -1 + 0{\cdot}8277$	$= \bar{1}{\cdot}8277$
Log 0·07214	$= -2 + 0{\cdot}8581$	$= \bar{2}{\cdot}8581.$
Log 0·008319	$= -3 + 0{\cdot}9201$	$= \bar{3}{\cdot}9201$

From these examples we see that the logarithm of a number (omitting powers of 10) consists of two parts—(*a*) a whole number or integral part ; (*b*) a decimal part.

The whole number part of a logarithm is called its *characteristic* and as we see may be either positive or negative or zero.

The decimal part of a logarithm is called its *mantissa* and is *always positive*.

Thus in $\log 438{\cdot}2 = 2{\cdot}6417$, 2 is the characteristic, 0·6417 is the mantissa.

And in $\log 0{\cdot}6725 = \bar{1}{\cdot}8277$, -1 or $\bar{1}$ is the characteristic, ·8277 is the mantissa.

Turning again to our graph of $y = 10^x$, we find when $y = 0{\cdot}67$ $x = -0{\cdot}17$.

Does this agree with the above, where we have

$$\log 0{\cdot}6725 = \bar{1}{\cdot}8277\ ?$$

Remember we said that *the mantissa of a logarithm is alway positive.*

Then
$$\begin{aligned} \text{Log } 0{\cdot}6725 &= -1 + 0{\cdot}8277 \\ &= -(1 - 0{\cdot}8277) \\ &= -0{\cdot}1723, \end{aligned}$$
[Note use of brackets.]

which in this form agrees with our graph.

The question now arises, Why write $\bar{1}{\cdot}8277$ in preference to $-0{\cdot}1723$?

The following will perhaps make this clear.

1. *The Mantissæ of the Logarithms of Numbers are the same, if the numbers have the same significant figures.*

[Reminder.—The significant figures of a number are those which remain when all ciphers at the beginning and end have been removed.

Example 1.—The significant figures of 0·003106, 31·06, 3·106, 3106000 are 3106.

Example 2.—Using the digits 3147 and in that order we may have the numbers 3·147, 31·47, 314·7, 3147, 0·3147, 0·03147, etc.]

Now $\log 3{\cdot}147 = 0{\cdot}4979$ since 3·147 lies between 1 and 10

and $31{\cdot}47 = 3{\cdot}147 \times 10$,

then $\log 31{\cdot}47 = \log 3{\cdot}147 + \log 10$ ($\log ab = \log a + \log b$)
$= 0{\cdot}4979 + 1 = 1{\cdot}4979.$

$314{\cdot}7 = 3{\cdot}147 \times 100.$

$\therefore \log 314{\cdot}7 = \log 3{\cdot}147 + \log 100$
$= 0{\cdot}4979 + 2 = 2{\cdot}4979.$

$3147 = 3{\cdot}147 \times 1000.$

$\therefore \log 3147 = \log 3{\cdot}147 + \log 1000$
$= 0{\cdot}4979 + 3 = 3{\cdot}4979.$

$0{\cdot}3147 = 3{\cdot}147 \div 10.$

$$\therefore \quad \log 0{\cdot}3147 = \log 3{\cdot}147 - \log 10 \quad \left(\log \frac{a}{b} = \log a - \log b\right)$$
$$= 0{\cdot}4979 - 1 = \bar{1}{\cdot}4979.$$

$$0{\cdot}003147 - 3{\cdot}147 \div 1000.$$
$$\therefore \log 0{\cdot}003147 = \log 3{\cdot}147 - \log 1000$$
$$= 0{\cdot}4979 - 3 = \bar{3}{\cdot}4979.$$

Thus the logs. of all numbers having 3147 as the significant figures have the same mantissæ 0·4979 with some positive or negative characteristic added, and the same table which gives us logs. of 3·147, 31·47, etc. (numbers greater than 1) can be utilised for the logs. of 0·3147, 0·03147, etc. (numbers less than 1).

As another example :

Log 6·247	= 0·7957	since	6·247	lies	between	1	and	10.
Log 62·47	= 1·7957	,,	62·47	,,	,,	10	,,	100.
Log 624·7	= 2·7957	,,	624·7	,,	,,	100	,,	1000.
Log 62470	= 4·7957	,,	62470	,,	,,	10000	,,	100000.
Log 0·6247	= $\bar{1}$·7957	,,	0·6247	,,	,,	0·1	,,	1.
Log 0·006247	= $\bar{3}$·7957	,,	0·006247	,,	,,	0·001	,,	0·01.

42. *Manipulation of Logarithms.*—The method of manipulating the logarithms in the following examples should be very carefully noticed.

Example 1.

Given log 6·875 = 0·8373 ; log 1·325 = 0·1222 ;
log 3·064 = 0·4863.

To find the value of log (0·6875 × 13·25).

$$\text{Log } (0{\cdot}6875 \times 13{\cdot}25) = \log 0{\cdot}6875 + \log 13{\cdot}25$$
$$= \bar{1}{\cdot}8373 + 1{\cdot}1222$$
$$= -1 + 0{\cdot}8373 + 1 + 0{\cdot}1222$$
$$= 0{\cdot}9595.$$

Here we must remember that the fractional part of a logarithm is always positive and that $\bar{1}$·8373 stands for $-1 + 0{\cdot}8373$.

Example 2.

Find the value of log (687·5 × 0·003064).

$$\text{Log } (687{\cdot}5 \times 0{\cdot}003064) = \log 687{\cdot}5 + \log 0{\cdot}003064$$
$$= 2{\cdot}8373 + \bar{3}{\cdot}4863$$
$$= 2 + 0{\cdot}8373 - 3 + 0{\cdot}4863$$
$$= (2 - 3) + (0{\cdot}8373 + 0{\cdot}4863)$$
$$= -1 + 1{\cdot}3236$$
$$= 0{\cdot}3236.$$

Note. that the addition is similar to the addition of two decimal quantities, remembering—

$\bar{2}\cdot8373$
$3\cdot4863$
$0\cdot3236$

i. The carrying figure from the sum of the mantissæ is positive.

ii. The addition of (-3) is equivalent to a subtraction of 3.

Example 3.

Find the value of $\log \frac{1\cdot325}{68\cdot75}$.

$$\begin{aligned}\text{Log } \frac{1\cdot325}{68\cdot75} &= \log 1\cdot325 - \log 68\cdot75\\ &= 0\cdot1222 - 1\cdot8373\\ &= -1 + (0\cdot1222 - 0\cdot8373)\\ &= -1 + (-0\cdot7151)\end{aligned}$$

Here the mantissa is negative and must be re-written so that the decimal part is positive.

$$\begin{aligned}&= -1 + (1 - 0\cdot7151 - 1)\\ &= -1 + (0\cdot2849 - 1)\\ &= \bar{2}\cdot2849.\end{aligned}$$

Without going through all these steps we get the same result by subtracting the numbers as they stand, thus :

$0\cdot1222$
$1\cdot8373$
$\bar{2}\cdot2849$

After subtracting 9 from 11 in the tenths column we must "pay back" (?) 1 to the lower line in the units column. This is of course positive. Then 1 and 1 make 2 and 2 from 0 gives $\bar{2}$.

Notice again.

$$\begin{aligned}\text{Log } \frac{0\cdot6875}{0\cdot03064} &= \log 0\cdot6875 - \log 0\cdot03064\\ &= \bar{1}\cdot8373 - \bar{2}\cdot4863\\ &= -1 + (0\cdot8373) - (-2 + 0\cdot4863)\\ &= -1 + 2 + (0\cdot8373 - 0\cdot4863)\\ &= 1 + 0\cdot3510\\ &= 1\cdot3510.\end{aligned}$$

Following the ordinary method of subtracting we have

$1\cdot8373$
$2\cdot4863$
$1\cdot3510$

Here there is no carrying figure after subtracting 5 from 8 in the tenths column and subtracting 2 from 1 gives $+1$ for $-(-2) = +2$.

Example 4.

Find the value of $\log (0\cdot6875)^3$.

$$\begin{aligned}\text{Log } (0\cdot6875)^3 &= 3 \log 0\cdot6875 \quad (\log x^n = n \log x)\\ &= 3\,(\bar{1}\cdot8373) = 3(-1 + 0\cdot8373)\\ &= -3 + 2\cdot5119 = 1\cdot5119.\end{aligned}$$

or using ordinary method for multiplying $\bar{1}\cdot8373$ by 3 we have

$\bar{1}\cdot8373$
3
$\bar{1}\cdot5119$

Remembering that the mantissa is positive, when the 8 in the tenths column is multiplied by 3 the carrying figure is + 2, then 3 times $\bar{1}$ gives − 3 and with the carrying figure + 2 we get $\bar{1}$.

Example 5.

Find the value of $\log \sqrt[3]{0\cdot3064}$.

$$\begin{aligned}\text{Log } \sqrt[3]{0\cdot3064} &= \log (0\cdot3064)^{\frac{1}{3}} = \tfrac{1}{3}\log 0\cdot3064\\ &= \tfrac{1}{3}(\bar{1}\cdot4863)^{*} = \tfrac{1}{3}(\bar{3} + 2\cdot4863)\\ &= \bar{1}\cdot8287.\end{aligned}$$

* Here we have to divide $\bar{1}\cdot4863$ by 3. The characteristic $\bar{1}$ is not exactly divisible by 3. Therefore we subtract 2 from the characteristic and add 2 to the mantissa, thus keeping the value of the logarithm unaltered.

Example 6.

Divide $\bar{1}\cdot4772$ by 4.

$$\frac{\bar{1}\cdot4772}{4} = \frac{-1 + 0\cdot4772}{4} = \frac{-4 + 3\cdot4772}{4}$$
$$= -1 + 0\cdot8693$$
$$= \bar{1}\cdot8693.$$

43. *Use of Tables of Logarithms.*—Common logarithms or logarithms to the base 10 have two great advantages.

(i) *The characteristics can be written down by inspection.* Hence the characteristics need not be tabulated.

(ii) *The mantissæ are the same for the logarithms of all numbers which have the same significant digits.* Hence the tables need only contain the mantissæ of the logarithms of integers.

In the four-figure logarithm table at the end of the book we have *the mantissæ* of the logarithms of numbers from 1 to 9999.

The decimal points are not inserted in the table, but are understood to lie immediately before the first figure in all the logarithms given.

To find the value of log 756·4. We find the first two figures 75 on the left-hand side of the table; the next figure 6 is found at the top of the table. In the horizontal line opposite to 75 and in the column headed 6 we find the digits 8785. To account for the next figure 4 we use the table of differences on the right-hand side of the tables. In the horizontal line opposite 8785 and in the difference column headed 4 we find

the figure 2 which = 0·0002. This must be added to 8785, and the mantissa of the logarithm we are seeking is

$$0{\cdot}8785 + 0{\cdot}0002 = 0{\cdot}8787.$$

Since 756·4 lies between 100 and 1000, the characteristic is 2.

$$\therefore \quad \text{Log } 756{\cdot}4 = 2{\cdot}8787.$$

It will be noticed that the arrangement is somewhat different for numbers between 1 and 2000.

The logarithms here increase so rapidly with the numbers that two sets of differences are required.

Thus to find log 16·38 we first find the mantissa for

$$163 = 0{\cdot}2122.$$

The difference for the fourth figure 8 must now be sought in the horizontal line opposite 2122 already found. This difference = 0·0022.

$$\therefore \quad \text{Mantissa for } 1638 = 0{\cdot}2122 + 0{\cdot}0022 = 0{\cdot}2144.$$

Since 16·38 lies between 10 and 100 the characteristic is 1.

$$\therefore \quad \text{Log } 16{\cdot}38 = 1{\cdot}2144.$$

On the other hand, *to find log* 16·78 we find the mantissa for 167 = 0·2227, and the difference for the fourth figure 8 must now be sought in the lower horizontal line opposite 2227. The required difference = 0·0021.

$$\therefore \quad \text{Log } 16{\cdot}78 = 1{\cdot}2248.$$

44. *Antilogarithms.* — When the logarithm of a number is known and we wish to find the *number* or the *antilogarithm*, we may of course use the table of logarithms, but it is more expeditious to use a *table of antilogarithms.*

This table gives the *significant digits* of the numbers the mantissæ of whose logarithms range from 0 to 0·9999.

The arrangement is similar to that of the table of logarithms. Thus to find the number whose logarithm is $\bar{1}{\cdot}3674$.

Here the mantissa = 0·3674.

We find 0·36 on left-hand side of the table of antilogarithms ; next follow the horizontal line opposite, to the column headed 7 and find the digits 2328.

In a horizontal line with this and in the difference column headed 4 we find the digit 2.

This we add to 2328 and obtain 2330.

Now the logarithm is $\bar{1}{\cdot}3674$, which is greater than $\bar{1}$ and less than 0.

Now log 1 = 0 and log 0·1 = $\bar{1}$.

Then number lies between 0·1 and 1.

$$\therefore \quad \text{Antilog } \bar{1}{\cdot}3674 = 0{\cdot}2330.$$

Also $\quad \text{Antilog } 1{\cdot}3674 = 23{\cdot}30.$

The mantissa (0·3674) is the same as before and the antilog. has the same significant digits, but since the characteristic is 1, then the number required must lie between 10 and 100.

45. *Calculations using a Table of Logarithms.*

EXAMPLE 1.

Evaluate $0{\cdot}3618 \times 0{\cdot}027 \times 217{\cdot}9$.

Let $x = 0{\cdot}3618 \times 0{\cdot}027 \times 217{\cdot}9$.

Then $\log x = \log 0{\cdot}3618 + \log 0{\cdot}027 + \log 217{\cdot}9$

$= 1{\cdot}5585 + \bar{2}{\cdot}4314 + 2{\cdot}3383$

$= 0{\cdot}3282$.

$\therefore \quad x = \text{antilog } 0{\cdot}3282 = 2{\cdot}129$.

Rough approximation $x = 0{\cdot}4 \times 0{\cdot}03 \times 200$

$= 0{\cdot}012 \times 200 = 2{\cdot}4$

A rough approximation will check the magnitude of the result and lessen the danger of giving the answer 0·2129 or 21·29.

EXAMPLE 2.

Find the value of $2{\cdot}644 \div 0{\cdot}2863$.

Let $x = \dfrac{2{\cdot}644}{0{\cdot}2863}$.

Then $\log x = \log 2{\cdot}644 - \log 0{\cdot}2863$

$= 0{\cdot}4223 - \bar{1}{\cdot}4569 = 0{\cdot}9654$.

$\therefore \quad x = \text{antilog } 0{\cdot}9654 = 9{\cdot}234$.

(Rough approximation $x = \dfrac{2{\cdot}6}{0{\cdot}3} = \dfrac{26}{3} = 9$.)

EXAMPLE 3.

Evaluate $\dfrac{31{\cdot}35 \times 0{\cdot}003621}{2{\cdot}534 \times 76{\cdot}57}$

Let $x = \dfrac{31{\cdot}35 \times 0{\cdot}003621}{2{\cdot}534 \times 76{\cdot}57}$.

Then

$\log x = \log \text{ of numerator} - \log \text{ of denominator}$

$= (\log 31{\cdot}35 + \log 0{\cdot}003621) - (\log 2{\cdot}534 + \log 76{\cdot}57)$

$= (1{\cdot}4962 + \bar{3}{\cdot}5588) - (0{\cdot}4038 + 1{\cdot}8841)$

$= \bar{1}{\cdot}0550 - 2{\cdot}2879 = \bar{4}{\cdot}7671$.

$\therefore \quad x = \text{antilog } \bar{4}{\cdot}7671 = 0{\cdot}0005849$.

(Roughly $x = \dfrac{31 \times 0{\cdot}0036}{3 \times 77} = 0{\cdot}0005$.)

Example 4.

The sum of all the edges of a cube is 73·96 cms. Find its volume.

A cube has 12 edges.

Then length of one edge $= \frac{73{\cdot}96}{12}$ cms.,

and if length of one edge $= l$ units of length,

Volume of cube $(V) = l^3$ units of volume.

$$\therefore \quad V = \left(\frac{73{\cdot}96}{12}\right)^3$$

and
$$\begin{aligned} \text{Log } V &= 3\{\log 73{\cdot}96 - \log 12\} \\ &= 3\{1{\cdot}8690 - 1{\cdot}0792\} \\ &= 3 \times 0{\cdot}7898 = 2{\cdot}3694. \end{aligned}$$

$$\therefore \quad V = \text{antilog } 2{\cdot}3694 = 234{\cdot}1 \text{ c.c.'s.}$$

(Rough estimate $V = \left(\frac{73{\cdot}96}{12}\right)^3 = \left(\frac{72}{12}\right)^3 = 6^3 = 216.$)

Example 5.

Evaluate $\sqrt[3]{0{\cdot}03615}$.

Let
$$\begin{aligned} x &= \sqrt[3]{0{\cdot}03615} \\ &= (0{\cdot}03615)^{\frac{1}{3}}. \end{aligned}$$

Then
$$\begin{aligned} \log x &= \tfrac{1}{3} \log 0{\cdot}03615 = \tfrac{1}{3}(\bar{2}{\cdot}5581) \\ &= \tfrac{1}{3}(\bar{3} + 1{\cdot}5581) = \bar{1}{\cdot}5194. \end{aligned}$$

$$\therefore \quad x = \text{antilog } \bar{1}{\cdot}5194 = 0{\cdot}3307.$$

(Rough check. Here it is difficult to estimate $\sqrt[3]{0{\cdot}03615}$ even roughly. But we may see if the answer *cubed* is nearly equal to 0·03615.

$x = 0{\cdot}3307 = 0{\cdot}3$ approx.

$x^3 = (0{\cdot}3)^3 = 0{\cdot}027.$)

Example 6.

A worsted warp which contains 2346 threads has to be woven in a 64/2 sett Bradford. What width should it be in the reed ?

Now 1's sett Bradford means 40 ends in 36 inches. Since Bradford system sleys 2 threads per split 64/2 sley sett Bradford means (64 × 40) ends in 36″ or 2560 ends in 36 inches.

But the warp has 2346 ends.

$$\therefore \quad \text{Width in reed} = \frac{2346}{2560} \text{ of } 36''.$$

Let $$x = \frac{2346 \times 36}{2560}$$

$$\log x = \log 2346 + \log 36 - \log 2560$$
$$= 3{\cdot}3703 + 1{\cdot}5563 - 3{\cdot}4082$$
$$= 4{\cdot}9266 - 3{\cdot}4082 = 1{\cdot}5184.$$

$$\therefore \quad x = \text{antilog } 1{\cdot}5184 = 32{\cdot}99.$$

Then width in reed = 33 inches.

CHAPTER V

The Slide Rule

46.—Elementary Explanation. 47.—Laws of Logarithms. 48.—To mark divisions on a scale proportional to the Logarithms of Numbers. 49.—How to use the Cardboard Scales for Multiplication. 50.—How to use the Cardboard Scales to find $9 \div 36$ 51.—How to use the Cardboard Scales to find $3{\cdot}5 \times 4$. 52.—How to use the Cardboard Scales to find $5{\cdot}6 \div 7$. 53. The Slide Rule—Description of. 54.—How the Slide Rule is used for : (*a*) Multiplication—Use of Cursor ; (*b*) Division ; (*c*) Combined Multiplication and Division. 55.—To use the Rule to Change Worsted Counts to Cotton Counts and *vice versa*. 56.—To use the Rule to change "Sett" Bradford to "Ends per inch" and *vice versa*. 57.—Squares and Square Roots using a Slide Rule. 58.—Cubes and Cube Roots using a Slide Rule.

46. *Elementary Explanation.*—If we have two rulers graduated alike we may make use of them for the addition and subtraction of lengths. In the diagram, AB and CD represent two rulers graduated in cms. and mms. Suppose AB is kept fixed whilst CD slides along the edge of AB.

If CD slides so that its zero end C coincides with the 4 cm.

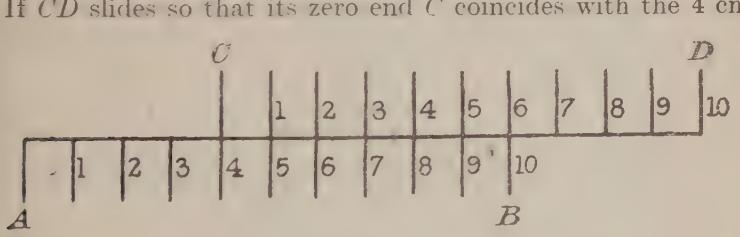


Fig. 6*a*.

mark on AB, then underneath the 1 on the slide CD we find on AB the result of $4 + 1$, *i.e.* 5 ; underneath the 2 of the slide we find the result of $4 + 2$, *i.e.* 6 ; underneath the 6 of the slide we find the result of $4 + 6$, *i.e.* 10, and so on.

In the same way we can subtract. If for example we wish to subtract 5 from 9 we move the slide CD until the 5 cm. mark on CD coincides with the 9 cm. mark on the fixed rule AB, then underneath the left-hand end of CD we find the result of $9 - 5$, *i.e.* 4 on AB.

The setting of the rulers in the diagram gives the result of (5 — 1), (6 — 2), (7 — 3), (8 — 4), (10 — 6).

47. *Laws of Logarithms.*—In the last chapter we learnt that :

$$(1)\ \log(abc) = \log a + \log b + \log c.$$

$$(2)\ \log \frac{a}{b} = \log a - \log b.$$

$$(3)\ \log a^2 = 2 \log a.$$

$$(4)\ \log a^3 = 3 \log a.$$

$$(5)\ \log \sqrt{a} = \tfrac{1}{2} \log a.$$

$$(6)\ \log \sqrt[3]{a} = \tfrac{1}{3} \log a.$$

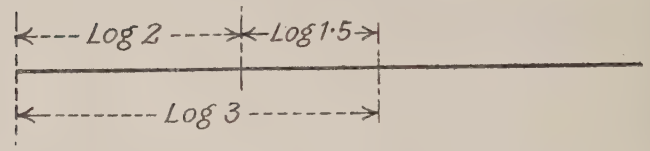


Fig. 6*b*.

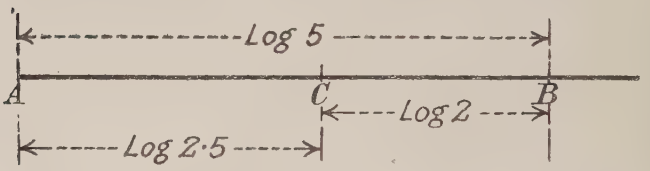


Fig. 6*c*.

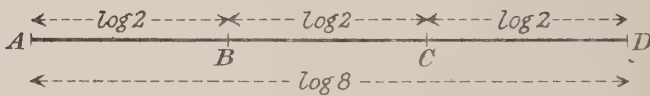


Fig. 6*d*.

So that in dealing with logarithms, multiplication is replaced by addition and division by subtraction.

Thus in Fig. (6*b*), if AB represents log 2 to some scale and BC represents log 1·5 to the same scale, then AC represents log 3, for

$$\begin{aligned} AC &= AB + BC \\ &= \log 2 + \log 1{\cdot}5 \\ &= \log 3. \end{aligned}$$

Similarly in Fig. (6*c*), if AB represents log 5 and BC represents log 2, then AC represents log 2·5, for

$$\begin{aligned} AC &= AB - BC \\ &= \log 5 - \log 2 \\ &= \log \tfrac{5}{2} \\ &= \log 2{\cdot}5. \end{aligned}$$

Again in Fig. (6*d*), if AB represents log 2, then AD represents log 8, for

$$AD = 3\,AB$$
$$= \log 2 + \log 2 + \log 2$$
$$= \log (2 \times 2 \times 2)$$
$$= \log 8.$$

And if AD represents log 8, AB represents $\frac{1}{3}$ log 8 or log $\sqrt[3]{8}$, *i.e.* log 2.

If then the rulers AB and CD (Fig. 6*a*) be graduated in the proportion of the logarithms of numbers, we can by sliding one scale along the other add the logarithms of two or more numbers, and the result will be the logarithm of their product. Subtraction can be performed, and in this case the difference will be the logarithm of the quotient of the two numbers.

48. *To mark divisions on a scale proportional to the Logarithms of Numbers.*—Use squared paper marked in cms. and mms.

Now $10^0 = 1$ and $10^1 = 10$

or $\log 1 = 0$ and $\log 10 = 1$.

Take 20 cms. or 200 squares to represent log 10, *i.e.* 1.

Then 20 squares represent 0·1

2 „ „ 0·01

1 square represents 0·005.

If AB is the line to be divided (Fig. 6*e*)

Mark the end A as 1,

„ „ „ B as 10,

since $AB = 1$, *i.e.* log 10.

Using a table of logarithms we mark off distances to represent log 2, log 3, log 4, etc.

Thus Log 2 = 0·3010, then find the point C 60 squares from A, and $AC = 0{\cdot}30$ of AB (the unit) and thus represents log 2.

Again Log 3 = 0·4771, then find the point D 95·4 squares from A, and $AD = 0{\cdot}477$ of the unit AB and thus represents log 3.

In a similar manner mark distances from A to represent log 4, 5, 6, 7, 8, 9, and draw the lines above and below AB.

Fig. 6*e*.

Notice the distances between succeeding marks diminish as we proceed from 1 to 10.

The distance from 1 to 3 is *not twice* the distance from 1 to 2, but

$$\frac{\text{Distance from 1 to 3}}{\text{Distance from 1 to 2}} = \frac{\log 3}{\log 2} = \frac{0{\cdot}4771}{0{\cdot}3010}.$$

The distance from 1 to 4 is *twice* the distance from 1 to 2, since

$$\log 4 = \log 2^2 = 2 \log 2,$$

The distance from 1 to 8 is *three times* the distance from 1 to 2, since $\log 8 = \log 2^3 = 3 \log 2$.

Intermediate graduations may now be obtained.

Thus $\log 1{\cdot}1 = 0{\cdot}0414$ represented by 8·2 squares (AK),
$\log 1{\cdot}2 = 0{\cdot}0792$,, ,, 15·8 ,, (AL),

and so on.

Since the distance between 1 and 2 is considerable we may with a little care mark off logs 1·01, 1·02, 1·03, 1·04, etc., so as to get 100 divisions between 1 and 2.

Between 2 and 3 or 3 and 4 we may get 50 divisions showing logs 2·02, 2·04, 2·06, 2·08, 2·1, 2·12, etc.

Between the rest we must be content with 10 divisions for clearness. Now cut the scale carefully along the line AB and paste the strips on two strips of cardboard and you have the parts of a *Simple Slide Rule.*

49. *How to use the Cardboard Scales for Multiplication.*—Let us use the two cardboard scales to find the value of $1{\cdot}2 \times 2{\cdot}75$.

(*a*) One of the scales is moved until the graduation 1 is over the graduation 1·2 of the lower scale ; then looking under the 2·75 graduation on the upper scale we find the product 3·3 on the lower scale.

The reason is clear from Fig. (6*f*).

Thus $\log 1{\cdot}2 = AB$ on lower scale
$\log 2{\cdot}75 = BC$ on upper ,,
$\therefore \quad \log (1{\cdot}2 \times 2{\cdot}75) = \log 1{\cdot}2 + \log 2{\cdot}75 = AB + BC$
$= AC = \log 3{\cdot}3.$
$\therefore \quad 1{\cdot}2 \times 2{\cdot}75 = 3{\cdot}3.$

(*b*) How shall we *set the scales to read the product* $1{\cdot}2 \times 27{\cdot}5$?

Referring to the lessons on Indices and Logarithms we have

$$1{\cdot}2 = 10^{0{\cdot}0792} \quad \text{or} \quad \log 1{\cdot}2 = 0{\cdot}0792$$
$$12 = 1{\cdot}2 \times 10 = 10^{0{\cdot}0792} \times 10$$
$$= 10^{(0{\cdot}0792+1)} \quad \text{or} \quad \log 12 = 1{\cdot}0792$$
$$0{\cdot}12 = \frac{1{\cdot}2}{10} = \frac{10^{0{\cdot}0792}}{10}$$
$$= 10^{(0{\cdot}0792-1)} \quad \text{or} \quad \log 0{\cdot}12 = \bar{1}{\cdot}0792, \text{ etc.}$$

In the figure (Fig. 6*g*) (not drawn to scale) the distance AA' represents 1 or log 10.

The distance AB represents 0·0792 or log 1·2
„ „ AC „ 0·4393 or log 2·75.

Continue the line to the right beyond A' and make $A'A'' = A''A''' = AA'$, and so to represent log 10. Make $A'B' = A''B'' = AB$ and $A'C' = A''C'' = AC$. Also produce AA' to the left to A_1 and make $AA_1 = AA'$; $A_1B_1 = AB$; $A_1C_1 = AC$.

Now if A_1A''' represents a scale graduated in proportion to the logarithms of numbers, let us see how we must number

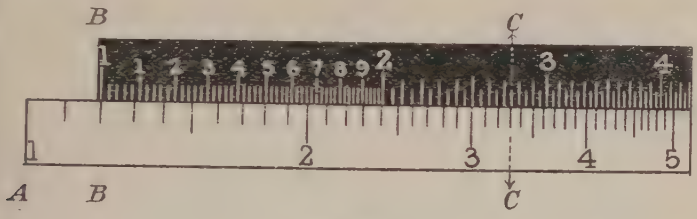


Fig. 6(*f*).

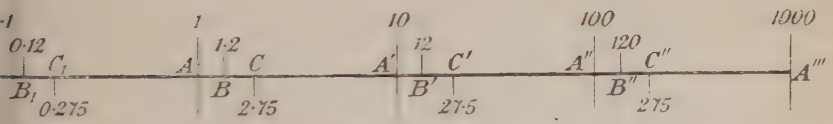


Fig. 6(*g*).

the graduation marks. A being the zero or starting point for the various measurements is marked 1, since log 1 = 0.

AA' is unit length and A' is marked 10 since log 10 = 1.

$AA'' = AA' + A'A'' = \log 10 + \log 10 = \log (10 \times 10)$ = log 100 and A'' is marked 100.

$AA''' = 3AA' = 3 \log 10 = \log (10 \times 10 \times 10) = \log 1000$ and A''' is marked 1000.

AA_1 being measured to the left represents -1 or $-\log 10$.

Now $-\log 10 = 0 - \log 10 = \log 1 - \log 10 = \log \frac{1}{10}$
$= \log 0{\cdot}1$,

and A_1 is marked 0·1.

Now $AB' = AA' + A'B' = AA' + AB$
$= \log 10 + \log 1{\cdot}2$
$= \log (10 \times 1{\cdot}2)$
$= \log 12$, and B' must be marked 12.

Similarly $AC' = AA' + A'C'$
$= AA' + AC$
$=$ log 10 + log 2·75
$=$ log 27·5 and C' must be marked 27·5.

Now reason out that B'' is marked 120 and C'' 275.

Thus we have a distance equivalent to log 10 between B and B'; C and C'; B' and B''; C' and C'', and also between B and B_1; C and C_1.

Now $AB_1 = AB - BB_1$

[*i.e.*, measure to the right of A a distance AB and then from B a distance BB_1 to the left]

$=$ log 1·2 $-$ log 10

$= \log \frac{1{\cdot}2}{10}$

$=$ log 0·12 and B_1 must be marked 0·12.

Similarly C_1 is marked 0·275.

Further, we may divide the lengths AA', $A'A''$, $A''A'''$ and AA_1 exactly as AB is divided in Fig. 6*e*, and corresponding points will be marked with numbers having the same significant digits but with the decimal point in different positions.

Thus 3·45 on AA' corresponds to
34·5 ,, $A'A''$
345 ,, $A''A'''$
0·345 ,, AA_1, and so on.

In AA' we shall have numbers from 1 to 10; in $A'A''$ 10 to 100; in $A'A'''$ 100 to 1000. Hence if the strips of cardboard (Fig. 6*f*) be made twice as long and marked with two sets of divisions; when the end 1 of the sliding strip is over 1·2 on the fixed strip, then 10 on the slide is over 12 on the fixed strip, and whereas 2·75 on the slide is over 3·3 on the lower strip, 27·5 on the extended slide will be over 33 on the extended fixed strip. The student will understand this if he will make these extended scales as shown, and then he will see that his first set of strips placed as in Fig. 6*f* will give him the products

$$\begin{aligned} 1{\cdot}2 \times 2{\cdot}75 &= 3{\cdot}3, \\ 1{\cdot}2 \times 27{\cdot}5 &= 33, \\ 12 \times 27{\cdot}5 &= 330, \\ 0{\cdot}12 \times 0{\cdot}275 &= 0{\cdot}033, \text{ etc.,} \end{aligned}$$

and just as our table of logarithms of numbers from 1 to 10 can be used to find log 5·876; log 58·76; log 587·6; log 0·5876, etc., so the single strips divided proportionally to the logarithms of numbers between 1 and 10 may be used to multiply or divide numbers less than 1 or greater than 10.

50. *To use the Cardboard Scales to find the Quotient* $\frac{9}{36}$.

Now $$\frac{9}{36} = \frac{9}{3{\cdot}6 \times 10} = \frac{9}{3{\cdot}6} \times \frac{1}{10}.$$

Move one of the scales until the graduation 3·6 is over the graduation 9 on the lower scale; then looking under the graduation 1 on the upper scale we find 2·5 on the lower scale.

From the figure we see (Fig. 6*h*)

$\log 9 = AB$ on the lower scale.

$\log 3{\cdot}6 = CB$ on the upper scale.

$$\therefore \quad \log \frac{9}{3{\cdot}6} = \log 9 - \log 3{\cdot}6$$
$$= AB - BC$$
$$= AC$$
$$= \log 2{\cdot}5.$$

$$\therefore \quad \frac{9}{3{\cdot}6} = 2{\cdot}5 \text{ and } \frac{9}{36} = 0{\cdot}25.$$

The scales give the answer 2·5, which is the quotient $\frac{9}{3{\cdot}6}$.

But 25 are the significant digits of

$\frac{9}{3{\cdot}6} = 2{\cdot}5$; $\frac{9}{36} = 0{\cdot}25$; $\frac{90}{36} = 2{\cdot}5$;

$\frac{90}{3{\cdot}6} = 25$, etc.

Also if we wish to find the product 42·5 × 12·4. We calculate mentally that the product = 43 × 12 = 516 approx. Then we proceed to add log 4·25 on the slide scale to log 1·24 on the fixed scale and find 5·27 for the product (4·25 × 1·24).

Hence product required is 527.

Fig. 6*h*.

51. *To use the Cardboard Scales to find the product* (3·5 × 4).—Fix the lower scale and move the upper scale till the graduation 1 is over the graduation 3·5 on the lower scale. Find the graduation 4 on the upper scale, and we notice it is beyond the end of the lower fixed scale as in Fig. 6*k*.

If the lower scale were extended so as to show graduation

marks for numbers between 10 and 100, then we should find the answer under the graduation 4 on the upper sliding scale.

Imagine this lower scale extended and we see that the graduation 10 of the upper scale will be above 35 on the extended lower scale. Now bring graduation 10 of the upper scale above the graduation 3·5 on the lower scale as in Fig. 7 and we have a picture of the upper scale in Fig. 6*k* as it would appear above the extension of the lower scale. Below graduation 4 on the upper scale (Fig. 7) we find 1·4 on the lower

Fig. 6*k*.

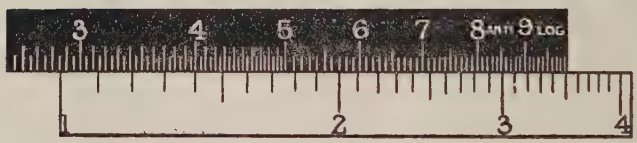


Fig. 7.

scale, which is the same distance from graduation 1 as 14 is from graduation 10,
since $\log 14 = \log (1{\cdot}4 \times 10) = \log 10 + \log 1{\cdot}4$.

Note.—Using the 10 graduation or right-hand end of the movable scale in place of the 1 graduation or left-hand end has the effect of dividing by 10, and we get 1·4 in place of 14.

52. *To use the Cardboard Scales to find the Quotient* $\frac{5{\cdot}6}{7}$.

—Move the upper scale so that the graduation 7 is over the graduation 5·6 on the lower scale.

Since $$\log \frac{5{\cdot}6}{7} = \log 5{\cdot}6 - \log 7$$
$$= AB - BC \text{ in Fig. 8,}$$

we look for the answer on the lower scale below the graduation 1 of the upper movable scale.

Here again we fail to find the answer in the ordinary way, since the graduation 1 on the slide is beyond the lower fixed scale.

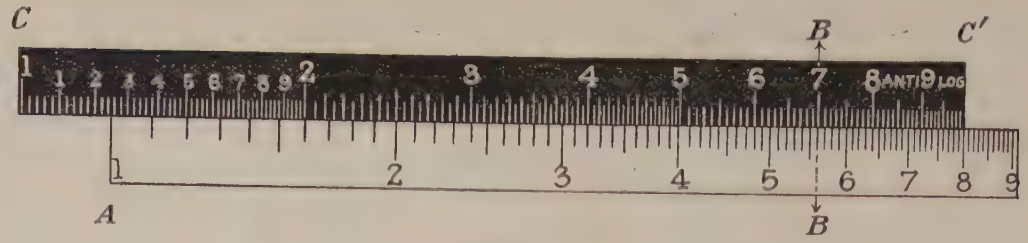


FIG. 8.

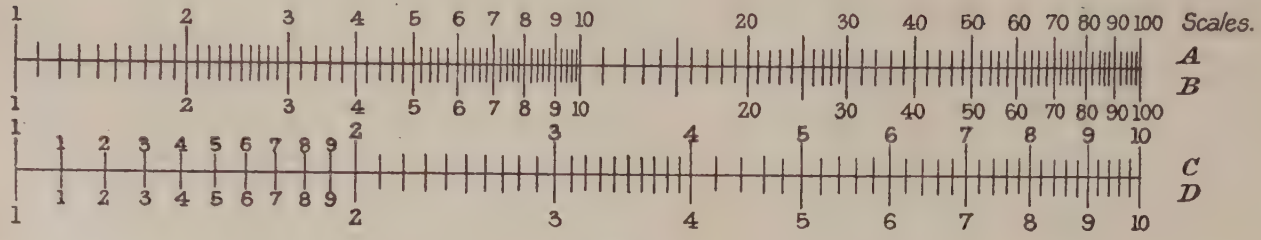


FIG. 9.

We get over the difficulty by adding log 10 or the length CC'.

For $$\log\left(\frac{5{\cdot}6}{7} \times 10\right) = \log 5{\cdot}6 - \log 7 + \log 10$$
$$= AB - BC + CC',$$

and we get an answer with the same significant figures but 10 times too large.

Then below the graduation 10 on the slide (C') we find 8 on the lower fixed scale,

and $$\frac{5{\cdot}6}{7} \times 10 = 8 \text{ or } \frac{5{\cdot}6}{7} = 0{\cdot}8.$$

Hence when dividing, if the left-hand end is off the fixed scale, we note the reading below the right-hand end.

Before proceeding further it will repay the student to use his cardboard scales to work numerous simple multiplications and divisions, the answer to which he can check mentally.

Thus he might try to work examples like the following :—

3×2 ; $1{\cdot}5 \times 3$; $2{\cdot}7 \times 3$; $6{\cdot}8 \times 2$; $12 \times 2{\cdot}5$; $1{\cdot}3 \times 7$; 27×4 ; 5×5 ; 8×8 ; $0{\cdot}8 \times 0{\cdot}7$; $0{\cdot}8 \times 1{\cdot}2$; $2{\cdot}5 \times 2{\cdot}5$.

$6 \div 2$; $9 \div 3$; $9{\cdot}5 \div 5$; $8{\cdot}4 \div 12$; $8{\cdot}4 \div 1{\cdot}2$; $9{\cdot}6 \div 1{\cdot}2$; $10 \div 2{\cdot}5$; $3{\cdot}6 \div 6$; $55 \div 11$; $72 \div 6$; and many more.

53. *The Slide Rule—Description of.*

(*a*) The instrument specially referred to here is the "Jackson-Gravet" Rule made by Messrs. Jackson Bros., Ltd., Armley, Leeds, and consists of three parts—the Ruler, the Slide, and the Cursor.

As it is of the ordinary type, most of what follows will apply to any ordinary slide-rule.

Supposing the student has one of these rules in front of him he will notice four scales on the front as in Fig. 9 (markings not fully shown).

These are simply tables of common logarithms plotted to scale.

The *two upper scales—Scale A* on the ruler and *Scale B* on the slide—are the logarithms of the numbers 1 to 100 plotted to the scale :—12·5 cms. represent 1·00, *i.e.* log 10.

The *two lower scales—Scale C* on the slide and *Scale D* on the ruler—are the logarithms of the numbers 1 to 10 plotted to the scale :—25 cms. represent 1·00, *i.e.* log 10.

It is important to observe that Scales C and D are marked on exactly *twice* the scale used for marking the upper scales A and B.

(*b*) *How to read the Scales.*—Take the *upper scales A and B.* On these we find the numerals

1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1.

These distances measured from the left-hand end of the scales represent the logarithms of

1 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 90 100,

or, as we have tried to show in § 49(*b*), they may represent the logarithms of

0·1, 0·2, 0·3 . . . 1, 2, 3, 4 . . . 10, *or*
10, 20, 30 . . . 100, 200, 300, 400, . . . 1000 etc.

The distance between 1 and 2 is divided into 50 parts (not equal parts—why ?) and if we commence at 1 these parts are read

1·00, 1·02, 1·04, 1·06, 1·08, 1·1, 1·12, . . . 1·98, 2·00.

Between 2 and 5 the graduations correspond to numbers which increase 0·05 at each step. Thus between 2 and 3 we have

2·00, 2·05, 2·10, 2·15, 2·20, 2·25 . . . 3·00.

Between 5 and 10 the graduations correspond to numbers which increase by 0·1 at each step.

Now notice the *lower scales C and D.*

On these we find distances marked proportional to the logarithms of

1 2 3 4 5 6 7 8 9 10 (marked 1).

Between 1 and 2 we have 100 divisions, and the graduations are read as

1·00, 1·01, 1.02, 1·03 . . . 1·05 . . . 1·10, 1·11 . . . 1·19, 1·20 . . . 1·25 . . . 2·00.

Between 2 and 4 the graduations correspond to numbers which increase 0·02 at each step and we read

2·00, 2·02, 2·04, 2·06, 2·08, 2·10 . . . 2·98, 3·00.

Between 4 and 10 the graduations correspond to numbers which increase 0·05 at each step.

The student will note that the divisions on all four scales A, B, C and D read to three figures, whilst four figure readings are obtainable by subdividing the spaces between the divisions " by the eye."

(*c*) *The Cursor* is a rectangular frame of aluminium with a glass front on which is engraved a black line at right angles to the length of the rule. The frame is made to slide freely in grooves and a small steel spring on the upper part of the framework ensures that the black hair line shall keep parallel to the graduation marks on the slide and ruler.

54. *How the Slide-Rule is used.*

(*a*) *Multiplication.*—For greater accuracy use scales C and D.

Example 1.

Find the value of $19{\cdot}6 \times 2{\cdot}74$.

Set the left-hand 1 on C over 1·96 on the D scale. Then find 2·74 on C and below we read 5·37 on D.

[Now $19{\cdot}6 \times 2{\cdot}74 = 20 \times 3 = 60$ approx.]

Hence $19{\cdot}6 \times 2{\cdot}74 = 53{\cdot}7$ to 3 sig. figs.

Example 2.

Find the value of $6{\cdot}85 \times 4{\cdot}85$.

Placing the left-hand 1 of the C scale over 6·85 on the D scale we find that 4·85 on the C scale is off the ruler. In a case like this we bring the right-hand 1 of the C scale (§ 51) over 6·85 on the D scale and under 4·85 on the C scale we read 3·322 on the D scale (*estimating the fourth figure*).

[Now $6{\cdot}85 \times 4{\cdot}85 = 7 \times 5 = 35$ (roughly).]

Hence $6{\cdot}85 \times 4{\cdot}85 = 33{\cdot}22$.

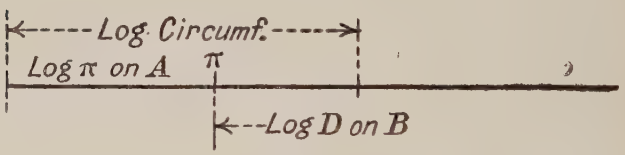


Fig. 9*a*.

Example 3.

Find circumferences of circles whose diameters are 12·5, 0·375 inches.

Let us use Scales A and B.

Bring the left hand 1 of B below π (special graduation mark) on A. Find 1·25 on B and above it read 3·92 on A.

[$12{\cdot}5 \times 3{\cdot}14 = 13 \times 3 = 39$ (roughly).]

Hence Circumf. $= 12{\cdot}5 \times \pi$ inches
$= 39{\cdot}2$ inches true to 3 figures.

Since for all circles

$$\text{Circumf.} = \pi \times \text{Diameter}$$
$$\text{Log C} = \log \pi + \log \text{D}.$$

If we bring the left-hand 1 of B under π on A and if we find the diameter on B we may read the circumference above it on the A scale (Fig. 9*a*).

The slide rule is thus *set* for finding the circumferences of circles given the diameters.

Thus if diameter = 0·375 inches

Circumf. = 1·177 estimating the fourth figure

= 1·18 to 3 figures.

Example 4.

(*b*) *Continued Multiplication—use of Cursor.*—Find the value of $2{\cdot}1 \times 3{\cdot}5 \times 2{\cdot}4 \times 4{\cdot}2$.

Use Scales C and D.

Place the left-hand 1 of C over 2·1 on D, then move the Cursor till the hair-line is over 3·5 on C; the partial product $(2{\cdot}1 \times 3{\cdot}5)$ is on D under the Cursor.

If now we move C until its left-hand 1 is under the Cursor we find that 2·4 on C is *off the rule*. In a case of this sort we bring the right-hand 1 of C under the Cursor. Now move the Cursor till it is over 2·4 on C. The partial product

$$(2{\cdot}1 \times 3{\cdot}5 \times 2{\cdot}4)$$

is under the Cursor on D.

Again move C till its left-hand 1 is under the Cursor and under 4·2 on C we read 7·4 on *D*.

[The product is approx. $2 \times 4 \times 2 \times 4 = 64$.]

$$\therefore \quad 2{\cdot}1\tfrac{1}{4} \times 3{\cdot}5 \times 2{\cdot}4 \times 4{\cdot}2 = 74.$$

(*c*) *Division.*

Example 1.

Divide 5·65 by 23·6.

Now $\frac{5{\cdot}65}{23{\cdot}6} = \frac{5{\cdot}65}{2{\cdot}36 \times 10}$, so that we shall obtain the significant figures of the required quotient if we divide 5·65 by 2·36.

(1) *Using Scales A and B.*

Move the slide so that 2·36 on B is below 5·65 on A and above 1 on B we read 2·4 on A.

(2) *Using Scales C and D.*

Move the slide so that 2·36 on C is above 5·65 on D and below the left-hand 1 on C we read 2·39 on D.

But $\frac{5{\cdot}65}{23{\cdot}6} = \frac{6}{24} = \frac{1}{4} = 0{\cdot}25$ roughly.

$\therefore$ Required quotient = 0·24 or 0·239 in the two cases respectively.

EXAMPLE 2.

Find the diameter of a circle whose circumference is 158 inches.

Now $\text{Diameter} = \frac{158}{\pi}$ inches.

Using Scales C and D.

Move the slide so that π (special marking) on C is above 1·58 on D and the left-hand 1 of C is off the rule. As explained when using the cardboard strips (§ 52) we use the right-hand 1 of C and find below it 5·03 on D.

But $\frac{158}{3{\cdot}1416} = \frac{158}{3} = 52$ approx.

$\therefore$ Required diameter = 50·3 inches correct to 3 sig. figs.

(*d*) *Combined Multiplication and Division.*

Find the value of $\frac{52{\cdot}4 \times 1{\cdot}41 \times 41{\cdot}5}{11{\cdot}3 \times 35{\cdot}6 \times 18{\cdot}9}$.

In a case like this the student will note the usefulness of the Cursor and he will find it *quicker* to divide and multiply alternately, thus :—

First Step.—Divide 5·24 *by* 1·13 by moving the slide so that 1·13 on C is over 5·24 on D. The quotient is on D under the left-hand 1 of C.

Second Step.—Multiply by 1·41 by *moving the Cursor* to 1·41 on C ; the product is on D under the Cursor.

Third Step.—Divide by 3·56 by *moving the slide* so that 3·56 on C is under the Cursor ; the quotient is on D under the left-hand 1 of C.

Fourth Step.—Multiply by 4·15 by *moving the Cursor* to 4·15 on C ; the product is on D under the Cursor.

Fifth Step.—Divide by 1·89 by *moving the slide* so that 1·89 on C is under the Cursor ; the final quotient is on D under the left hand 1 of C.

The final result is 4·03.

[The approx. value $= \frac{50 \times 1 \times 40}{10 \times 40 \times 20} = 0{\cdot}25$.]

Then Required value = 0·403.

55. *To use the Rule to change Worsted Counts to Cotton Counts and* vice versa.—Suppose we wish to change x's worsted counts to their equivalent in cotton counts.

x's worsted means 560 x yards weigh 1 lb.

The cotton hank is 840 yards and the number of these hanks which weigh 1 lb. is the count.

$$\therefore \quad \text{Equivalent cotton counts} = \frac{560\,x}{840} = \frac{2 \times x}{3}.$$

Hence given any value for x (worsted counts) we must multiply by 2 and divide by 3.

Then move the slide till 3 on C is over 2 on D.

If $x = 60$ we find the equivalent cotton counts on D under 6 on C.

The number found is 4 and equivalent cotton counts = 40's.

If $x = 24$ we find 1·6 on D under 2·4 on C,

and 24's worsted = 16's cotton.

To change 36's cotton to worsted counts we find 5·4 on C over 3·6 on D, and 36's cotton = 54's worsted, and so on.

With this setting of the slide and ruler,

If any number on C be taken as worsted counts, then the number on D immediately under will be the equivalent cotton count and *vice versa*.

56. *To use the Rule to change "Sett Bradford" to Ends per inch and* vice versa.—x's "sett Bradford" means x setts of 40 ends in a width of 36 inches.

$$\therefore \quad \text{Number of ends per inch} = \frac{40 \times x}{36} = \frac{10 \times x}{9}.$$

Hence given any value for x (sett Bradford) we must multiply by 10 and divide by 9.

Then move the slide so that 9 on C is over 10 on D.

If $x = 63$ (sett Bradford), then at 6·3 on C we find 7 on D.

$\therefore$ Number of ends per inch = 70.

If $x = 40$ (sett Bradford) under 4 on C we find 4·44 on D.

$\therefore$ Ends per inch = 44·4.

To change 50 ends per inch to Bradford setts we find 5 on D and immediately over it we have 4·5 on *C*.

Then 50 ends per inch = 45's sett Bradford.

Hence with this setting of the slide and ruler,

If any number on C be taken as sett Bradford, then the number on D immediately under will be the corresponding number of ends per inch, and *vice versa*.

57. *Squares and Square Roots: using a Slide Rule.*—In Fig. 10 we have placed Scale D immediately below scale A. In our drawing D is drawn to a scale of 20 cms. represent 1 or log 10, whilst A is drawn to half this scale, *i.e.*, 10 cms. represent 1 or log 10.

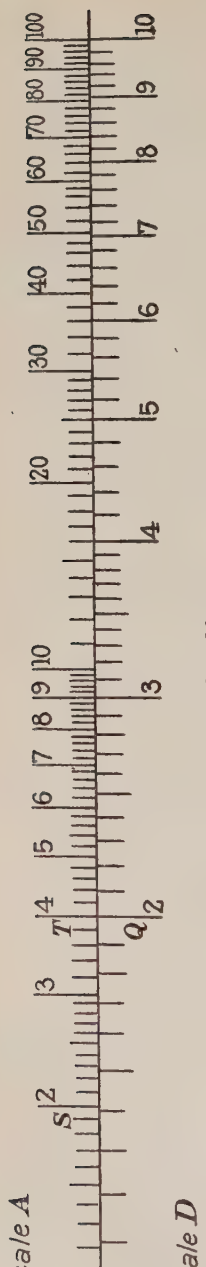


FIG. 10.

Then we notice—

(1) *RS* (representing log 2) on Scale A is only half the distance *PQ* (representing log 2) on Scale D.

(2) *RT* on Scale A $= 2 \times RS = 2 \log 2 = \log 4 = \log 2^2$.

(3) *RT* (representing log 2^2 on Scale A) $= PQ$ (representing log 2 on Scale D).

In Fig. 10*a* if *PQ* on Scale D represents log *a*, *RT*, which is the same length as *PQ*, will represent log a^2 on Scale A.

(i) Thus *above any graduation on the D Scale we find its square on the A Scale.*

EXAMPLES.

Place the cursor over 5 on D and it is over 25 on A.

Place the cursor over 2·5 on D and it is over 6·25 on A.

Place the cursor over 5·4 on D and it is over 29·2 on A.

Note.—Scale A reads from 1 to 100.

(ii) *Square Root.*—Readings on Scale A being *squares* of opposite readings on Scale D, then readings on Scale D are the *square roots* of opposite readings on A.

EXAMPLE 1.

Find the square root of 650.

Here we have to decide whether to set the cursor over 6·5 or 65 on A.

If we set the cursor over 6·5 on A we find 2·55 on D.

If we set the cursor over 65 on A we find 8·06 on D.

This small difficulty always occurs, but is easily met if we picture the extended slide-rule again.

Since we can use the single Scale A on the ruler for numbers which occur in the extended scale (§ 49 (*b*)), the diagram (Fig. 11) shows that the *first half* of A must be *used for numbers* 0·01 to 0·1; 1 to 10; 100 to 1000; 10,000 to 100,000, whilst

the *second half* of A must be *used for numbers* 0·1 to 1 ; 10 to 100 ; 1000 to 10,000 ; 100,000 to 1,000,000, etc.

Returning to our example : — 650 lies between 100 and 1000. So we must use 6·5 on the first half of A, and opposite on D we read 2·55.

But remember when A reads from 100 to 10,000, D reads from 10 to 100.

Hence $\sqrt{650} = 25{\cdot}5$.

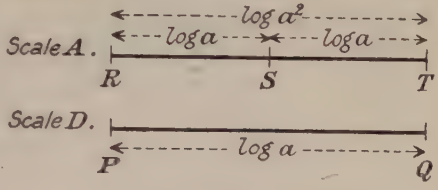


FIG. 10a.

EXAMPLE 2.

Find the square root of 0·76.

Here 0·76 lies between 0·1 and 1, then we must use the second half of Scale A.

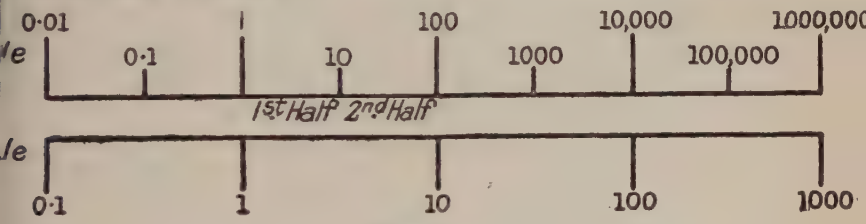


FIG. 11.

Place the cursor over 76 on A and we find it is over 8·72 on D.

But when A reads from 0·01 to 1, D reads from 0·1 to 1.

$\therefore \quad \sqrt{0{\cdot}76} = 0{\cdot}872.$

N.B.—In most instances the student will be able to obtain an approximate square root by inspection, thus :

In Example 1 $25^2 = 625 \quad \therefore \quad \sqrt{650} = 25$ approx.

In Example 2 $0{\cdot}8^2 = 0{\cdot}64 \quad \therefore \quad \sqrt{0{\cdot}76} = 0{\cdot}8$ approx.

He thus determines *which half of Scale A to use.*

EXAMPLE 3.

Find the area of a circle 2·5 feet diameter.

$$\text{Area of Circle} = \pi R^2$$
$$= \pi \frac{D^2}{4}$$
$$= 0{\cdot}7854\ D^2.$$

Set the cursor to 2·5 on Scale D and under it on Scale A we find 6·25 which is $(2{\cdot}5)^2$. Move the slide so that 10 on Scale B is under 6·25 on A (or under the cursor), then above 7·854 on B (special mark) we find 4·9 on A.

$\therefore$ Area required $= 0{\cdot}7854 \times (2{\cdot}5)^2 = 0{\cdot}8 \times 6 = 4{\cdot}8$ approx.
$= 4{\cdot}9$ square feet.

Example 4.

Find the diameter of a circle whose area is 25 square feet.

Since $\text{Area} = 0{\cdot}7854\, D^2$

$$D^2 = \frac{\text{Area}}{0{\cdot}7854}$$

$$\text{and } D = \sqrt{\frac{\text{Area}}{0{\cdot}7854}}.$$

We must first divide 25 by 0·7854 thus :—

Move the slide till 7·854 (special mark) on Scale B is under 25 on Scale A. Above the right-hand 100 on B place the cursor and the quotient is under the cursor on A.

The square root of this quotient is under the cursor on Scale D.

Then Diameter $= \sqrt{\frac{25}{0{\cdot}7854}} = \sqrt{\frac{25}{0{\cdot}8}} = \sqrt{30} = 5{\cdot}5$ approx.
$= 5{\cdot}64$ feet.

58. *Cubes and Cube Root: using a Slide Rule.*

To set the Rule to find 3^3 and $\sqrt[3]{27}$.

Method 1.—Refer to Fig. 12.

(*a*) To find 3^3.

Move the slide so that left-hand 1 of Scale C is over 3 on Scale D, then 1 on Scale B is below 9 or 3^2 on Scale A.

For *RS* representing log 9 = *PQ* representing log 3 on a drawing of double scale.

Now find 3 on Scale B and immediately above on Scale A we read 27.

For since Scales B and A are alike,

$$RT = RS + ST = \log 9 + \log 3 = \log 27.$$

(*b*) To find $\sqrt[3]{27}$ we reverse the process.

Set the cursor to 27 on A. Bring a rough approximation to the root on Scale B under the cursor and move the slide to the right or left until the number on B under the cursor is the same as that found on D below the left-hand (or right-hand) 1 on C.

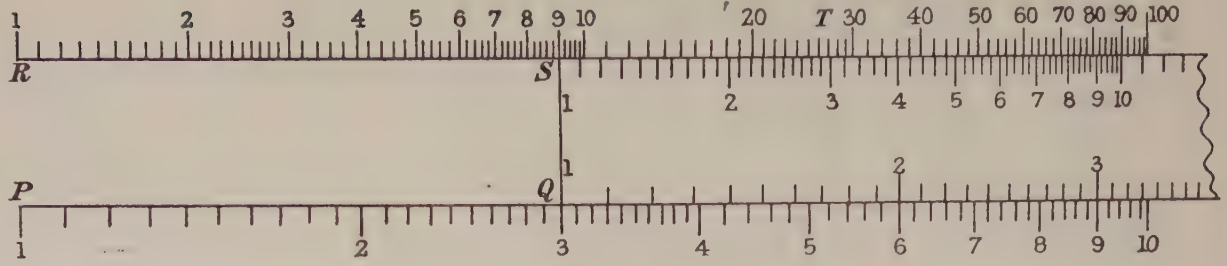

1 2 3 4 5 6 7 8 9 10 20 T 30 40 50 60 70 80 90 100
R
S
1 2 3 4 5 6 7 8 9 10
1 2 3
P
Q
1 2 3 4 5 6 7 8 9 10

FIG. 12.

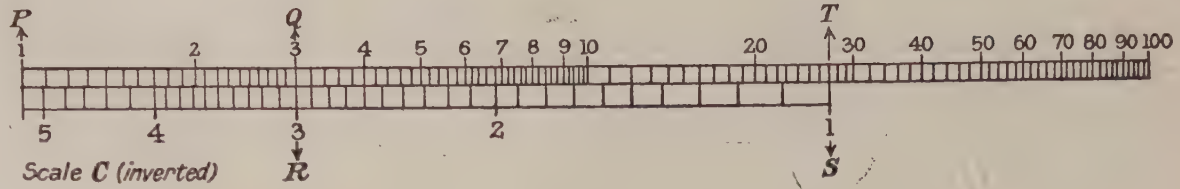

P
Q
T
1 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 90 100
5 4 3 2 1
Scale C (inverted)
R
S

FIG. 13.

Method 2. By inverting the slide.

(*a*) To find 3^3.

Pull out the slide, invert it, and replace it. Then Scale C will be under Scale A with its numbers inverted.

Refer to Fig. 13, which shows Scale A and Scale C (inverted).

Put 3 on Scale C under 3 on Scale A. Then opposite the right-hand (or left-hand) 1 of the Scale C will be found the cube required on Scale A—in this case 27.

Reason.—*PQ* on Scale A represents log 3.
RS on Scale C „ log 3 on its own scale or 2 log 3 on Scale A.
Then *PT* on Scale A represents log 3 + 2 log 3 or 3 log 3 or log 27.

(*b*) To find $\sqrt[3]{27}$.

Invert the slide as in (*a*). Bring the right-hand 1 of inverted Scale C under 27 on Scale A and then look what number on Scale C coincides with the same number on Scale A. This number is the cube root required. If the right-hand 1 of C does not supply the answer, use the left-hand 1.

When the foregoing examples are thoroughly understood, the student should have no difficulty in applying the "rule" to the special calculations which occur in his business work. Practice alone is necessary to *gain skill* in using it and *confidence* in the results obtained. A few exercises are given to afford this practice.

CHAPTER VI

Mensuration of Regular Figures

59.—Introductory Geometry. 60.—Relation between the Circumference and Diameter of a Circle. 61. Worked Examples on the Relation between C and D. 62.—Units of Length and Area. 63.—Area of a Rectangle. 64.—Area of a Parallelogram. 65.—Area of a Triangle. 66.—Area of a Trapezium. 67.—Pythagoras' Theorem. 68.—Area of a Circle. 69.—Area of a Circular Ring. 70.—Area of a Sector of a Circle. 71.—To show that $R^2 - r^2 = (R + r)(R - r)$. 72.—Area of an Irregular Figure. 73.—Worked Examples on Areas. 74.—Units of Volume and Capacity. 75.—Volume of a Rectangular Prism. 76.—Volume of a Cylinder. 77.—Volume of a Cone. 78.—Area of Curved Surface of a Cone. 79.—Volume of a Sphere. 80.—Area of Surface of a Sphere. 81.—Card Clothing—Fineness of. 82.—Worked Examples on Volumes and Surface Areas.

59. *Introductory Geometry.*—It is assumed that the student has been through a course of Practical Mathematics and Drawing and is acquainted with the meanings of such geometrical terms as Angle, Acute angle, Right angle, Obtuse angle, Parallel lines, Parallelogram, Rectangle, Square, Circle, Semicircle, etc. He will have learnt how to use (*a*) his protractor in the construction and measurement of angles; (*b*) his set squares in drawing parallel lines, perpendiculars, and

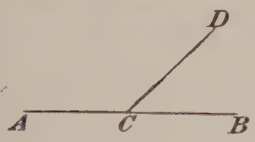


Fig. 14.

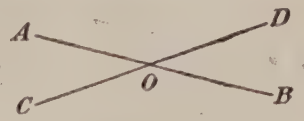


Fig. 15.

certain angles; (*c*) his compasses, in drawing circles, copying angles, etc.

He should be acquainted with the following *geometrical truths* which may be verified by drawing and measurement.

1. When two straight lines AB and CD (Fig. 14) meet, the *adjacent angles, BCD* and *DCA, are together equal to two right angles or* 180°.

2. When two straight lines AB and CD (Fig. 15) cross one

another at O, the *vertically opposite angles* AOC and BOD *are equal.* The other pair of vertically opposite angles AOD, BOC are also equal.

3. If a straight line PS cuts two parallel straight lines AB and CD (Fig. 16) and the eight angles be measured, it will be noticed that certain pairs of angles are equal.

Thus Angles PQB, QRD } are termed pairs of *corresponding angles* and are equal pairs.
,, PQA, QRC }
,, SRC, RQA }
,, SRD, RQB }

Angles AQR, QRD are equal } these are termed pairs of
,, BQR, QRC ,, } *alternate angles.*

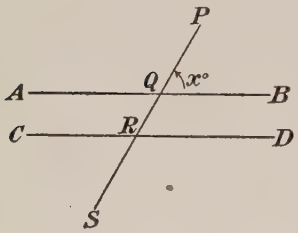


Fig. 16.

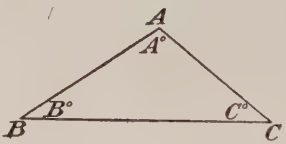


Fig. 17.

Fig. 18.

Hence if one of the angles is known, all the other angles can at once be calculated.

Thus if angle $PQB = x^0$
,, $PQA = (180 - x)^0$ using Fact (1) above.
,, $AQR = x^0$,, Fact (2) ,,
,, $BQR = (180 - x)^0$,, Fact (2) ,,
,, $QRD = x^0$ corresponding angle to PQB—Fact (3).
,, $QRC = (180 - x)^0$,, ,, PQA
,, $CRS = x^0$,, ,, AQR
,, $SRD = (180 - x)^0$,, ,, BQR

4. Draw any triangle ABC (Fig. 17) and measure the angles ABC, BCA, CAB, and notice *the sum of the three angles of a triangle is two right angles or* 180°.

Thus if angle $BAC = A^0$; angle $ABC = B^0$; angle $BCA = C^0$; $A + B + C = 180$.

Hence if two angles of a triangle be known, the third angle is at once calculated.

5. Draw an Isosceles triangle, *i.e.* a triangle with two of its sides equal. In Fig. 18 sides AB and AC are equal. Measure the angles ABC, ACB and notice they are equal.

Then "*the angles at the base of an isosceles triangle are equal.*"

Also bisect the angle BAC by a line AD.

Notice (1) AD is perpendicular to BC.
(2) AD bisects BC.

6. Draw an equilateral triangle ABC (Fig. 19), *i.e.* make all the sides the same length (2″). Measure the angles and notice *all the angles of an equilateral triangle are equal, each angle being* 60°.

7. (*a*) Draw two triangles ABC, DEF (Fig. 20), making $AB = DE = 2{\cdot}0''$; $BC = EF = 3{\cdot}0''$; $CA = FD = 2{\cdot}5''$.

Measure the angles of each triangle and notice the triangles are equiangular and the equal angles are opposite the equal sides. Cut the triangles out and notice they will fit on each other.

Then "*triangles which have their sides equal are equal in all respects.*"

(*b*) Draw two triangles ABC, DEF (Fig. 20), making $AB = DE = 2{\cdot}0''$; $BC = EF = 2{\cdot}5''$; angle ABC = angle $DEF = 56°$.

Measure the third sides AC and DF and also the other angles

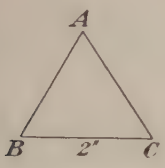


Fig. 19.

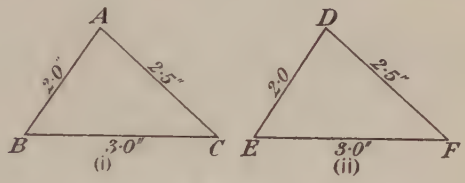


Fig. 20.

of each triangle. Also cut the triangles out and try to fit them, then we learn: "*if two triangles have two sides of the one equal to two sides of the other, each to each, and have also the angles included by those sides equal, then the triangles are equal in all respects.*"

(*c*) Again draw triangles ABC, DEF, making $BC = EF = 3{\cdot}0''$; angle ABC = angle $DEF = 56°$; angle ACB = angle $DFE = 41°$.

Measure the remaining sides and angle in each triangle and again try to fit them together.

From this we learn: "*if two triangles have two angles of the one equal to two angles of the other, each to each, and one side of one triangle equal to the corresponding side of the other, then the triangles are equal in all respects.*"

Hence the shape and size of a triangle is fixed if we know either (*a*) three sides, or (*b*) two sides and the included angle, or (*c*) one side and two angles.

8. Draw any parallelogram (Fig. 21) using your set squares. Measure the sides and angles, and verify that *the opposite sides and angles of a parallelogram are equal.*

Cut out your figure and then cut along a diagonal AC and notice the triangle ABC fits exactly on the triangle ADC. Then *the diagonal of a parallelogram bisects it.*

9. Draw a good-sized semicircle, ACB (Fig. 22), say of radius 6 cms. On the semicircumference take three or four points $C, D, E, \ldots$ and join each to A and B. Measure the angles

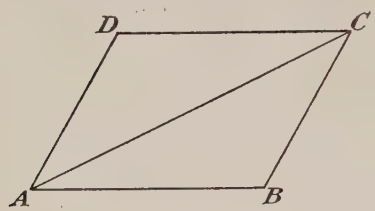


Fig. 21.

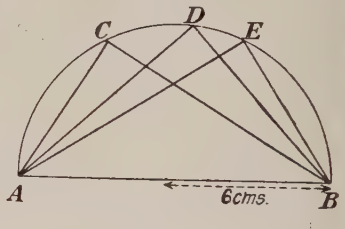


Fig. 22.

ACB, ADB, AEB, and learn that *the angle in a semicircle is a right angle.*

60. *Relation between the Circumference and Diameter of a Circle.*—Measure the circumference and diameter of some cylindrical object, *e.g.* a small pulley, a round ruler or roller, a penny.

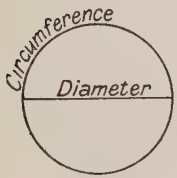


Fig. 23.

Find how many times the length of the diameter is contained in the length of the circumference.

A good average of several measurements is 3·14, which is roughly $3\frac{1}{7}$.

Now the number of times that the length of the circumference of a circle contains the length of its diameter cannot be expressed exactly, but theory shows that it is nearly

3·1415926 . . . (roughly $3\frac{1}{7}$).

The number 3·1416 is sufficiently accurate for practical purposes, and when four-figure logarithms are used we count it as 3·142.

This number is so important that a special symbol is used to denote it—the Greek letter π.

Hence if the circumference of a circle is C units long, and the diameter of the same circle is D units long,

Then $C = D \times 3{\cdot}1416 = D \times \pi$

$= \pi \,.\, D = 2\pi R$ if the radius $= R$ units.

Thus if D be known, C may be at once calculated,
also „ C „ D „ „ „ „
for $$D = \frac{C}{\pi}.$$

61. *Worked Examples on the Relation between the Circumference and Diameter of a Circle.*

EXAMPLE 1.

The diameter of a doffer of a carding engine is 40·5 inches. (*a*) What is its circumference? (*b*) If it is making 5 revolutions per minute, find the surface speed of the doffer.

(*a*) Circumference = Diameter $\times \pi$.
$(C) = 40{\cdot}5 \times 3{\cdot}1416$ inches.

Taking logs. of both sides of the equation,
$\text{Log } C = \text{Log } 40{\cdot}5 + \log 3{\cdot}142$
$= 1{\cdot}6075 + 0{\cdot}4972 = 2{\cdot}1047.$
$\therefore \quad C = \text{Antilog } 2{\cdot}1047 = 127{\cdot}3$ inches
$= 10{\cdot}61$ feet to nearest $\frac{1}{100}$ foot.

(*b*) By the surface speed of the doffer we mean the distance travelled by a point on the surface in a minute. If the cylinder be rolled on the floor till a mark on the surface, which is in contact with the floor, comes again in contact with the floor, and if there be no slipping, then the distance travelled along the floor is equal to the circumference of the cylinder or 10·61 feet in this case.

Distance travelled in 5 revolutions $= 10{\cdot}61 \times 5$ feet,
or Surface speed of doffer $= 10{\cdot}61 \times 5 = 53{\cdot}05$
$= 53$ feet per minute.

EXAMPLE 2.

Find the radius of a cylinder whose circumference is 14 feet.

If Circumference $= C$ feet.
Diameter $= D$ feet and radius $= R$ feet,
we have $D = 2R$ or $R = \frac{D}{2}$.

Now $$D = \frac{C}{\pi}.$$

Then $$R = \tfrac{1}{2} \times \frac{C}{\pi} = \frac{14}{2 \times 3{\cdot}1416} = \frac{7}{3{\cdot}1416} \text{ foot}$$

Taking logs., $\log R = \log 7 - \log 3{\cdot}142$
$= 0{\cdot}8451 - 0{\cdot}4972 = 0{\cdot}3479.$
$\therefore \quad R = \text{Antilog } 0{\cdot}3479 = 2{\cdot}228$ feet
$= 2$ ft. 3 ins. to nearest inch.

Example 3.

The diameter of a front roller of a worsted roving box is 4 inches. It makes 35 revolutions per minute. How many worsted hanks of roving will be delivered to each spindle in 48 hours, assuming *there are no stoppages?*

Every time the roller revolves it will deliver a length of roving equal to its circumference, and

$$\text{Circumference} = 4 \times \frac{22}{7} \text{ inches (here } \pi \text{ is taken as } 3\tfrac{1}{7}\text{).}$$

Roller makes 35 revolutions per minute.

$$\therefore \text{ Length delivered per minute} = \frac{4 \times 22 \times 35}{7} \text{ inches}$$

$$= 440 \text{ inches.}$$

$$\text{Length delivered per hour} = 440 \times 60 \text{ inches}$$

and $$\text{Length delivered in 48 hours} = 440 \times 60 \times 48 \text{ inches}$$

$$= \frac{440 \times 60 \times 48}{36} \text{ yards.}$$

Now the worsted hank is 560 yards.

$$\therefore \text{ Number of hanks delivered} = \frac{440 \times 60 \times 48}{36 \times 560} = 62\tfrac{6}{7}.$$

N.B.—Always proceed in stages, but leave the working, *i.e.* cancelling and taking logs., till the finish. Simplifying at each stage is a waste of time.

62. *Units of Length and Area.*

(*a*) The *standard unit of length* in the British system is the yard.

For small measurements we use smaller units which are fractions of the standard, *viz.*, the foot ($\frac{1}{3}$ yard) and the inch ($\frac{1}{36}$ yard).

For measuring long distances we use larger units which are multiples of the standard, *viz.*, the chain (22 yards), the furlong (220 yards), the mile (1760 yards).

In the Metric System, the standard unit of length is the metre, with its fractions decimetre (0·1 metre), centimetre (0·01 metre), millimetre (0·001 metre), and its multiples dekametre (10 metres), hectometre (100 metres), kilometre (1000 metres).

(*b*) The unit of area or surface is made to depend upon the unit of length.

If we cut out a square piece of cardboard making each side of the square one of the above-mentioned units of length (say 1 foot), then on whatever flat surface (floor, ceiling, window-

pane, etc.) we place the cardboard, it covers 1 square foot of surface.

If $ABCD$ represents a square with each side 1 yard in length, then the area of the surface of paper enclosed by the four sides is a *square yard*.

A *square centimetre* is the area of the surface enclosed by a square each side of which is one centimetre long.

63. *Area of a Rectangle.*—If $ABCD$ (Fig. 24) is a rectangle of length 7 units, breadth 3 units, then we may divide AB into 7 equal parts, each of unit length, and AD into 3 equal parts, each of unit length. If we then draw lines through the points of division parallel to the sides of the rectangle we shall form 3 rows (7 in each row) of squares like $PQRS$.

Since PS is 1 unit long,

Area of $PQRS$ = 1 unit of area.

Fig. 23a.

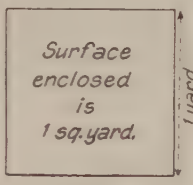


Fig. 23b.

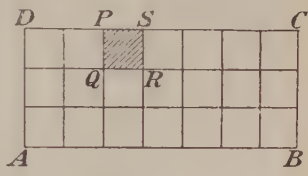


Fig. 24.

If unit of length = 1 inch or 1 foot or 1 cm.,
Area of $PQRS$ = 1 square inch or 1 square foot or 1 square cm.

and Area of $ABCD = 7 \times 3 = 21$ { square inches or square feet or square cms. }

Again, if the length of $ABCD$ be 15 inches and the breadth 9 inches, it is easy to see that we may divide the rectangle into squares 9 rows (15 in each row): the area of each square will be 1 square inch, and area of $ABCD = 15 \times 9 = 135$ square inches.

If $AB = 3{\cdot}7$ ins.; $AD = 2{\cdot}4$ ins. (Fig. 25), we may take 0·1 inch as our unit of length.

In this case area of square $DEFG$ = 1 square inch and area of shaded square whose side is 0·1 inch is seen to be 0·01 square inch.

The rectangle $ABCD$ contains 24 rows (37 in a row) of these small squares.

Hence Area of rectangle $ABCD = 37 \times 24 \times 0{\cdot}01$ square inches.
$= 3{\cdot}7 \times 2{\cdot}4$ square inches

By choosing a unit of length small enough we may express the length and breadth of a rectangle as whole numbers. So that if length be L units and breadth be B units, we shall obtain $L \times B$ squares of unit area.

If A is the number of units of area in the rectangle,

$$A = L \times B \quad \text{(i)}.$$

From this equation we can deduce

$$L = \frac{A}{B} \text{ (ii) and } B = \frac{A}{L} \text{ (iii)}.$$

A square is a rectangle with length and breadth the same. Then if the side of a square $= L$ units of length and if the area of the square $= A$ units of area,

we have $A = L \times L = L^2$ (iv)

and $L = \sqrt{A}$ (v).

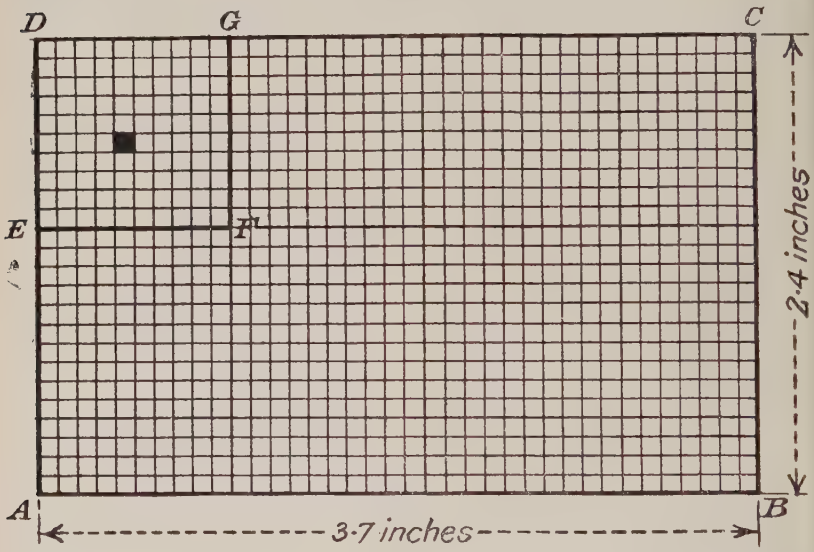


FIG. 25.

64. *Area of a Parallelogram.*—Let $ABCD$ be a parallelogram (Fig. 26). Draw BG and AH perpendicular to CD. Then $ABGH$ is a rectangle.

If a tracing be made of the triangle BGC we may show that it is equal to the triangle AHD in all respects. Or we may reason thus :—

In the two triangles BGC, AHD,
Angle BGC = Angle AHD (each is a right angle).
„ BCG = „ ADH (corresponding angles. § 59.)
Side BC = Side AD (opposite sides of a parallelogram).
∴ Triangles are equal in all respects, *i.e.* they are equal in area.

Hence Area of parallelogram $ABCD$ = Area of rectangle $ABGH$
$= AB \times BG.$
= Base × Altitude.

65. *Area of a Triangle.*—1. The area of a triangle is half the area of a rectangle having the same base and the same altitude.

Let ABC be a triangle and AD its altitude. Through A

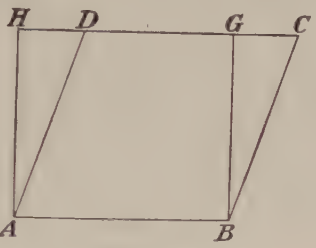


Fig. 26.

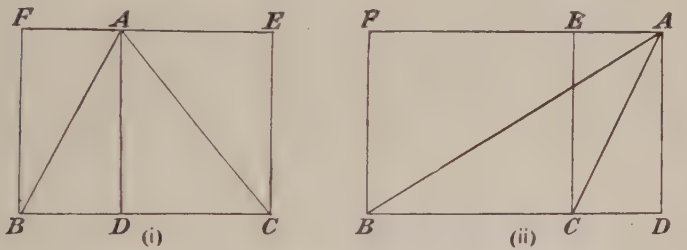


Fig. 27.

draw a line parallel to BC and draw perpendiculars from B and C to meet this line at F and E.

Then $BDAF$ and $DCEA$ are rectangles with diagonals BA and AC. Since the diagonal of a parallelogram bisects the parallelogram,

Area of triangle $ABD = \frac{1}{2}$ area of rectangle $BDAF$
$= \frac{1}{2} \times BD \times DA$
and Area of triangle $ADC = \frac{1}{2}$ area of rectangle $DCEA$
$= \frac{1}{2} \times DC \times DA.$

Adding these areas in Fig. 27 (i) and subtracting in Fig. 27 (ii) we have

Area of triangle $ABC = \frac{1}{2} \times BD \times DA + \frac{1}{2} \times DC \times DA$ Fig. 27 (i)

or

$= \frac{1}{2} \times BD \times DA - \frac{1}{2} \times DC \times DA$ (Fig. 27 (ii)

$= \frac{1}{2} \times DA \times (BD + DC)$

or

$(BD - DC)$

$= \frac{1}{2} BC \times DA$

$= \frac{1}{2} \times$ Base $\times$ Altitude.

2. Now any side of a triangle may be considered as its base and the perpendicular from the opposite angle to that side is the altitude.

Thus in Fig. 28 if base is BC the altitude is AD; if base is CA the altitude is BE; if base is AB the altitude is CF.

Then *Area of triangle* $= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} CA \times BE = \frac{1}{2} AB \times CF.$

The student should draw a triangle and its three altitudes. Measure each side and its corresponding altitude and calcu-

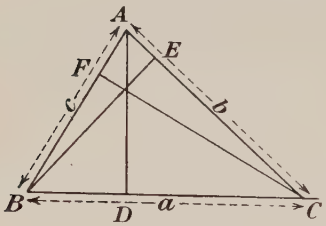


Fig. 28.

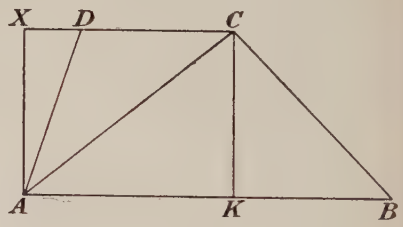


Fig. 29.

late the area of the triangle in three ways and compare the answers.

3. If the lengths of the sides BC, CA, AB be a, b and c units long respectively,

Then $\frac{a + b + c}{2}$ = half the perimeter

$= s$

and

Area of the triangle $= \sqrt{s(s - a)(s - b)(s - c)}$ units of area.

[The proof of this is beyond the scope of this book.]

66. *Area of a Trapezium.* — A trapezium is a four-sided figure with one pair of opposite sides parallel.

Let $ABCD$ be a trapezium (Fig. 29) having the opposite sides AB and CD parallel.

Draw the diagonal AC.

Draw CK perpendicular to AB and AX perpendicular to CD. Then the figure $AKCX$ is a rectangle and therefore its opposite sides CK and AX are equal.

Now Area of triangle $ACD = \frac{1}{2} CD \times AX$ units of area

" " $ACB = \frac{1}{2} AB \times CK$ " "

Then *Area of Trapezium* $= \Delta ACD + \Delta ACB$

$= (\frac{1}{2} CD \times AX) + (\frac{1}{2} AB \times CK)$ units of area

$= \frac{1}{2}(CD + AB) \times CK$ " "
[since $AX = CK$]

$= \frac{1}{2}$(Sum of the parallel sides) $\times$ (The perpendicular distance between them).

67. "The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the sides containing the right angle." The truth of this geometrical proposition may be proved experimentally.

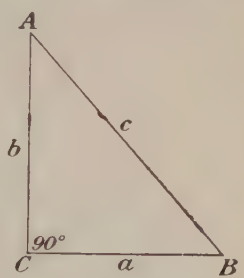


FIG. 30.

EXPERIMENT 1.

Draw right-angled triangles ABC each right-angled at C, given the lengths of the sides containing the right angle.

Measure the hypotenuse AB in each case and fill up the following table.

Length of BC a units.	Length of CA b units.	Length of AB c units.	Area of square on AB c^2 units of area.	Sum of the squares on BC and CA $(a^2 + b^2)$ units of area.
3″	4″			
6 cms.	8 cms.			
4″	0·9″			
2·7″	3·6″			
4·8 cms.	6·4 cms.			
11·9 cms.	12·0 cms.			

What relation is there between c^2 and $(a^2 + b^2)$?

Experiment 2.

Draw any right-angled triangle on squared paper. Describe squares on the sides and on the hypotenuse. Count the number of small squares in each square and verify as in Experiment 1 that $c^2 = a^2 + b^2$.

Then if $c^2 = a^2 + b^2$ or $c = \sqrt{a^2 + b^2}$ . . . . . (i

From this we deduce that

$a^2 = c^2 - b^2$ or $a = \sqrt{c^2 - b^2}$ . . . . . (ii

$b^2 = c^2 - a^2$ or $b = \sqrt{c^2 - a^2}$ . . . . . (iii

68. *Area of a Circle.*

(*a*) "The *area of a circle is* π *times the area of the square described on its radius.*"

Thus if Radius of circle $= R$ units,

then Area of square on radius $= R^2$ units of area

and Area of circle $= \pi \times R^2$ units of area

$= \frac{22}{7} \times R^2$,, ,, (roughly)

$= 3{\cdot}1416\ R^2$,, ,, (for greater accuracy)

This may be proved experimentally thus :—

On squared paper marked in inches and tenths draw quadrants of circles *OAP*, *OBQ*, *OCR*, etc., as in (Fig. 31) with radii 1″, 2″, 3″, etc.

Count the squares in each quadrant, reckoning halves and squares greater than halves as whole ones and neglecting those less than halves, and tabulate the results thus.

Note.—Area of each small square = 0·01 square inch.

Radius of circle R (units).	Area of square on radius R^2.	Number of squares in quadrant.	Number of squares in circle.	Area of circle A (units of area).	$\left(\frac{\text{Area of circle}}{\text{Area of square on radius}}\right)$
Inches.	Square inches.			Square inches.	
1	1				
2	4				
2·5	6·25				
3	9				
3·5	12·25				
4	16				
4·5	20·25				
5	25				

You should obtain on an average

$$A \div R^2 = 3{\cdot}14.$$

5) If radius of a circle $= R$ units,
If diameter of a " $= D = 2R$ units,
If area of a " $= A$ units of area,

hen $$A = \pi R^2 = 3{\cdot}1416\, R^2$$

$$= 3{\cdot}1416 \times \frac{D^2}{4} \text{ since } R = \frac{D}{2} \text{ and } R^2 = \frac{D^2}{4}$$

$$= 0{\cdot}7854 \times D^2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

lso $$D^2 = \frac{A}{0{\cdot}7854} \text{ or } D = \sqrt{\frac{A}{0{\cdot}7854}} \quad . \quad . \quad (2)$$

Similarly $$R = \sqrt{\frac{A}{3{\cdot}1416}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

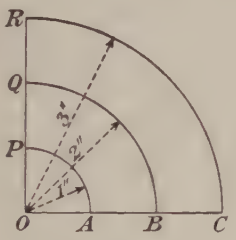


Fig. 31.

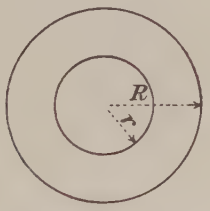


Fig. 32.

). *Area of a Circular Ring.*—In Fig. 32,

If radius of outer circle $= R$ units
If " inner " $= r$ "
Area of outer circle $= \pi R^2$ units of area
" inner " $= \pi r^2$ " "

Then the area of the shaded portion will be the difference tween the two areas, or

Area of Circular Ring $= \pi R^2 - \pi r^2$ units of area
$= \pi(R^2 - r^2)$ " "
$= \pi(R + r)(R - r)$ " (see § 71.)

If we are given the diameters of the circles and the diameter the outer circle $= D$ units, the diameter of the inner rcle $= d$ units.

Then Area of Circular Ring $= \pi . \frac{D^2}{4} - \pi \frac{d^2}{4}$ units of area

$= \frac{\pi}{4}(D^2 - d^2)$,, ,,

$= 0{\cdot}7854(D + d)(D - d)$,,

[*Note.*—$(R + r)(R - r)$ and $(D + d)(D - d)$ are suitable fo calculation by logs. or slide rule.]

70. *Area of a Sector of a Circle.*—A sector of a circle is portion of a circle enclosed by two radii and the arc inter cepted by those radii.

Thus in the figure *OAB*, *OCD*, *OEF*, etc., are sectors.

Now the area of a sector depends upon the size of the angl

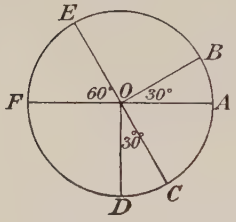


Fig. 33.

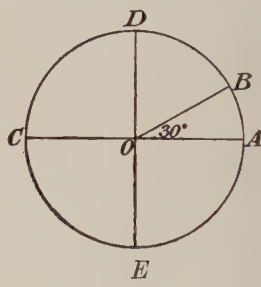


Fig. 34.

between the bounding radii, and to prove this experimentall draw a large circle as in Fig. 33. Make $\hat{AOB} = 30°$; $\hat{COD} = 3$ $\hat{EOF} = 60°$.

Cut out the sectors and observe that sector *AOB* will fit sector *COD*, whilst these two sectors together just cover t sector *EOF*.

Thus we learn that in the same circle, (*a*) sectors with eq angles are equal in area; (*b*) if we double the angle of sector, its area is doubled, etc.

Hence in Fig. 34, sectors *AOD*, *DOC*, *COE*, *EOA* are eq in area, since angle $AOD = D\hat{O}C = C\hat{O}E = E\hat{O}A =$ and each sector is $\frac{1}{4}$ area of the circle.

Now angle $AOB = 30°$.

Then Area of sector *AOB* $= \frac{1}{3}$ area of sector *AOD*

$= \frac{1}{12}$,, circle

$= \frac{30}{360}$,, circle.

Generally, if radius of circle $= R$ units,

if angle of sector $= D^0$,

Then Area of sector $= \frac{D}{360} \times$ area of the circle

$= \frac{D}{360} \times \pi R^2$ units of area.

1. To show graphically that

$$R^2 - r^2 = (R + r)(R - r).$$

Draw a square $ABCD$ (Fig. 35a), each side being R units long

" " $AKEF$ " " " r " "

Then Area of $ABCD = R^2$ units of area.

" $AKEF = r^2$ " "

Cut out the square $AKEF$ and leave an L-shaped figure $CDFEK$ (Fig. 35*b*).

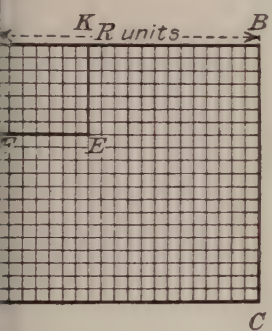


Fig. 35*a*.

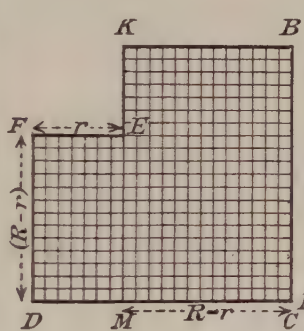


Fig. 35*b*.

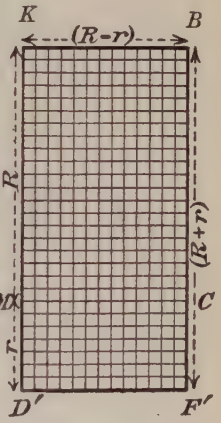


Fig. 35*c*.

Then area of L-shaped figure $= (R^2 - r^2)$ units of area.

In Fig. 35*b* $DF = (R - r)$ units of length.

$EF = r$ " "

Cut this L-shaped figure along the line EM, thus obtaining two rectangles $DFEM$ and $BCMK$.

Since $MC = EM = (R - r)$ units of length,

Rectangle $DFEM$ may be fitted to the rectangle $BCMK$ giving a rectangle $KBF'D'$ as in Fig. 35*c*,

and Area of rectangle $KBF'D' = BF' \times KB$

$= (R+r)(R-r)$ units of area.

But this rectangle is the L-shaped figure transformed.

Hence Area of L-shaped figure = area of rectangle (Fig. 35*c*)

or $R^2 - r^2 = (R + r)(R - r)$.

72. *Area of an Irregular Figure.*

Method 1.—When the boundaries are straight lines (Fig. 36), the area may be obtained by dividing the figure into a number of triangles and obtaining the area of each triangle separately. The sum of the areas of all the triangles into which the figure has been divided will be the area of the figure.

Method 2.—If one or more of the boundaries be curved lines, we may trace the figure on transparent tracing paper, and lay the tracing paper on squared paper ruled in inches and tenths of an inch. Count the number of small squares covered by the trace of the figure, neglecting any square obviously less than one-half and reckoning as a whole square any one which is equal to or greater than one-half. Remember that the area of a small square of side 0·1 inch is 0·01 square inch.

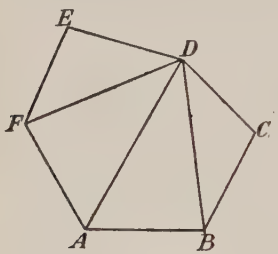


Fig. 36.

Method 3.—By weighing. Cut the figure out in cardboard and weigh the cardboard as accurately as possible. From the same cardboard cut a square (2-inch side) or a rectangle (3 inches by 2 inches) and weigh this. It is obvious that if 1 square inch of cardboard weighs 0·5 gramme, then 2 square inches of the same cardboard will weigh (0·5 × 2) grammes and so on. Then knowing the weight of the rectangle whose area is 6 square inches and the weight of the cardboard figure we may calculate the area of the figure.

Note.—There are other and more exact methods when the figure has one or more curved boundaries, but we shall not deal with them in this book.

73. *Worked Examples on Areas.*

Example 1.

A wool drying table is 20 feet long. Its rectangular horizontal surface is 3 feet broad and its sloping side surfaces are 3 feet 6 inches broad. What is the area of the drying surface

Area of horizontal surface $= 20 \times 3$ square feet
$= 60$ square feet.

Area of each sloping surface $= 20 \times 3\frac{1}{2} = 70$ square feet.

$\therefore$ Total drying surface $= 60 + (2 \times 70) = 200$ sq. feet

Example 2.

The figure 36*a* represents the side of a lean-to shed of dimensions as shown. Find the area of the wood which is necessary to cover this side.

The shape of the side is a trapezium with parallel sides 12 feet and 18 feet, and the perpendicular distance between them is 12 feet.

Then

$$\text{Area of wood required} = \tfrac{1}{2}\left\{\begin{matrix}\text{sum of}\\ \text{parallel}\\ \text{sides}\end{matrix}\right\} \times \left\{\begin{matrix}\text{perp. distance}\\ \text{between}\\ \text{them}\end{matrix}\right\}$$

$$= \tfrac{1}{2}(12 + 18) \times 12 \text{ square feet.}$$

$$= \tfrac{1}{2} \times 30 \times 12 \quad \text{,,} \quad \text{,,}$$

$$= 180 \text{ square feet.}$$

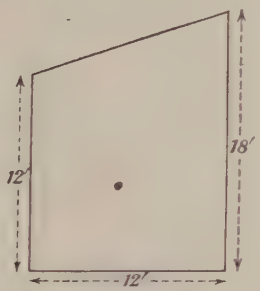


Fig. 36*a*.

Example 3.

The area of a triangle is 24 square inches. If the base is 7·5 inches, find the corresponding height.

Now for a triangle $A = \tfrac{1}{2}bh$

or $2A = bh.$

Dividing both sides by b we have

$$h = \frac{2A}{b}.$$

Substituting the given values of A and b,

$$\text{Height} = \frac{2 \times 24}{7{\cdot}5} = \frac{2 \times 8}{2{\cdot}5}$$

$$= \frac{16}{2{\cdot}5} = \frac{64}{10} = 6{\cdot}4 \text{ feet.}$$

Example 4.

Find the diagonal of a square field whose area is 8 acres.

If $ABCD$ represents the field (Fig. 36*b*), then AC represents the diagonal.

1. To *find the length of a side of the field.*

Now 1 acre = 10 square chains.

$\therefore$ Area = 80 ,, ,,

and Side $AB = \sqrt{80}$ chains.

Taking logs., Log $AB = \frac{1}{2}$ log 80

$= \frac{1}{2} \times 1{\cdot}9031 = 0{\cdot}9516.$

$\therefore$ AB = antilog $0{\cdot}9516 = 8{\cdot}945$ chains.

2. *To find the diagonal AC.*

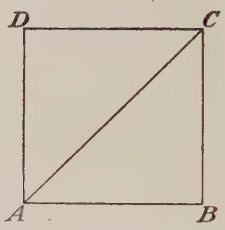


Fig. 36*b*.

In the right-angled triangle ABC,

$$AC^2 = AB^2 + BC^2$$
$$= (8{\cdot}945)^2 \times 2 \text{ square chains,}$$

or $AC = 8{\cdot}945 \times \sqrt{2}$ chains.

$$\text{Log } AC = \log 8{\cdot}945 + \tfrac{1}{2} \log 2$$
$$= 0{\cdot}9516 + 0{\cdot}1505 = 1{\cdot}1021.$$

$\therefore$ AC = antilog $1{\cdot}1021 = 12{\cdot}65$ chains.

Example 5.

The diameter of the piston of an engine is 2 feet. The pressure of the steam is 80 lbs. per square inch. Find the total thrust on the piston.

$$\text{Area of end of piston } (A) = \pi \frac{D^2}{4}$$
$$= 0{\cdot}7854\, D^2$$
$$= 0{\cdot}7854 \times 24^2 \text{ square inches}$$

On every square inch of area we have a pressure of 80 lbs.

$\therefore$ Total pressure (P) on piston $= 0{\cdot}7854 \times 24^2 \times 80$ lbs.

Taking logs., $\text{Log } P = \log 0{\cdot}7854 + 2 \log 24 + \log 80$
$= \bar{1}{\cdot}8951 + 2{\cdot}7604 + 1{\cdot}9031$
$= 4{\cdot}5586.$
$P = \text{antilog } 4{\cdot}5586$
$= 36190$ lbs. to 4 significant figures.

Example 6.

Find the area of the face of a circular washer of 2·5″ diameter if the diameter of the hole is $\frac{1}{2}$ inch (Fig. 36*c*).

Here outside radius $= 1{\cdot}25'' = R$,
radius of hole $= 0{\cdot}25'' = r$.

Then Area $= \pi (R^2 - r^2)$
$= \pi (R + r)(R - r)$
$= 3{\cdot}1416\ (1{\cdot}25 + 0{\cdot}25)(1{\cdot}25 - 0{\cdot}25)$ sq. inches
$= 3{\cdot}1416 \times 1{\cdot}5 \times 1$
$= 4{\cdot}71$ square inches to nearest $\frac{1}{100}$ square inch.

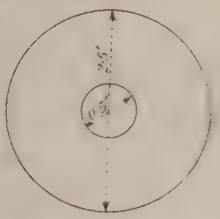


Fig. 36*c*.

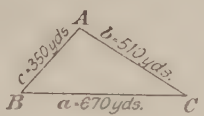


Fig. 37.

Example 7.

Find the area of a triangular field whose sides are 350 yards, 510 yards, 670 yards (Fig. 37).

Here we use the formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = \frac{1}{2} \times$ perimeter of the triangle.

Now $s = \frac{670 + 510 + 350}{2} = \frac{1530}{2} = 765$ yards.
$s - a = 765 - 670 = 95$ yards.
$s - b = 765 - 510 = 255$,,
$s - c = 765 - 350 = 415$,,

Then Area $= \dfrac{\sqrt{765 \times 95 \times 255 \times 415}}{4840}$ acres.

$$\text{Taking logs., } \log A = \tfrac{1}{2}\{\log 765 + \log 95 + \log 255 + \log 415\} - \log 4840$$
$$= \tfrac{1}{2}\{2{\cdot}8837 + 1{\cdot}9777 + 2{\cdot}4065 + 2{\cdot}6180\} - 3{\cdot}6848$$
$$= 4{\cdot}9430 - 3{\cdot}6848 = 1{\cdot}2582.$$
$$\therefore \quad A = \text{antilog } 1{\cdot}2582 = 18{\cdot}12 \text{ acres.}$$

74. *Units of Volume and Capacity.*—An object shaped like a brick or a box or a book occupies space in three directions or has three dimensions of length and breadth and thickness. The amount of room or space occupied by the object is called its *volume*. Since liquids and gases occupy space in three directions, they are treated geometrically as solids and must be measured by the same units.

The *unit of volume is the space occupied by a cube each edge of which is of unit length.*

A cubic foot is the space occupied by a cube with edges 1 foot.

A cubic inch is the space occupied by a cube with edges 1 inch.

A cubic centimetre is the space occupied by a cube with edges 1 cm., and so on.

Hollow vessels like water-troughs, oil-tanks, and gasometers have the power to hold liquids and gases which take up room in three dimensions and thus possess volume. Thus hollow vessels will hold so many cubic inches, cubic feet, or cubic centimetres of a liquid or gas. This "holding power" of, or "volume contained" by, a hollow vessel is termed its "*capacity*" or "*cubical content.*"

The British unit of capacity is the gallon with its fractions, the quart, the pint, and the gill.

The metric unit of capacity is the litre, which is the volume contained in a hollow cube each edge of which is a decimetre. It is thus equivalent to 1000 c.cms.

The relation between the gallon and the cubic foot is found as follows :—

1 gallon of water weighs 10 lbs. approx.
1 cubic foot of water weighs $62\frac{1}{2}$ lbs. approx.

Then 1 gallon is equivalent to $\dfrac{10}{62\frac{1}{2}}$ cubic foot or 0·16 cubic foot.

The tables give 1 gallon = 0·1604 cubic foot.

75. *Volume of a Rectangular Prism.*—This solid may be built up from cubes of unit volume.

Now a cube with edges 1 inch covers an area of 1 square inch and has a volume of 1 cubic inch. If the base of the prism be l inches long and b inches broad its area is $(l \times b)$

square inches, and we shall require lb cubes for the first layer.

If the height of the prism is h inches we shall require h layers of cubes, and to build up the prism we shall require $(lb \times h)$ cubes each with a volume of 1 cubic inch.

If $l = 3{\cdot}5''$, $b = 2{\cdot}3''$, $h = 1{\cdot}6''$, we may use cubes of 0·1 inch edge. Then each cube covers 0·01 square inch of surface and has a volume of 0·001 cubic inch, for it is easy to see that 1000 of these small cubes will build a cube with each edge 1 inch.

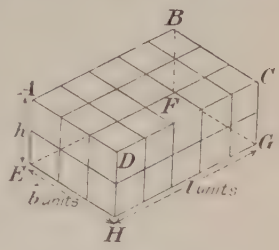


FIG. 38.

For the bottom layer we shall require (35×23) cubes, and there will be 16 layers to give a height of 1·6 inches.

Hence volume of prism $= 35 \times 23 \times 16 \times 0{\cdot}001$ cubic ins.
$= 3{\cdot}5 \times 2{\cdot}3 \times 1{\cdot}6$ cubic inches
$= l \times b \times h$ units of volume.

Note that the number of cubes required for the bottom layer is always the number of units of area in the base of the prism and the number of layers required is the number of units in the height of the prism.

Hence $\left.\begin{matrix}\text{Volume of} \\ \text{rect. prism}\end{matrix}\right\} = \left\{\begin{matrix}\text{Number of} \\ \text{units in } \textit{area} \\ \textit{of base}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Number of} \\ \text{units in} \\ \textit{height}\end{matrix}\right\}$

It may be shown that
the Volume of any right prism = Area of base × height.

76. *Volume of Cylinder.*—(*a*) It is shown in Geometry that the cylinder may be regarded as a prism with a circular base.

Then if radius of the base $= R$ units,
if height of cylinder $= H$,,
Volume of cylinder = Area of base × height
$= \pi R^2 . H$ units of volume.

(*b*) *Surface of Cylinder.*—The surface of a cylinder consists of two parts, (1) the curved surface and (2) the two ends which are plane surfaces. Now a rectangular sheet of paper of width equal to the height of the cylinder (H units) can be wrapped round the curved surface of a metal cylinder so that its ends just meet. When this is the case, the length of the sheet of paper will be equal to the circumference of the cylinder ($2\pi R$ units).

Then Area of curved surface of cylinder
= Area of sheet of paper
= Circumference × height
= $2\pi R \,.\, H$ units of area.

To obtain the total surface area, we must add the areas of the two ends which are circles.

∴ Total surface of cylinder = $2\pi R \,.\, H + 2\pi R^2$
= $2\pi R\,(H + R)$ units of area.

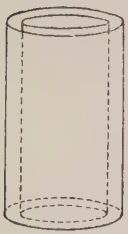


Fig. 38*a*.

(*c*) *Hollow Cylinder.*—Suppose we require the volume of lead in a piece of lead pipe, we first find the volume if it were a solid lead cylinder and then subtract the volume of the hole which could be bored through.

If R and r units denote the radii of the outer and inner circles respectively, D and d units the corresponding diameters, and if length of pipe be H units (Fig 38*a*), we have

Volume of solid cylinder (radius R) = $\pi R^2 \,.\, H$ units of volume.

Volume of cylindrical hole (radius r) = $\pi r^2 \,.\, H$,, ,,

∴ Volume of lead in the pipe
= $\pi R^2 H - \pi r^2 H$ units of volume
= $\pi(R^2 - r^2) \,.\, H$,, ,,
= $\pi(R + r)(R - r) \,.\, H$,,

In terms of D and d, we have

$$R = \frac{D}{2}\ ;\ r = \frac{d}{2}$$

$$R^2 = \frac{D^2}{4}\ ;\ r^2 = \frac{d^2}{4}$$

and Volume of lead = $\pi \,.\, \frac{D^2}{4} \,.\, H - \pi \,.\, \frac{d^2}{4} \,.\, H$ units of volume.

(or any other material of which the hollow cylinder is made)

= $\frac{\pi}{4}(D^2 - d^2) \,.\, H$. units of volume
= $0{\cdot}7854\,(D + d)(D - d) \,.\, H$,,

Notice we use $(R + r)(R - r)$ in place of $R^2 - r^2$ if we wish to use logarithms.

77. *Volume of a Cone.*—(*a*) "*The volume of a cone is one-third that of a cylinder having the same base and the same altitude.*"

This result may be checked in the following experimental way.

Given a hollow cylindrical vessel (Fig. 38*b*) and a metal cone having the same base and altitude, you fill the hollow vessel with water from a measuring cylinder and note the volume of water used. Pour out the water, put in the cone and again fill with water, and note the volume of water required this time. The difference between the two quantities of water will measure the room taken up by the cone, *i.e.* the volume of the cone. Here are some results, obtained in this way.

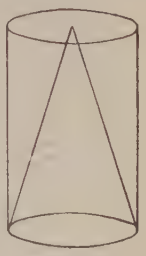


Fig. 38*b*.

Volume of water required to fill the hollow vessel = 43 c.c.'s.

Volume of water required when the cone was in = 28·6 c.c.'s.

Difference = 14·4 c.c.'s

= Volume of cone.

$$\therefore \frac{\text{Volume of cone}}{\text{Volume of cylinder}} = \frac{14{\cdot}4}{43} = 0{\cdot}335 = \tfrac{1}{3} \text{ approx.}$$

Hence if the radius of the base $= R$ units,

and if the height of the cone $= H$,,

Volume of cone $= \frac{1}{3}$ (area of base $\times$ height)

$= \frac{1}{3} \times \pi R^2 . H$ units of volume.

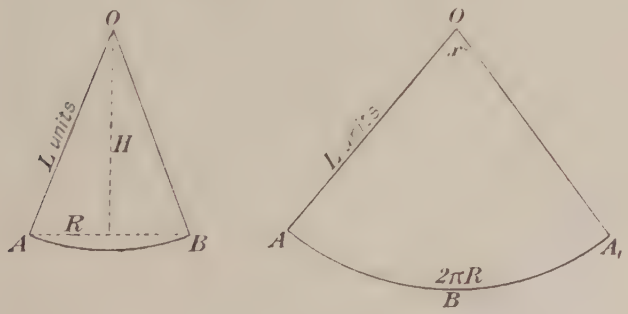
78. *Area of Curved Surface of a Cone.*

Let Fig. 39*a* represent a cone ;

Radius of base $= R$ units.

Altitude $= H$,,

Slant height $(OA) = L$ units.



(*a*). Fig. 39. (*b*).

If we cut a piece of thin paper to cover the curved surface of the cone and open it out we obtain a sector of a circle of radius L units (Fig. 39*b*).

The length of the arc AA' = circumference of base of cone
$= 2\pi R$ units.

If the angle of the sector is x^0,

Then Arc $AA' = \left\{ \frac{x}{360} \times \text{circumference of circle with centre } O \right\}$

$$= \left\{ \frac{x}{360} \times 2\pi L \right\} \quad \ldots\ldots\ldots \quad (A)$$

Also Area of sector $= \frac{x}{360} \times$ area of circle with centre O

$$= \frac{x}{360} \times \pi L^2$$

$$= \left\{ \frac{x}{360} \times 2\pi L \right\} \times \frac{L}{2}$$

$$= \text{Arc } AA' \times \frac{L}{2} \text{ [from (A) above]}$$

$$= 2\pi R \times \frac{L}{2}.$$

Then

$$\left.\begin{matrix}\text{Area of curved} \\ \text{surface of cone}\end{matrix}\right\} = 2\pi R \times \frac{L}{2}$$

$$= \tfrac{1}{2} \left\{ \begin{matrix}\text{Circumference} \\ \text{of the base}\end{matrix} \right\} \times \left\{ \text{Slant height} \right\}.$$

Note.—If we know R and H

$$L^2 = R^2 + H^2 \text{ or } L = \sqrt{R^2 + H^2}.$$

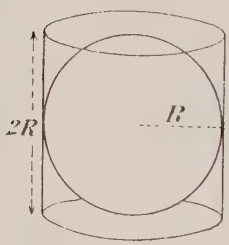


FIG. 40.

79. *Volume of a Sphere.*— "The volume of a sphere is two-thirds the volume of its circumscribing cylinder."

If we substitute a brass sphere with the same radius for the cone in Experiment § 77, we may obtain such results as the following :—

$$\left.\begin{matrix}\text{Volume of water required} \\ \text{to fill the hollow cylinder}\end{matrix}\right\} = 43{\cdot}0 \text{ c.c.'s.}$$

$$\left.\begin{matrix}\text{Volume of water required} \\ \text{when the sphere was in}\end{matrix}\right\} = 14{\cdot}6 \text{ c.c.'s.}$$

Difference $= 28{\cdot}4$ c.c.'s.
$=$ Volume of sphere.

Then $\dfrac{\text{Volume of sphere}}{\text{Volume of circumscribing cylinder}} = \dfrac{28{\cdot}4}{43{\cdot}0}$

$= 0{\cdot}660 = \frac{2}{3}$ approx.

Hence referring to Fig. 40,

Radius of sphere = Radius of base of cylinder
= R units.

Height of cylinder = Diameter of sphere
= $2R$ units.

Then Volume of sphere = $\frac{2}{3}$ × volume of cylinder
= $\frac{2}{3} \times \pi R^2 \times 2R$
= $\frac{4}{3} \pi R^3$ units of volume.

If Diameter of sphere = D units,

Then $$R = \frac{D}{2} \text{ and } R^3 = \left(\frac{D}{2}\right)^3 = \frac{D^3}{8}$$

and Volume of sphere $= \frac{4}{3} \times \pi \times \frac{D^3}{8} = \frac{\pi D^3}{6}$
= 0·5236 D^3 units of volume.

80. *Area of Surface of a Sphere.*—It may be proved mathematically that "the *area of the surface of a sphere is equal to the area of the curved surface of the circumscribing cylinder.*"

Referring again to Fig. 40 we have,

Area of surface of sphere = area of curved surface of cylinder
= Circumference of base × height
= $2 \pi R \times 2R$
= $4 \pi R^2$ units of area.

Substituting D for $2R$ or D^2 for $4R^2$,

then Area of surface of sphere = πD^2 units of area.

81. *"Card Clothing" for the Cylinders of the Woollen and Worsted Carding Engines.*

(*a*) This clothing is an arrangement of bent wire teeth set in a foundation of leather or woven material, the latter being strengthened by a thin layer of vulcanised rubber.

The clothing is made of two forms, one known as "filleting" and the other as "sheeting."

(1) Filleting is in strip form which varies from 1 to 3 inches in width and is wound round the various rollers and cylinders, leaving no space on the whole of the surface.

(2) Sheeting is of a standard width 5 inches and supplied in lengths to suit various lengths of the rollers which vary from 5 to 6 feet.

To calculate the length of filleting or the number of sheets required to cover any given roller we must use our knowledge of the mensuration of the cylinder.

Example 1.

A swift 50 inches diameter is 60 inches long. Find the length of filleting 2 inches wide required to cover the surface.

Now the area of the filleting required is equal to the area of the curved surface of the cylinder.

If the length required is x inches, then area of filleting $= 2x$ square inches.

Also Area of curved surface of cylinder

$$= \text{Circumference} \times \text{length}$$
$$= 3{\cdot}1416 \times 50 \times 60 \text{ sq. inches.}$$

Hence $$2x = 3{\cdot}1416 \times 50 \times 60$$

or $$x = \frac{3{\cdot}1416 \times 50 \times 60}{2}$$
$$= 3{\cdot}1416 \times 1500$$
$$= 4712{\cdot}4.$$

$\therefore$ Length of filleting $= 4712{\cdot}4$ inches $= 392{\cdot}7$ feet.

Example 2.

How many sheets would be required for the cylinder mentioned in Example 1?

The sheets are 5 inches wide and for this cylinder will be 60 inches long. Each sheet as it is put on will cover 5 inches of the circumference for the whole length of the cylinder. Then if we know the circumference and divide by 5 we shall obtain the number of sheets required.

Then $$\left.\begin{matrix}\text{Number of sheets} \\ \text{required}\end{matrix}\right\} = \frac{3{\cdot}1416 \times 50}{5} = 31{\cdot}4 = 31\tfrac{1}{2}$$

practically.

Now the sheets are fastened round the cylinders with tacks, and when they have been nailed on in this manner, a small space of about $\frac{1}{2}$ inch will remain uncovered between one sheet and another.

If in Fig. 41 the inner circle represents the roller and the outer marks represent the sheets of clothing (distance between sheets greatly exaggerated) we see that the number of spaces left is the same as the number of sheets required.

Returning to our problem and allowing a space of $\frac{1}{2}$ inch between each sheet and the next :

If we require x sheets there will be x spaces. The x sheets will cover $5x$ inches of the circumference. The x spaces will require $\frac{1}{2}x$ inches of the circumference.

Hence $$5x + \tfrac{1}{2}x = \text{circumference of roller}$$
$$= 3{\cdot}1416 \times 50$$

or $$\frac{11x}{2} = 3{\cdot}1416 \times 50.$$

Divide both sides by $\frac{11}{2}$ and

$$x = \frac{3{\cdot}1416 \times 50 \times 2}{11} = 28{\cdot}56.$$

Then Number of sheets required = 29.

(*b*) *Fineness of Card Clothing.*—Clothing varies in counts or the number of wire points per square inch, according to the work to be done by the cylinder in the carding process.

The terms "*Counts and Crowns*" are used to indicate the fineness of the clothing.

By "*Counts*" we mean the number of wires in 5 inches counted down the clothing.

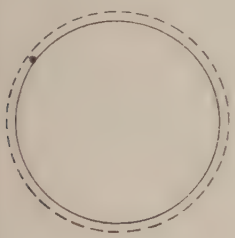


Fig. 41.

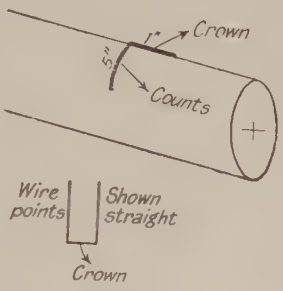


Fig. 42.

By "*Crowns*" we mean the number of "heads" or "crowns" in 1 inch measured across the back of the clothing.

Example 1.

Thus in 140's/12's, *i.e.* 140's counts and 12's crowns, we have that 140's counts means 140 points in 5 inches down the clothing and 12's crowns means 12 crowns in 1 inch across the back.

Now each crown carries 2 points, then 12's crowns means 24 points per inch across the front.

$\therefore$ In 5 square inches there are 140×24 points

and in 1 square inch there are $\frac{140 \times 24}{5}$,,

and Number of points per square inch $= \frac{140 \times 24}{5} = 28 \times 24 = 672.$

Example 2.

The second swift in a scribbler is covered with clothing of 125's counts and 11's crowns. Find the number of points of wire per square inch.

Here we have 125 points in 5 inches or 25 points per inch measured down the clothing, and 11 crowns or 22 points per inch across the clothing.

Hence Number of points per square inch $= 25 \times 22 = 550$.

82. *Worked Examples on Volumes and Surface Areas.*

Example 1.

The Factory Act requires 250 cubic feet of space for each person. If the length, breadth, and height of a mending room be 45 feet, 30 feet, and 12 feet respectively, what number of menders would be allowed in this room ?

The room is a rectangular prism in shape.

$\therefore$ Air capacity of the room = area of base $\times$ height
$= 45 \times 30 \times 12$ cubic feet.

Since each person requires 250 cubic feet of space.

$$\text{Number of persons allowed} = \frac{45 \times 30 \times 12}{250} = 64.$$

Example 2.

A wool washing bowl is 30 feet by 3 feet by 4 feet. How many gallons of water must be run in to fill it to a depth of 3 feet ? (1 gallon is equivalent to 0·1604 cubic foot.)

The shape of the water will be that of a rectangular prism

$\therefore$ Volume of water $= 30 \times 3 \times 3$ cubic feet

$$= \frac{30 \times 3 \times 3}{0{\cdot}1604} \text{ gallons}$$

$$= \frac{270}{0{\cdot}1604} \text{ gallons.}$$

Taking logs., Log volume $= \log 270 - \log 0{\cdot}1604$
$= 2{\cdot}4314 - \bar{1}{\cdot}2052 = 3{\cdot}2262.$

$\therefore$ Volume = antilog $3{\cdot}2262 = 1684$ gallons

Example 3.

A wool washing bowl contains 1000 gallons of scour. If it is 18 feet long and 3 feet broad, what is the depth of the liquor ?

Since 1 gallon $\equiv$ 0·1604 cubic foot
1000 gallons $\equiv$ 160·4 cubic feet.

If depth of scour $= x$ feet,

Volume of scour $= 18 \times 3 \times x = 54\,x$ cubic feet.

Then $54\,x = 160{\cdot}4$

or $$x = \frac{160{\cdot}4}{54} = 2{\cdot}97.$$

$\therefore$ Depth of scour $= 3{\cdot}0$ feet to nearest $\frac{1}{10}$ foot.

Example 4.

Water flows through a cylindrical pipe at the rate of 4·75 feet per second. If the pipe has an internal diameter of 12 inches, find the supply in gallons per minute.

The water as it flows through the pipe is the shape of the pipe, and during one second a volume of water will issue equal in volume to that of a cylinder 4·75 feet long and 12 inches in diameter.

$$\text{Volume of a cylinder} = \frac{\pi D^2}{4} . H = 0{\cdot}7854\,D^2 . H.$$

$\therefore$ Volume of water issuing per sec. $= 0{\cdot}7854 \times 1^2 \times 4{\cdot}75$ cubic feet

and Volume supplied per minute $(V) = 0{\cdot}7854 \times 1 \times 4{\cdot}75 \times 60$ cubic feet

$$= \frac{0{\cdot}7854 \times 4{\cdot}75 \times 60}{0{\cdot}1604} \text{ gallons.}$$

Then $\log V = \log 0{\cdot}7854 + \log 4{\cdot}75 + \log 60 - \log 0{\cdot}1604$

$= \bar{1}{\cdot}8951 + 0{\cdot}6767 + 1{\cdot}7782 - 1{\cdot}2052$

$= 2{\cdot}3500 - \bar{1}{\cdot}2052 = 3{\cdot}1448.$

$\therefore$ $V = \text{antilog } 3{\cdot}1448 = 1396$ gallons.

$$\left[\text{Rough check } V = \frac{0{\cdot}8 \times 5 \times 60}{0{\cdot}2} = 1200.\right]$$

N.B.—Always check your answer roughly to avoid mistakes in the position of the decimal point when using logs. and slide rule.

Example 5.

A hollow steel shaft is 20 feet long. Its external diameter is 20 inches and its internal diameter is 16 inches. If 1 cubic inch of steel weighs 0·288 lb., find the weight of the shaft in tons.

Volume of hollow cylinder $= \frac{\pi}{4}(D^2 - d^2) . H.$

$= 0{\cdot}7854(D + d)(D - d) . H.$

Then Volume of shaft $= 0{\cdot}7854(20 + 16)(20 - 16) \times 240$ cubic inches.

Since 1 cubic inch weighs 0·288 lb.

Weight of shaft (W) $= 0{\cdot}7854 \times 36 \times 4 \times 240 \times 0{\cdot}288$ lbs.

$$= \frac{0{\cdot}7854 \times 144 \times 240 \times 0{\cdot}288}{2240} \text{ tons.}$$

$\log W = (\log 0{\cdot}7854 + \log 144 + \log 240 + \log 0{\cdot}288) - \log 2240.$

$= (\bar{1}{\cdot}8951 + 2{\cdot}1584 + 2{\cdot}3802 + \bar{1}{\cdot}4594) - 3{\cdot}3502.$

$= 3{\cdot}8931 - 3{\cdot}3502 = 0{\cdot}5429.$

$\therefore \quad W = \text{antilog } 0{\cdot}5429 = 3{\cdot}49$ tons.

$$\left[\text{Roughly } W = \frac{0{\cdot}8 \times 150 \times 200 \times 0{\cdot}3}{2000} = 15 \times 0{\cdot}24 = 3{\cdot}6.\right]$$

Example 6.

A 12-inch metal cube is melted down and made into a sphere. Find the radius of the sphere.

Volume of cube $= 12 \times 12 \times 12.$

$= 1728$ cubic inches.

$=$ Volume of sphere.

If R (ins.) be radius of sphere,

Volume of sphere $= \frac{4}{3}\pi R^3.$

Hence $\frac{4}{3}\pi R^3 = 1728.$

Divide by $\frac{4}{3}\pi$, and $R^3 = \frac{1728 \times 3}{4 \times 3{\cdot}1416} = \frac{432 \times 1}{1 \times 1{\cdot}0472}$

and $$R = \sqrt[3]{\frac{432}{1{\cdot}0472}}$$

$\log R = \frac{1}{3}(\log 432 - \log 1{\cdot}047)$

$= \frac{1}{3}(2{\cdot}6355 - 0{\cdot}0200)$

$= \frac{1}{3} \times 2{\cdot}6155 = 0{\cdot}8718.$

$\therefore \quad R = \text{antilog } 0{\cdot}8718 = 7{\cdot}444$

$= 7{\cdot}44$ inches to nearest $\frac{1}{100}$ inch.

Example 7.

If the specific gravity of wool be taken as 1·3, find the diameter of a 1/16's worsted thread, and also the number of threads which may be laid side by side in a space of one inch.

1/16's worsted means that (16 × 560) yards weigh 1 lb., or

1 yard weighs $\frac{1}{16 \times 560}$ lb. . . . . (A)

Take 1 yard of the thread and consider it as a cylinder. Suppose its diameter $= D$ inches and its length $= 36$ inches.

Then Volume of thread $= \frac{\pi D^2}{4} \times 36$ cubic inches.

$$= \frac{0{\cdot}7854 \times 36 \times D^2}{1728} \text{ cubic feet.}$$

Now 1 cubic foot of water weighs 62·5 lbs., and specific gravity of wool = 1·3.

∴ 1 cubic foot of wool weighs (62·5 × 1·3) lbs.

[For Sp. Gr. see § 100.]

And Weight of thread in 1 yard $= \frac{0{\cdot}7854 \times 36 \times D^2 \times 62{\cdot}5 \times 1{\cdot}3}{1728}$ lbs. (B)

Equating results (A) and (B),

$$\frac{0{\cdot}7854 \times 36 \times D^2 \times 62{\cdot}5 \times 1{\cdot}3}{1728} = \frac{1}{16 \times 560}$$

or

$$D^2 = \frac{1728}{0{\cdot}7854 \times 36 \times 62{\cdot}5 \times 1{\cdot}3 \times 16 \times 560}$$

$$= \frac{1}{0{\cdot}2618 \times 62{\cdot}5 \times 728}.$$

$$\therefore \quad D = \frac{1}{\sqrt{0{\cdot}2618 \times 62{\cdot}5 \times 728}} = \frac{1}{109{\cdot}1} \text{ inch (by logs.).}$$

Hence 109 threads may be laid side by side in the space of one inch.

CHAPTER VII

Use of Squared Paper—Graphs

83.—Certain Facts relating to Triangles. 84.—To fix the Position of a Point on a Plane Surface. 85.—Coordinates of a Point. 86.—The Straight Line Graph. 87.—Functions. 88.—Graphs of Functions of x. 89.—The Linear Graph. 90.—Meaning of the Constants a and b in $y = ax + b$. 91.—Some Applications of Graphs. (i) Graph to convert Worsted Counts into Cotton Counts and *vice versa*. (ii) Graph to convert Yorkshire Skeins to American "Runs" and *vice versa*. (iii) Graph to convert Worsted Counts to Counts in Various Systems. (iv) Graph to convert "Ends per Inch" to various "Sett" Systems and *vice versa*. 92.—Simultaneous Equations—Introduction. 93.—Graphic Solution of Simultaneous Equations 94.—Algebraical Solution of Simultaneous Equations.

83. *Certain Facts relating to Triangles.*—For a better understanding of what follows, the student should repeat exercises 7(*a*), (*b*), and (*c*), § 59, p. 113.

In these exercises are given the three different methods of constructing a triangle. In each case *three* dimensions are given, namely :

(i) The lengths of the *three sides.*
(ii) The lengths of *two sides* and the size of the *included angle.*
(iii) The length of *one side* and the sizes of *two angles.*

In each case the triangle formed is unique. In other words each set of dimensions is sufficient to fix the size and shape of the triangle.

84. *To fix the Position of a Point on a Plane Surface.*—Suppose A and B (Fig. 43) represent the positions of two trees in a garden, and a person hides some treasure at a point in the garden marked C. Let us consider what measurements should be made so as to locate the position C.

1. Suppose AC *alone* is measured (*e.g.*, 10 feet). What positions might the point C occupy ?

2. Suppose BC *alone* measured (*e.g.*, 7 feet). Show on a drawing all the positions C could possibly take.

3. If distances AC (10 feet), BC (7 feet) were noted, what re the possible positions of C?

From these results we observe that *to locate* C, we must now the two *distances* AC *and* BC, and make a mental note s to whether C is above or below (North or South) the line oining the trees A and B.

The position of C is fixed, for we know the lengths of the hree sides of the triangle ABC.

4. If we measure the distance AC and the angle BAC, oticing whether the turn from AB to AC is clockwise or ounter-clockwise, we again fix the position of C, for we know wo sides, AB and AC, and the included angle BAC of the riangle ABC.

5. We may measure the angles BAC and ABC, with a note to the direction of the turns, and again fix the position f C, for we know one side AB and the sizes of two angles of he triangle ABC.

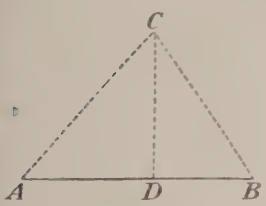


Fig. 43.

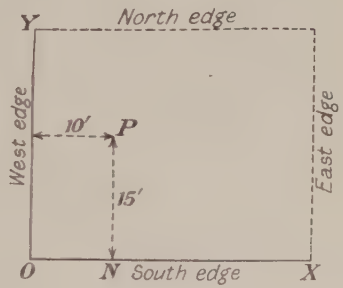


Fig. 44.

6. Again we may measure the distance AD and the distance DC at right angles to AD, noting the direction in which the perpendicular is drawn, and in this way fix the position of C, or we know two sides AD, DC of the triangle ADC, and the ngle included by those two sides.

From the above considerations we find that to fix the osition of a point (C) on a plane surface, relative to two fixed oints (A and B), we must make at least two measurements, ith a note as to the direction in which these measurements re made.

7. Now let us try to *fix the position of a point on a rectangular awn.*

Suppose the figure (Fig. 44) represents the rectangular awn and OX and OY represent the South and Western edges

of the lawn. We wish to fix a point P relative to the edges OX and OY.

(*a*) Suppose we know that P is 10 feet from OY, where may the point P be ? Clearly anywhere along a line parallel to OY and 10 feet distant from it. Then this measurement alone is not sufficient to fix P.

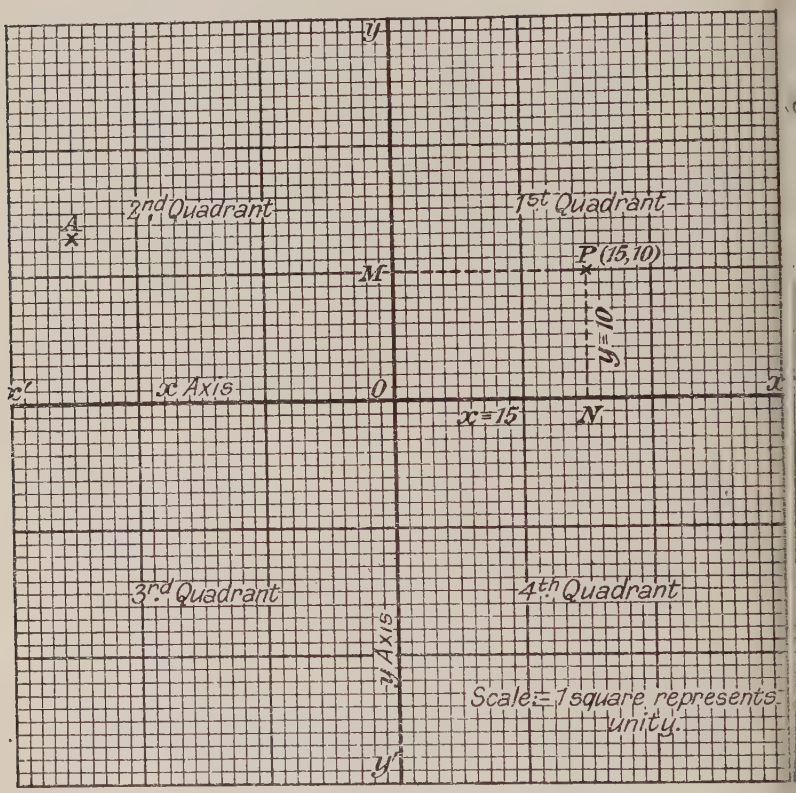


FIG. 45.

(*b*) If we know that P is 15 feet from OX, then the *locus* o P (*i.e.*, the line which passes through all possible positions c P) is a straight line parallel to OX and 15 feet distant from it, and this measurement of itself is not sufficient to locate th point.

(*c*) But if we know that P is 10 feet from OY and 15 fee from OX, then P must lie on each of the two lines mentioned

in (*a*) and (*b*), that is, it must be at the point where these two lines cross, and this is a fixed position.

Referring to the figure, we know the lengths of *ON* and *NP*, two sides of a triangle *OPN* and the included angle *ONP*.

Thus again we need to know *two* measurements to fix a point with regard to a rectangular surface. Account must also be taken of the direction of these measurements, in other words we must say from which edges of the lawn we make the measurement.

(*d*) Is it possible to fix the point *P* by any other two measurements ?

(*e*) Think what measurements would be required to fix the positions of (1) a point on the floor ; (2) a pencil on the desk ; (3) a chair on the floor ; (4) a book on the desk ; (5) the bob of a pendulum hung from the ceiling.

85. *Coordinates of a Point.*—Draw two straight lines XOX', YOY' (Fig. 45) intersecting at right angles at the point *O*, thus dividing the plane of the paper into four parts YOX, YOX', $X'OY'$, $Y'OX$ which are known as the first, second, third, and fourth quadrants respectively. If a point *A* be marked on the paper, we have seen in the preceding paragraph that its position is exactly determined if we know (1) its distances from the lines XOX', YOY', and (2) the directions in which these distances are measured (*i.e.*, above or below XOX' ; right or left of YOY').

Thus in locating the position of a place on the earth's surface, we require to know (1) its latitude, (2) its longitude, and it is not sufficient to say that the latitude is 30°, and the longitude 45°, we must also know whether the latitude 30° is measured towards the North or South starting from the equator, and whether the longitude 45° is measured to the East or West of the Meridian passing through Greenwich.

On the map we find that Huddersfield is at latitude 53° 35′ N, and longitude 1° 40′ W.

The lines XOX', YOY' (Fig. 45) are usually drawn horizontally and vertically respectively ; they are taken as lines of reference and correspond to the equator and the Meridian through Greenwich.

The line XOX' is known as the "*Axis of x*" or the "*x axis.*"
„ „ YOY' „ „ *Axis of y* „ *y axis.*

The point *O* at which they intersect is called *the* ORIGIN, and from this point the measurements are taken. The distances of a point from these axes and measured at right angles to them are called the *Rectangular Coordinates* of the point.

The distance of a point P (Fig. 46) (PM or ON) from the *y axis* is known as its *x coordinate* or its *abscissa*.

The distance of the point P (OM or NP) from the *x axis* is known as its *y coordinate* or its *ordinate*.

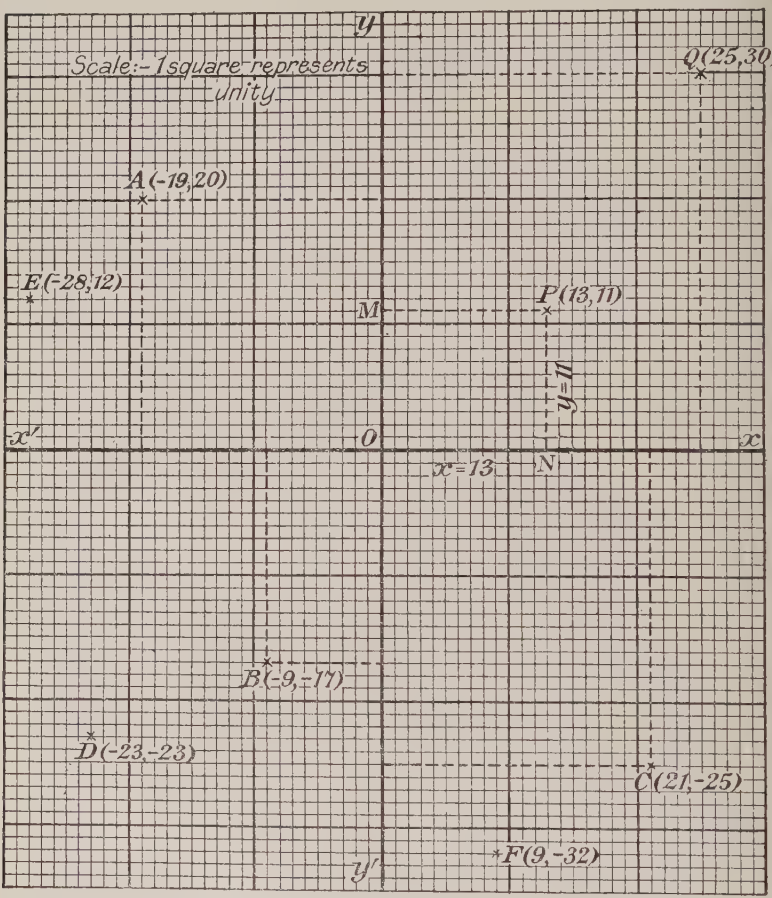


Fig. 46.

Thus the coordinates of P are $x = 13$, $y = 11$, *or* P is described as the point (13, 11).

Note.—The distance from YOY' ($x = 13$) measured along OX is given first and the distance from XOX' ($y = 11$) is given second.

Similarly, Q is described as the point (25, 30).

Now the point A is 19 units *to the left* of YOY', and to indicate this we say that *its x coordinate* is (-19). It is 20 units *above* XOX' and its *y coordinate* is 20. Then A is the point $(-19, 20)$.

Again the point B is 17 units *below* XOX', and to indicate this we say that its *y coordinate* is (-17), and since its x coordinate is (-9), then B is the point $(-9, -17)$.

Similarly, C is the point $(21, -25)$; D is the point $(-23, -23)$; E is the point $(-28, 12)$; F is the point $(9, -32)$.

Then

Measurements along XOX' to the right of O are positive.
" " " " left of O " negative.
" " YOY' upwards from O are positive.
" " " downwards " " negative.

86. *The Straight Line Graph.*

1. The following table gives the coordinates of a series of points.

Values for x . .	-3	-2	$-1{\cdot}5$	0	1	2·5	4
Corresponding values for y .	-6	-4	-3	0	2	5	8

Plot these points as in Fig. 47 and you notice they lie on a straight line—AB in the diagram.

Notice that for all the points given to be plotted, the y *coordinate is twice the* x *coordinate.*

Now take any point P on the line AB. Its coordinates are $x = 2$, $y = 4$.

Write down the coordinates of the points Q and R.

Do you find as before that the y coordinate is twice the x coordinate ?

The line AB may be extended if the paper will allow, and for every point on AB we shall find the ordinate is twice the abscissa.

Hence if x and y are the coordinates of any point on the line AB, we know that $y = 2x$, and $y = 2x$ is called the EQUATION OF THE LINE AB.

2. Plot the following set of points.

Values for x . .	-4	-2	$-0{\cdot}5$	0	4	7	8
Corresponding values for y .	$+2$	$+1$	$+0{\cdot}25$	0	-2	$-3{\cdot}5$	-4

These, again, you notice lie on a straight line—CD in diagram (Fig. 47).

Comparing the coordinates of the given points, we notice : (1) when x is positive, y is negative, and *vice versa* ; (2) the numerical value of y is always half that of x.

What are the coordinates of the points S and T ?

Do you find the same observations as before ? Then if any point (x, y) be taken on the line CD, we find that $y = -\frac{x}{2}$ and we say :

The EQUATION OF THE LINE CD is $y = -\frac{1}{2}x$.
or $y = -0{\cdot}5\,x$.

3. Plot the points whose coordinates are :—

Values for x .	5	5	5	5	5	5	5
Corresponding values for y .	0	2	3·5	5·25	− 1	− 3·75	− 5

These we notice lie on a straight line EF (Fig. 47), which is parallel to YOY' and 5 units distant from it.

For all points on this line the abscissa (x coordinate) is the same, whatever the value of the ordinate (y coordinate).

Then The *equation of the line* EF is $x = 5$.

Similarly The equation of the line GH (Fig. 47) is $x = -4{\cdot}75$.

4. In the same way if we plot a number of points whose ordinates are all equal to 7·25 and join them, we get a straight line KL parallel to XOX', and we say the line is the graph of $y = 7{\cdot}25$.

Similarly The equation of the line MN (Fig. 47) is $y = -1{\cdot}25$.

87. *Functions.*—Suppose a train is travelling at a constant rate, say 45 miles per hour, then in 2 hours it covers a distance of (45×2) miles ; in $5\frac{1}{2}$ hours it covers a distance of $(45 \times 5\frac{1}{2})$ miles ; in t hours it covers a distance of $(45 \times t)$ miles, and we notice that the distance travelled becomes known when the *time* of travel is known.

If distance travelled in t hours be s miles, then $s = 45\,t$.

Given any value of t there is a definite value for s corresponding to it.

We express this relation by saying that s is a *function of* t.

Again, it has been seen that

Circumference of a circle $= \pi \times$ diameter,

and for any particular value of the diameter there is a definite value for the circumference corresponding to it. Thus if D represents the number of units in the diameter, the value of

πD depends upon the value of D, and we say that

The circumference (πD) is a *function of* D.

Returning to the line AB in (Fig. 47), we said its equation was $y = 2x$.

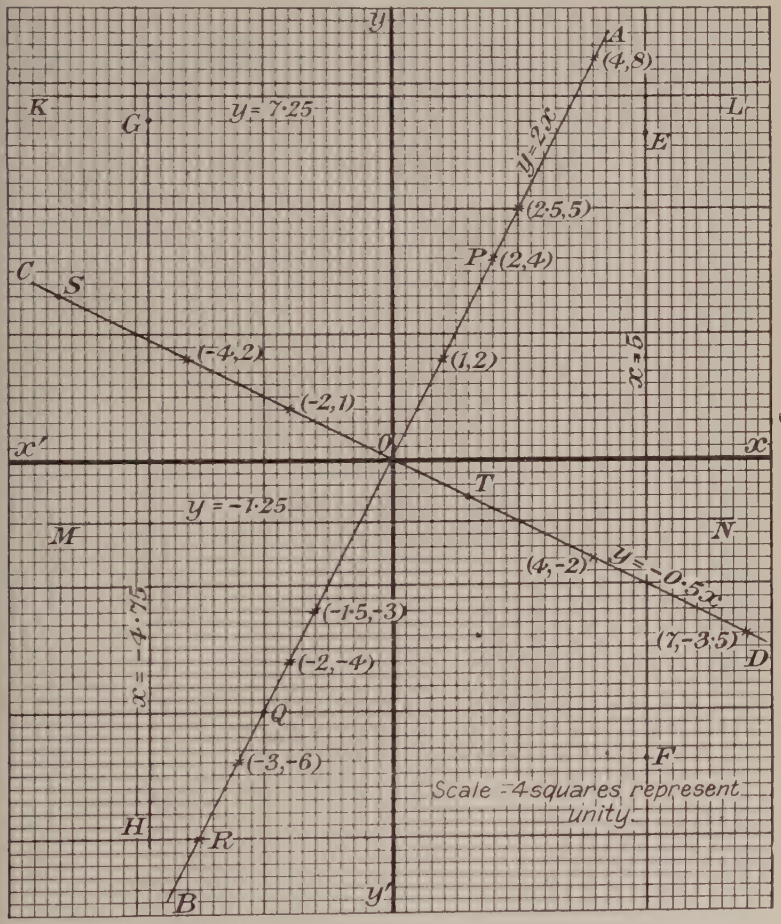


Fig. 47.

and the value of y or $2x$ depends on the value of x, then y or $2x$ is a *function of* x.

Similarly, $-0{\cdot}5x$ is a function of x.

The lines AB and CD (Fig. 47) show us how the functions of x alter in value as x alters.

88. *Graphs of Functions of x.*

EXAMPLE 1.

To draw the graph of $(\frac{4}{3}x + 5)$,

$(\frac{4}{3}x + 5)$ is a function of x of the first degree, that is, the highest power of x in the expression is the first power.

Let $y = \frac{4}{3}x + 5$.

Values for x . .	-12	-9	-3	0	3	6	10	12
Corresponding values for y or function of x	-11	-7	1	5	9	13	$18\frac{1}{3}$	21

Plotting these corresponding values of x and y on squared paper we get a series of points which lie on a straight line AB (Fig. 48).

1. Now consider any point P on the line.

Its coordinates are $(x = 2\,;\ y = 7\frac{2}{3})$.

Now when $x = 2$, the value of the function of x

$$(\tfrac{4}{3}x + 5) = (\tfrac{4}{3} \times 2) + 5.$$
$$= 2\tfrac{2}{3} + 5.$$
$$= 7\tfrac{2}{3}.$$

That is, the y coordinate of P shows us the value of the function when the value of x is the same as the x coordinate of P.

The student should test in the same way the coordinates of other points as Q, R and S.

2. Within the limits of the paper the graph AB will tell us the values of $(\frac{4}{3}x + 5)$ for any value of x, and, conversely, we may find the value of x for which the function $(\frac{4}{3}x + 5)$ has some particular value.

Thus suppose we wish to find the value of x which makes the function have the value $10\frac{1}{3}$.

We require the point whose y coordinate is $10\frac{1}{3}$.

Measure OD along the y axis $= 10\frac{1}{3}$ units.

Draw DC parallel to the x axis meeting the graph at C. Then the coordinates of C are $(x = 4\,;\ y = 10\frac{1}{3})$.

That is when $x = 4$,

$$\frac{4x}{3} + 5 = 10\tfrac{1}{3}.$$

Note.—We have solved the simple equation $\frac{4}{3}x + 5 = 10\frac{1}{3}$ by means of the graph.

Exercise.—Use the graph to solve:

1. $\frac{4}{3}x + 5 = 3\frac{2}{3}$; 2. $\frac{4}{3}x + 5 = 0$; 3. $\frac{4}{3}x + 5 = -9\frac{2}{3}$.

3. Find the coordinates of the points M and N and others not on the graph. See if these values satisfy the equation

$$y = \tfrac{4}{3}x + 5.$$

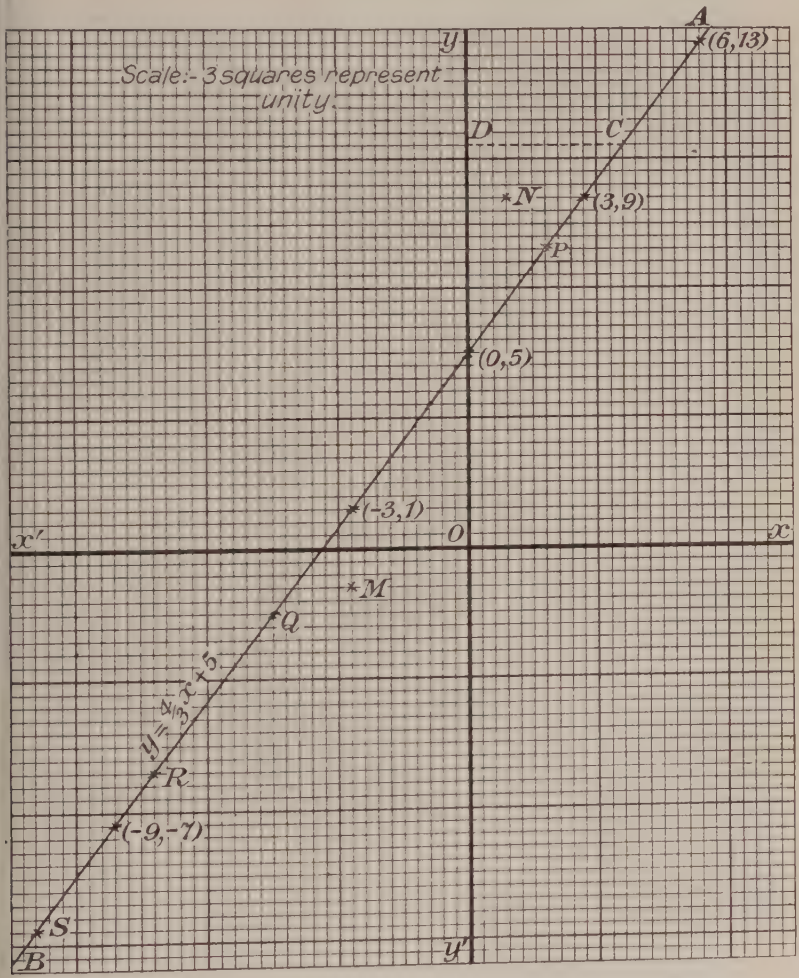


FIG. 48.

In this way you will be able to trust the statement that "For all points on the line AB and no others, the y coordinate is equal to $\frac{4}{3}$ (the x coordinate) + 5. Or we may say, the y

coordinate gives the value of the function of x when x has the value of the x coordinate of the point."

Thus THE LINE IS THE GRAPH OF $(\frac{4x}{3} + 5)$, or THE EQUATION OF THE LINE is $y = \frac{4}{3}x + 5$.

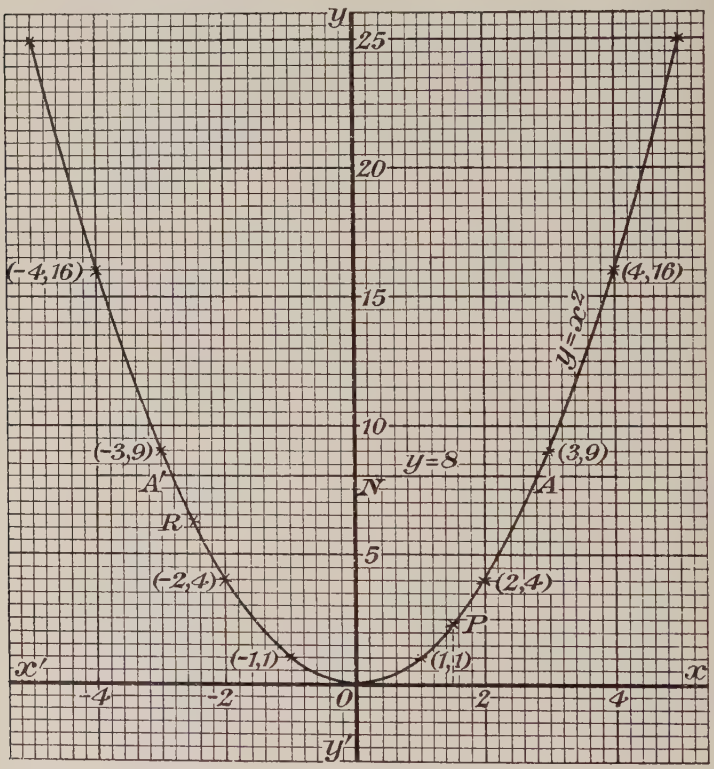


FIG. 49.

EXAMPLE 2.

Draw the graph of x^2.

x^2 is a function of x of the second degree.

Let the value of the function be y, then $y = x^2$.

Values for x . .	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$
Corresponding values for y .	$+16$	$+9$	$+4$	$+1$	0	$+1$	$+4$	$+9$	$+16$

Plotting these values on squared paper we find that the points lie on a curve (Fig. 49).

1. Observe that whether x is positive or negative, y is always positive, and the curve lies in the first and second quadrants.

2. Consider any point P on the curve.
Its coordinates are $(x = 1{\cdot}5 : \ y = 2{\cdot}25)$.
Now when $x = 1{\cdot}5$, $x^2 = 1{\cdot}5 \times 1{\cdot}5 = 2{\cdot}25$.
Hence the y coordinate of P = (the x coordinate)^2.

3. Find coordinates of any other points on curve as Q and R, and verify that $y = x^2$.

4. Find the value of x which makes $x^2 = 8$.
We must find the point whose y coordinate is 8.
Measure ON (8 units) along the y axis. Through N draw line ANA' parallel to the X axis.
The x coordinate of $A = 2{\cdot}82$, and for A', $x = -2{\cdot}82$.
Hence when $x^2 = 8$, $x = \pm 2{\cdot}82$.
Note.—We have found the $\sqrt{8}$ by means of the graph.

5. Use the graph to find

$$\sqrt{10};\ \sqrt{2},\ \sqrt{12},\ \sqrt{14{\cdot}5},\ \sqrt{3{\cdot}25},\ (0{\cdot}6)^2,\ (-2{\cdot}5)^2.$$

Example 3.

Draw the graph of $(x^2 - 1{\cdot}5x - 4{\cdot}5)$ between the values $x = -5$ and $x = 5$.

$(x^2 - 1{\cdot}5x - 4{\cdot}5)$ is a FUNCTION OF x OF THE SECOND DEGREE since it contains a term x^2.

Let the value of the function be represented by y.
Then $y = x^2 - 1{\cdot}5x - 4{\cdot}5 = x^2 - (1{\cdot}5x + 4{\cdot}5)$.

Values for x	-5	-4	-3	-2	-1	0
„ „ x^2	25	16	9	4	1	0
„ „ $1{\cdot}5x$	$-7{\cdot}5$	-6	$-4{\cdot}5$	-3	$-1{\cdot}5$	0
„ „ $(1{\cdot}5x + 4{\cdot}5)$	-3	$-1{\cdot}5$	0	$1{\cdot}5$	3	$4{\cdot}5$
„ „ $y = \{x^2 - (1{\cdot}5x + 4{\cdot}5)\}$	28	$17{\cdot}5$	9	$2{\cdot}5$	-2	$-4{\cdot}5$

Values for x	1	2	3	4	5
„ „ x^2	1	4	9	16	25
„ „ $1{\cdot}5x$	$1{\cdot}5$	3	$4{\cdot}5$	6	$7{\cdot}5$
„ „ $(1{\cdot}5x + 4{\cdot}5)$	6	$7{\cdot}5$	9	$10{\cdot}5$	12
„ „ $y = \{x^2 - (1{\cdot}5x + 4{\cdot}5)\}$	-5	$-3{\cdot}5$	0	$5{\cdot}5$	13

1. Take any point P on the curve (Fig. 50). Its coordinates are $x = 4{\cdot}4$; $y = 8{\cdot}2$.

Now if $x = 4{\cdot}4$

$$y = (4{\cdot}4)^2 - (1{\cdot}5 \times 4{\cdot}4) - 4{\cdot}5.$$
$$= 19{\cdot}36 - 6{\cdot}6 - 4{\cdot}5.$$
$$= 8{\cdot}26.$$

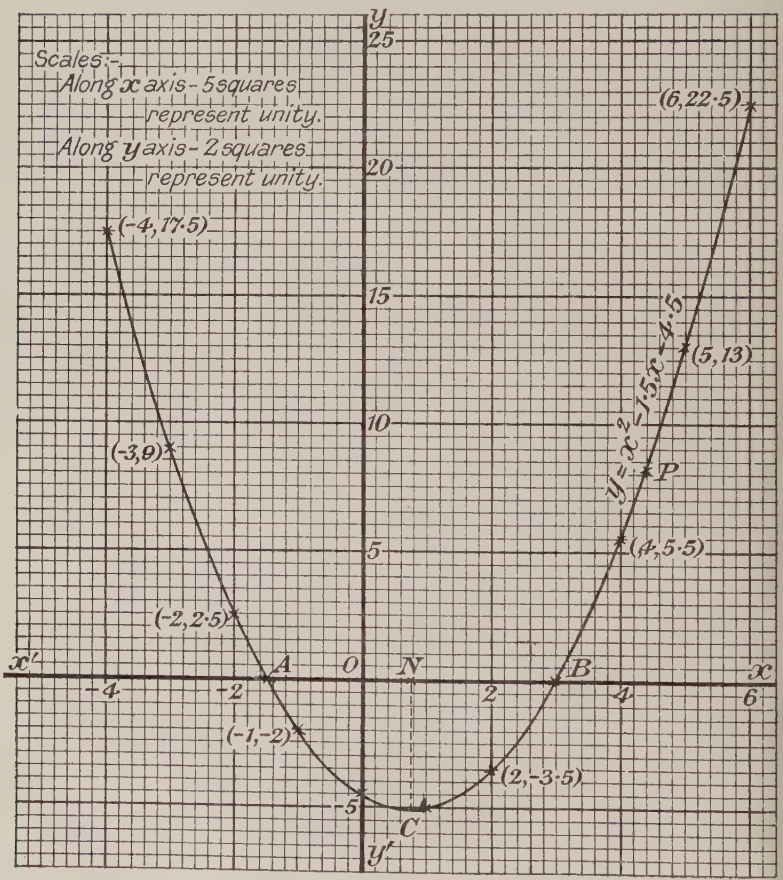


Fig. 50.

Then the coordinates of P satisfy the relation

$$y = x^2 - 1{\cdot}5x - 4{\cdot}5.$$

Now try other points on the curve.

2. Notice at the points A and B the curve crosses the X axis. The y coordinate of each of these points is zero.

And we learnt above :

The y coordinate of any point on the curve represents the alue of the function of x $(x^2 - 1{\cdot}5x - 4{\cdot}5)$ for that par- icular value of x given by the x coordinate of the point.

The x coordinate of $B = 3$.

Then when $x = 3$. function x $(x^2 - 1{\cdot}5x - 4{\cdot}5) = 0$, nd this we found when making the table.

The x coordinate of A is $-1{\cdot}5$, and when $x = -1{\cdot}5$,

$$x^2 - 1{\cdot}5x - 4{\cdot}5 = 2{\cdot}25 + 2{\cdot}25 - 4{\cdot}5 = 0.$$

Thus $x^2 - 1{\cdot}5x - 4{\cdot}5 = 0$, when $x = 3$ or $-1{\cdot}5$.

THE GRAPH ENABLES US TO SOLVE THE EQUATION OF THE ECOND DEGREE IN x.

$$x^2 - 1{\cdot}5x - 4{\cdot}5 = 0.$$

3. From the graph, solve the following equations :

(a) $x^2 - 1{\cdot}5x - 4{\cdot}5 = 10.$
(b) $x^2 - 1{\cdot}5x - 4{\cdot}5 = 17{\cdot}5.$
(c) $x^2 - 1{\cdot}5x - 10 = 0.$

4. *Notice.*—As the value of x increases from (-5) to $(-1{\cdot}5)$, he value of the function (y) decreases, becoming zero when $= -(-1{\cdot}5)$, and as x continues to increase, the function ontinues to decrease, becoming negative up to a certain point, when it begins to increase till it is again zero, when $x = 3$. Also, as x increases indefinitely beyond 3, the function increases ndefinitely.

If C marks the turning point of the curve, then NC (the coordinate of C) gives the MINIMUM VALUE of $x^2 - 1{\cdot}5x - 4{\cdot}5$, which is $(-5{\cdot}1$ approx.), and ON (the x coordinate of C) gives the value of x, which makes the function of x a minimum, nd this value of $x = 0{\cdot}75$.

The student should now give to x values a little below and little above 0·75, and see if the value of the function is greater han the minimum value given by the graph.

Thus :

Values for x . . .	0·6	0·65	0·7	0·75
,, ,, x^2 . . .	0·36	0·4225	0·49	0·5625
,, ,, $1{\cdot}5x$ . . .	0·9	0·975	1·05	1·125
,, ,, $(1{\cdot}5x+4{\cdot}5)$.	5·4	5·475	5 55	5·625
,, ,, y or $\{x^2-(1{\cdot}5x+4{\cdot}5)\}$	−5·04	−5·0525	−5·06	−5·0625

Values for x . . .	0·8	0·85	0·9
,, ,, x^2 . . .	0·64	0·7225	0·81
,, ,, $1{\cdot}5x$ . . .	1·2	1·275	1·35
,, ,, $(1{\cdot}5x+4{\cdot}5)$.	5·7	5·775	5·85
,, ,, y or $\{x^2-(1{\cdot}5x+4{\cdot}5)\}$	−5·06	−5·0525	−5·04

We notice as x increases from 0·6 to 0·75, the value of y decreases from $-5{\cdot}04$ to $-5{\cdot}0625$. And as x increases from 0·75 to 0·9, y increases from $-5{\cdot}0625$ to $-5{\cdot}04$.

Now try the following values for x,

0·74, 0·745, 0·75, 0·755, 0·76,

and again the values for y are greater than $-5{\cdot}0625$. The student who is interested will try values for x, such as 0·749, 0·75, 0·751, or 0·74999, 0·75, 0·75001, and in this way accept the fact shown by the curve, that "when $x = 0{\cdot}75$ $x^2 - 1{\cdot}5x - 4{\cdot}5$ has its least value $-5{\cdot}0625$, and that for all other values of x the value of the function is greater than $-5{\cdot}0625$."

89. *The Linear Graph.*—The student will now have noticed that the graphs of functions of x of the first degree are straight lines.

Thus y (or function x) $= 2x + 3$,
$y = 1{\cdot}5 + 2{\cdot}3x$,
$y = 1{\cdot}5 - 2{\cdot}7x$,
$4y = 12 - 3x$,
have all straight line graphs.

Hence an expression of this form is called a LINEAR EXPRESSION.

The general form of an expression of the first degree in x is $ax + b$, where a and b represent some numerical constants.

Thus in $2x + 3$ $a = 2$; $b = 3$
in $1{\cdot}5 + 2{\cdot}3x$ $a = 2{\cdot}3$; $b = 1{\cdot}5$
and in $1{\cdot}5 - 2{\cdot}7x$ $a = -2{\cdot}7$; $b = 1{\cdot}5$.

Conversely, we may say that any straight line whatever may be represented algebraically by an equation of the form

$$y = ax + b.$$

Of course it is not necessary to use the letters a and b. We may write the equation

$$y = mx + c \quad \text{or} \quad y = px + q,$$

or $y = (\textit{some constant}) \times x + (\textit{another constant})$.

Thus assuming that the graph of an expression of the form $y = ax + b$ is a straight line, the graph of such an expression may be plotted rapidly by finding the values of y for *any two* values of x, plotting the points whose coordinates are thus determined, and joining these points by a straight line. This will save time and labour in succeeding examples.

90. *Meaning of the constants* a *and* b *in the equation to a straight line.*

I. Consider a *series of equations in which the* a's *are always the same and the* b's *are different.*

Fig. 51 shows the graphs of the straight lines:
$= \frac{2}{3}x$ (1); $y = \frac{2}{3}x + 5$ (2); $y = \frac{2}{3}x - 8$ (3); $y = \frac{2}{3}x + 7{\cdot}5$ (4).
For line (1) If $x = 3$ then $y = \frac{2}{3} \times 3 = 2$
,, $x = -9$,, $y = \frac{2}{3}(-9) = -6$.
Then the points (3, 2); (− 9, −6) fix the first line.

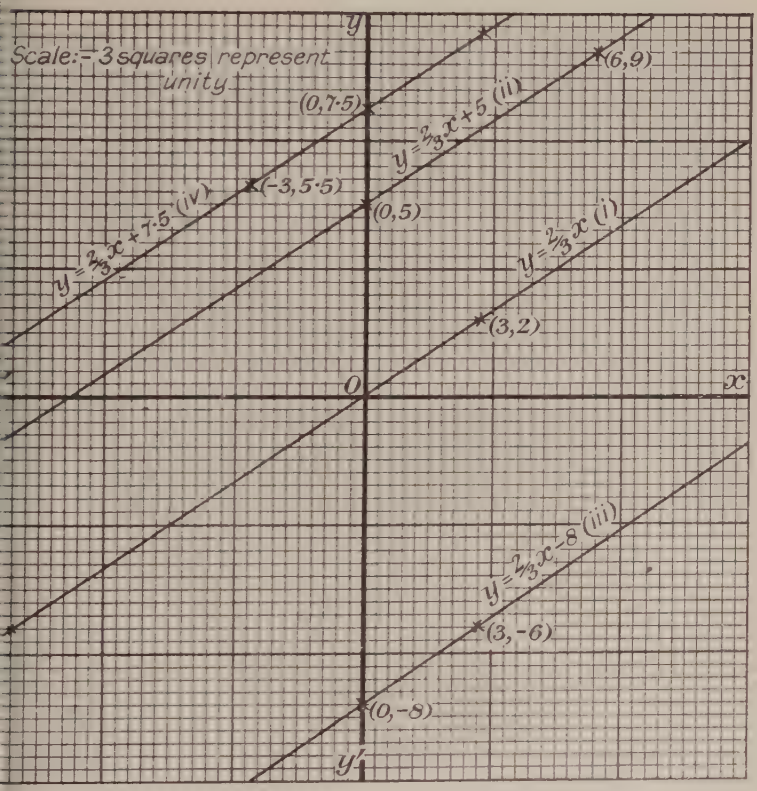


Fig. 51.

For line (2) If $x = 6$ then $y = \frac{2}{3}(6) + 5 = 9$.
,, $x = -3$,, $y = \frac{2}{3}(-3) + 5 = 3$.
Then the points (6, 9); (− 3, 3) fix the second line.
Points on the other lines are found in a similar manner
d plotted. Examining these graphs we notice:—
1. *The lines run in the same direction or are parallel to each*
er.

2. In line (1) $y = \frac{2}{3}x$; $b = 0$, and the line passes through the origin.

3. If b is positive, as in lines (2) and (4), the lines cut the y axis above the origin.

4. If b is negative, as in line (3), the line cuts the y axis below the origin.

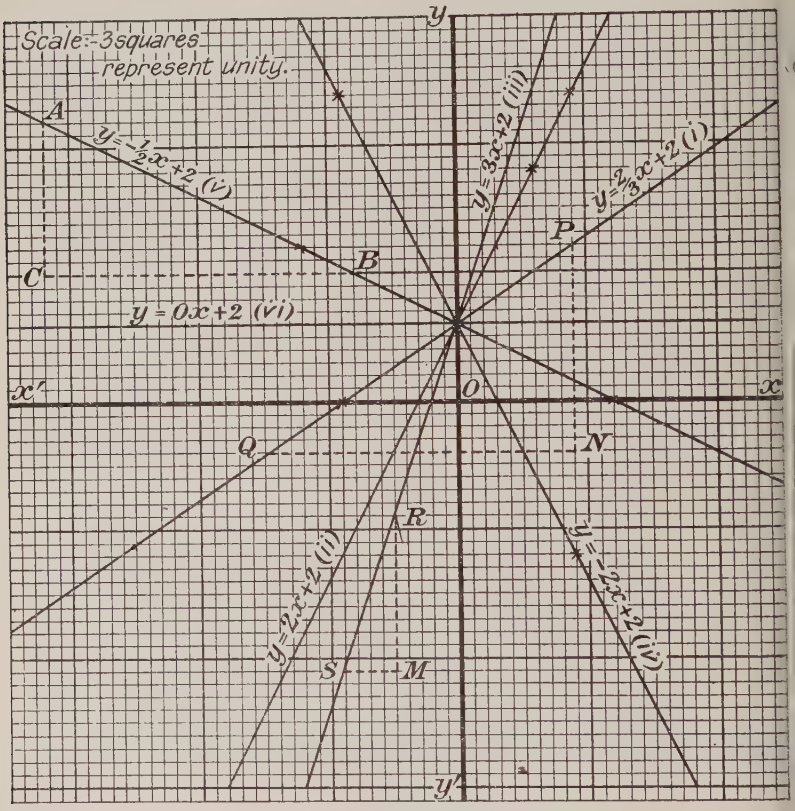


Fig. 52.

5. In each case the value of b gives the value of the y co ordinate of the point at which the line crosses the y axis.

This may also be seen by considering the equations.

When $x = 0$ $y = 5$ in line (2),
$y = -8$,, (3),
$y = 7{\cdot}5$,, (4).

And in the general equation $y = ax + b$,
hen $x = 0$; $y = b$.

II. Consider a *series of equations in which the* b's *are always he same and the* a's *vary.*

Fig. 52 shows the graphs of the straight lines:

$y = \frac{2}{3}x + 2$ (1) $y = 2x + 2$ (2) $y = 3x + 2$ (3)
$y = -2x + 2$ (4) $y = -\frac{1}{2}x + 2$ (5) $y = 0x + 2$ (6).

As before, the coordinates of two points on each line have een found and the lines drawn through these points.

Examining these graphs we notice :—

1. *These lines all pass through one common point* (0, 2), for each equation if $x = 0$ then $y = 2$; in other words, $b = 2$ each equation.

2. *The lines have different slopes* or *they make different angles ith the axis of* x.

3. When a is large [$a = 3$ in line (3)] and positive, the line opes steeply upwards.

As a gets smaller and still positive [lines (2) and (1)] the line ecomes less steep, and

When a is zero [line (6)] the slope of the line is zero; in other ords, the line is parallel to the axis of x.

When a is negative, the line slopes downwards more and ore as a increases numerically.

4. *Slope of a Line.*—If we take any two points P and Q ig. 52) on line (1) [$y = \frac{2}{3}x + 2$], and draw lines QN and N parallel to the axes of x and y respectively, then the *lue of the fraction* $\frac{PN}{QN}$ *is called the slope of the line*; PN and N, being measured according to the scale proper to the axis which it is parallel and in the positive direction along that xis.

PN is 16 squares long and represents $5\frac{1}{3}$ units (3 squares represents unity).

QN is 24 ,, ,, and ,, 8 ,,

$$\therefore \frac{PN}{QN} = \frac{5\frac{1}{3}}{8} = \frac{16}{24} = \frac{2}{3} = a \text{ in equation (1).}$$

Let us consider this from a different standpoint.

(*a*) Line (1) [$y = \frac{2}{3}x + 2$] (Fig. 52) shows us how the function x ($\frac{2}{3}x + 2$) grows as x grows, and we notice that as x continuly increases y or the value of the function ($\frac{2}{3}x + 2$) ntinually increases.

At the point Q, $x = -5$ and y or $(\frac{2}{3}x + 2) = -\frac{4}{3}$
,, ,, P, $x = 3$,, y or $(\frac{2}{3}x + 2) = 4$.

Then, as x increases by an amount QN, y increases by an amount PN, and the value of the fraction $\frac{PN}{QN}$ shows us the increase in y for a *unit* increase in x, or the slope of the straight line measures the rate of increase of y with respect to x.

(*b*) Again in line (3) $[y = 3x + 2]$:

At the point S, $x = -3$ and y or $(3x + 2) = -7$.

„ „ R, $x = -\frac{5}{3}$ „ y or $(3x + 2) = -3$.

Then, as x increases by an amount SM, y also increases by an amount RM, and

$$\text{Increase of } y \text{ per unit increase of } x = \frac{RM}{SM}$$
$$= \frac{4}{\frac{4}{3}} = \frac{3}{1}$$
$$= a \text{ in equation (3).}$$

(*c*) In line (5) $[y = -\frac{1}{2}x + 2]$ (Fig. 52) take any two points A and B.

At the point A, $x = -10\frac{2}{3}$ and y or $(-\frac{x}{2} + 2) = 7\frac{1}{3}$.

„ „ B, $x = -2\frac{2}{3}$ „ y or $(-\frac{x}{2} + 2) = 3\frac{1}{3}$.

Here, as x increases by an amount CB, y *decreases* by an amount AC.

Now if an increase be written as a positive quantity, then a decrease may be indicated by a negative quantity. Thus we say :—

$$\textit{The slope of the line } \text{AB} = \frac{\text{Increase of } y}{\text{Increase of } x}$$
$$= -\frac{AC}{CB} = -\frac{4}{8}$$
$$= -\frac{1}{2} = a \text{ in equation (5).}$$

(*d*) In line (6) $[y = 0x + 2]$ $a = 0$ and the line is parallel to the axis of x.

5. The points P, Q, R, S, etc., on the lines have been chosen at random, and the student should copy the figure and choosing other points verify that

The slope of the line = the VALUE OF a in the equation of the line.

If whilst x increases steadily, y also increases steadily, then the line slopes upward, and is said to have a *positive gradient* and its value a is positive.

If whilst x increases steadily, y decreases steadily, then the line slopes downward and may be said to have a *negative gradient*, and its value a is negative.

III. Collecting our conclusions from the series of equations I., and II., we have—

If the equation for the line AB (Fig. 53) be of the form

$$y = ax + b,$$

(1) $OC = b$ = the y coordinate of the point at which AB cuts the y axis.

(2) $a = \dfrac{PN}{NQ}$ = gradient of the line AB.

= increase of y per unit increase of x.

(3) b is positive or negative according as C is above or below the axis of x.

(4) a is positive or negative according as the line slopes upward or downward.

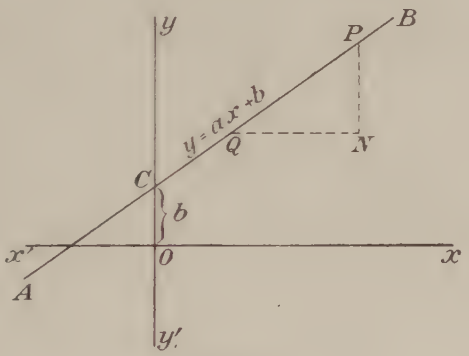


FIG. 53.

(5) If $b = 0$, the line passes through the origin.

(6) If $a = 0$, the line is parallel to the x axis and at a distance of b units from it.

EXAMPLE 1.

Plot the points (4, 0) ; (− 4, − 6). Join these by a straight line and find its equation.

In Fig. 54, A is the point (4, 0) ; B is the point (− 4, − 6).

The equation of the line AB is of the form

$$y = ax + b$$

when $x = 0$; $y = b$.

The line AB crosses the y axis at the point C ; the x coordinate of C is zero.

$\therefore$ $b = OC = -3$.

Again, the gradient of the line $= \frac{AN}{CN} = \frac{3}{4}$ (1).

$= \frac{AM}{BM} = \frac{6}{8} = \frac{3}{4}$ (2).

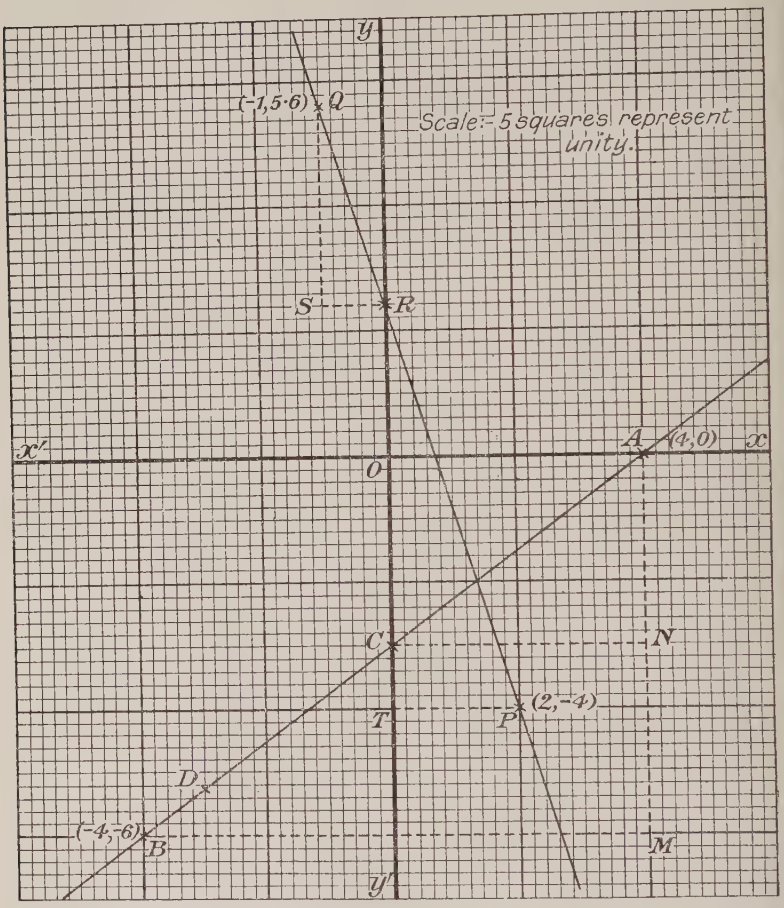


Fig. 54.

And since y increases as x increases, the line has a positive slope or gradient.

$\therefore \qquad a = +\frac{3}{4},$

and the equation for the line AB is

$$y = \tfrac{3}{4}x - 3 \quad \text{(I)}.$$

Multiply both sides by 4, and this may be written

$$4y = 3x - 12 \quad \text{(II)}.$$

Now take any point D on the line. Write down its co-ordinates, and check your result by noticing that the co-ordinates of D satisfy the equation obtained.

Thus Coordinates of D are $x = -3$; $y = -5{\cdot}25$.

Then $4y = 4(-5{\cdot}25) = -21$.

$3x - 12 = -9 - 12 = -21$.

Then $4y = 3x - 12$, when $x = -3$; $y = -5{\cdot}25$.

Example 2.

Find the equation of the line joining the points $(2, -4)$ and $(-1, 5{\cdot}6)$.

In Fig. 54, P is the point $(2, -4)$; Q is the point $(-1, 5{\cdot}6)$.

The equation of the line PQ is of the form

$$y = ax + b.$$

As in Example 1 $b = OR = 2{\cdot}4$.

The gradient of the line $= \dfrac{QS}{SR} = \dfrac{3{\cdot}2}{1}$ (1).

$= \dfrac{RT}{TP} = \dfrac{6{\cdot}4}{2} = \dfrac{3{\cdot}2}{1}$ (2).

And since y decreases as x increases, the line has a negative slope.

$\therefore \quad a = -3{\cdot}2,$

and the equation for the line is

$$y = -3{\cdot}2x + 2{\cdot}4,$$

or $$y + 3{\cdot}2x = 2{\cdot}4.$$

91. *Some Applications of Graphs.*—A graph accurately drawn to a suitable scale may often be used as a "*Ready Reckoner.*"

It is very important that the student should draw his diagrams on a sufficiently large scale, and that he should be careful in the choice of units. These *should always be clearly marked on the axes.*

Example 1.

To construct a graph which will enable you to convert, at sight, worsted counts into cotton counts and *vice versa*.

Taking any particular yarn.

Suppose its counts in worsted denomination $= x$.

" " " cotton " $= y$.

Now worsted counts being based on a hank of 560 yards,
and cotton ,, ,, ,, ,, ,, 840 yards,
x's worsted means $560x$ yards weigh 1 lb.

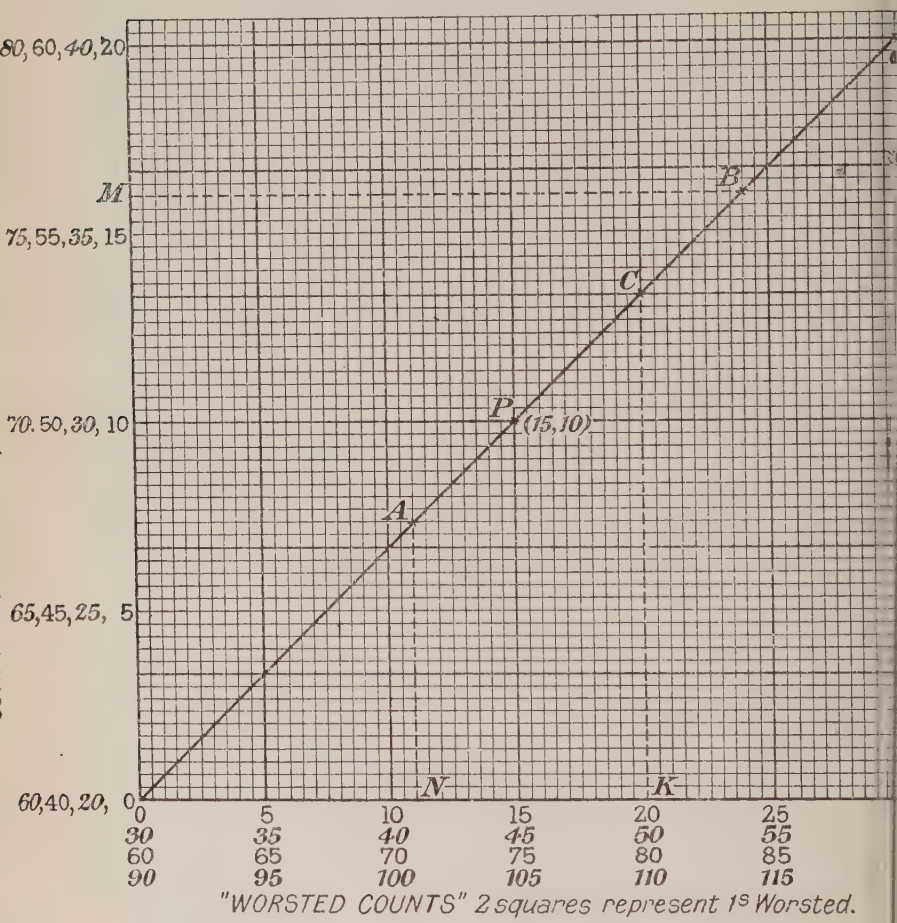


FIG. 55.—To change Worsted Counts to Cotton Counts and *vice versa*.

Number of hanks (840 yards long) in $560x$ yards $= \frac{560\,x}{840}$

or Corresponding cotton counts $= \frac{560\,x}{840} = \frac{2}{3}x$.

But corresponding cotton counts $= y$.

$\therefore \quad y = \frac{2}{3}x.$

Therefore, if we draw the graph of this equation, the abscissæ of points on the graph will give us the counts in the worsted denomination, whilst the corresponding ordinates will give us the corresponding cotton counts.

The graph of $y = \frac{2}{3}x$ is a *straight line through the origin.*

When $x = 15$; $y = 10$, and the required graph is obtained at once by joining the origin to the point P, whose coordinates are (15, 10). [Fig. 55.]

1. *To change* 11's *worsted to its equivalent cotton counts.*

In Fig. 55 the x coordinate of the point $A = ON = 11$.

,, y ,, ,, ,, $A = AN = 7\frac{1}{3}$.

$\therefore$ From graph 11's worsted are equivalent to $7\frac{1}{3}$'s cotton.

2. *To change* 16's *cotton to worsted counts.*

In Fig. 55 the y coordinate of the point $B = OM = 16$.

,, x ,, ,, ,, $B = MB = 24$.

$\therefore$ From graph 16's cotton are equivalent to 24's worsted.

3. *To change* 50's *worsted to cotton counts.*

Now the coordinates of the point Q, the extremity of our graph, are (30, 20), which show us that 30's worsted are equivalent to 20's cotton, and we require a point whose x coordinate is 50.

We have seen that the coordinates of all points on a straight line satisfy the equation for that line, so that if OQ be produced any length, the coordinates of any point on the extension will give us corresponding worsted and cotton counts. Imagine the line moved along itself so that the point Q occupies the position O, and a point R on the line will occupy the position of Q. The student will readily see that :—

Since the coordinates of Q are (30, 20),

The x coordinate of $R = (30 + 30) = 60$.

,, y ,, $R = (20 + 20) = 40$.

And by numbering the unit divisions along the axes, beginning at 30 along the X axis and 20 along the Y axis, we may obtain the coordinates for all points on the line between Q and R.

This is shown in Fig. 55 in different type.

Thus the x coordinate of the point $C = 30 + 20 = 50$.

,, y ,, ,, ,, $C = 20 + 13\frac{1}{3} = 33\frac{1}{3}$.

$\therefore$ 50's worsted are equivalent to $33\frac{1}{3}$'s cotton.

4. Similarly, by commencing the numbering (*a*) along the x axis at 60, 90, 120, 150, etc.; (*b*) along the y axis at 40, 60, 80,

100, etc., the graph may be used to convert any number of worsted counts into cotton counts.

Use the graph to :
(*a*) Convert into cotton counts, 40's, 70's, 110's, 120's worsted.
(*b*) Convert into worsted counts, 50's, 70's, 90's and 100's cotton.

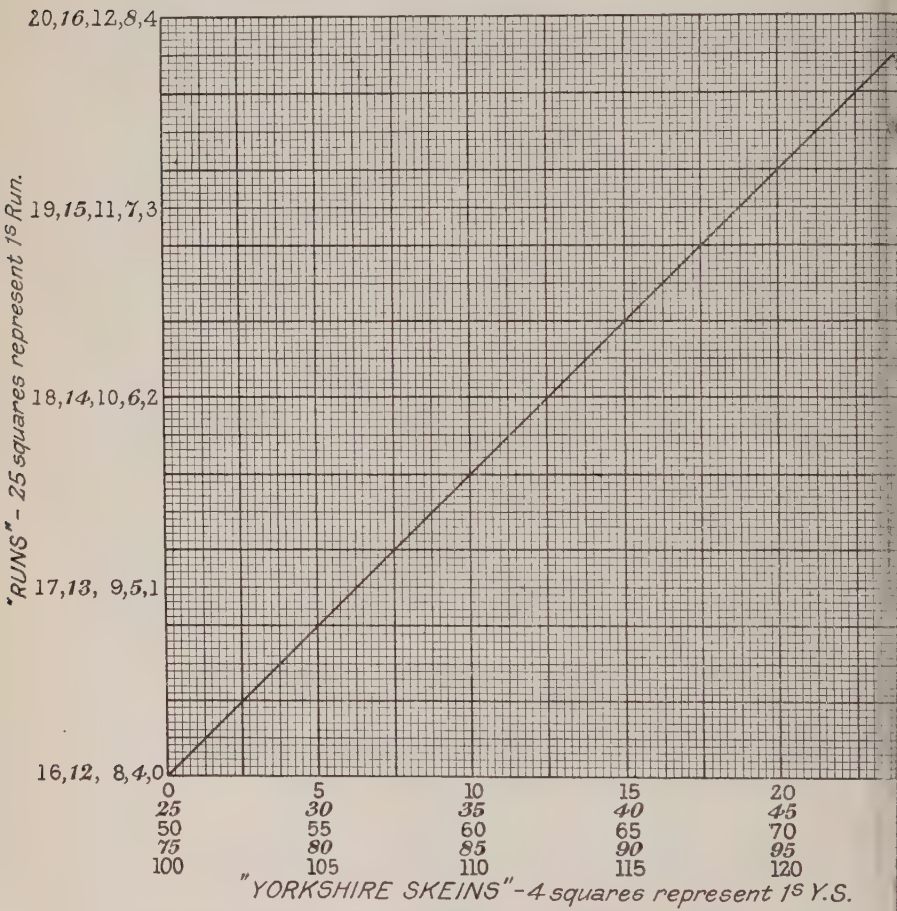


FIG. 56.—To convert Yorkshire Skeins to American "Runs" and *vice versa*.

EXAMPLE 2.

Draw a graph by which you can convert Yorkshire skeins woollen to American "Runs," and *vice versa*.

The "Yorkshire skein" system is based upon the number

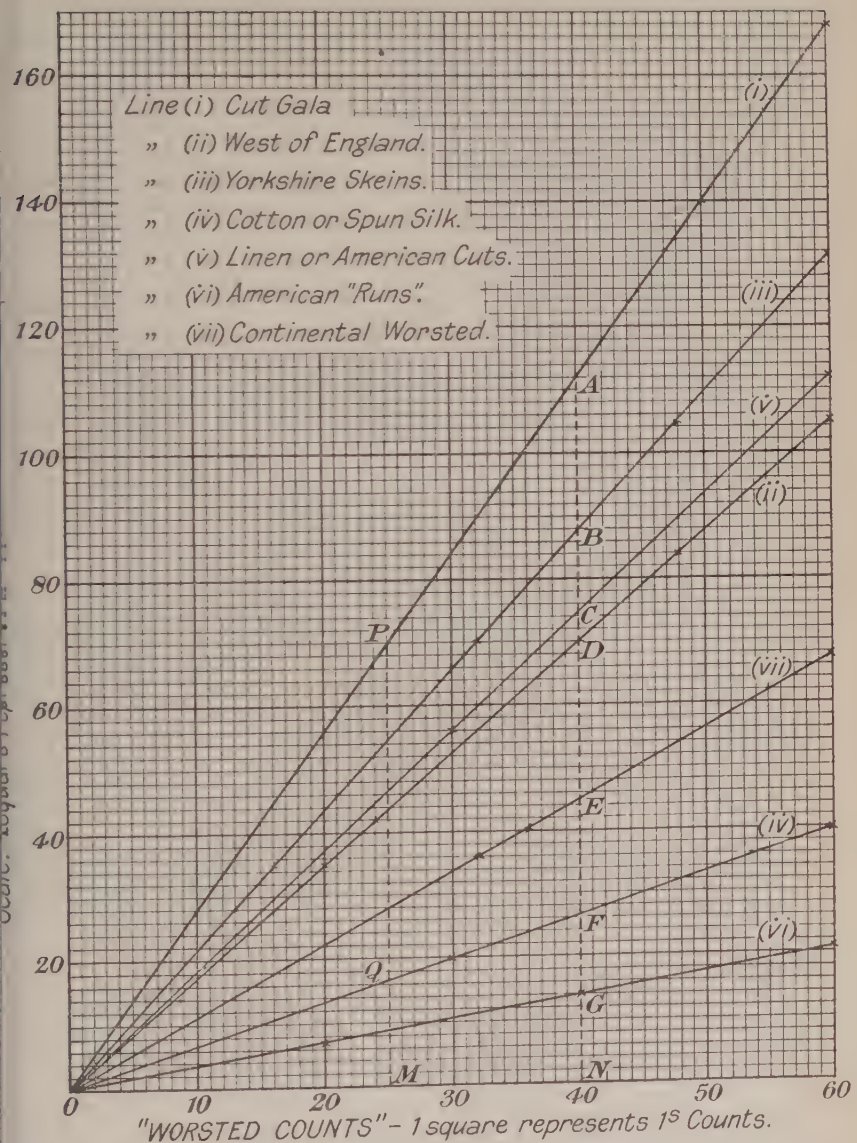


FIG. 37.—To convert Worsted Counts into their Equivalents in Various Systems and *vice versa*.

of skeins of 1536 yards each in one "warten" (6 lbs.) or, for convenience, the number of skeins of 256 yards per lb.

The "*Run*" is based upon 100 yards as unit of length and 1 ounce as the standard weight.

Taking any particular yarn.

Suppose its counts in Yorkshire skein system $= x$'s.

" " " American "Run" system $= y$'s.

Then x's Yorkshire skeins means 256 x yards weigh 1 lb.

And since 1's Run means 100 yards weigh 1 oz., or 1600 yards weigh 1 lb.

$\therefore$ Number of hanks (1600 yards long) in 256 x yards $= \dfrac{256x}{1600}$.

$\therefore$ Counts in Run system $= \dfrac{256\,x}{1600}$.

And, according to our supposition, this is y.

Hence $$y = \frac{256\,x}{1600}$$

or $$y = \tfrac{4}{25}x.$$

The graph is thus a straight line through the origin.

When $x = 25$; $y = 4$.

Plot the point A (25, 4). Join this point to the origin, and the graph is drawn as in Fig. 56.

By a system of numbering as explained in Example 1, we may use the graph over a wide range of numbers.

Use the graph to:

(*a*) Convert 30's, 40's, 2/36's, 80's Yorkshire skeins to "Runs."

(*b*) Convert 3's, 6's, 16's, 18's, 24's "Runs" to Yorkshire skeins.

Example 3.

To draw a graph which will enable you to convert worsted counts into (1) Cut Gala, (2) West of England (woollen), (3) Yorkshire skeins (woollen), (4) cotton or spun silk, (5) linen or American "Cuts" (woollen), (6) American "Runs" (woollen), (7) Continental worsted.

On referring to the various systems of counting yarns in Chapter I, and following the method of examples 1 and 2, the student may check the following equations for the graphs.

(1) $y = \tfrac{14}{5}x.$
(2) $y = \tfrac{7}{4}x.$
(3) $y = \tfrac{35}{16}x.$
(4) $y = \tfrac{2}{3}x.$
(5) $y = \tfrac{28}{15}x.$
(6) $y = \tfrac{7}{20}x.$
(7) $y = \tfrac{55}{21}x.$

In each equation x is the value of the counts in the worsted system, whilst y gives the corresponding value of the counts in the system to which we are converting them.

The graphs thus drawn are shown in Fig. 57.

1. Given 40's worsted and using the graph, its equivalent counts are :—

1. American "Runs"	$= GN$	$= 14$'s.
2. Cotton or spun silk	$= FN$	$= 26\frac{2}{3}$'s.
3. Continental worsted	$= EN$	$= 45\frac{1}{2}$'s.
4. West of England	$= DN$	$= 70$'s.
5. Linen or American "Cuts"	$= CN$	$= 74\frac{2}{3}$'s.
6. Yorkshire skeins	$= BN$	$= 87\frac{1}{2}$'s.
7. Cut Gala	$= AN$	$= 112$'s.

2. Each of these 7 is equivalent to any one of the other. Thus

112's Cut Gala $= 70$'s West of England,
$26\frac{2}{3}$'s cotton $= 74\frac{2}{3}$'s linen, etc.,

since each one is equivalent to 40's worsted.

3. The graph may be used to change counts in any one of the 8 denominations to equivalent counts in any other denomination.

For example : What are the equivalent cotton counts of 70's Cut Gala ? On graph (1) find the point P whose y coordinate $= 70$. This is the line PM.

Now PM crosses graph (4) at the point Q. The y coordinate of $Q = QM = 16\frac{2}{3}$.

The points P and Q have the same abscissa (x coordinate), viz. : $OM = 25$.

$\therefore$ 70's Cut Gala are equivalent to $16\frac{2}{3}$'s cotton, since each is equivalent to 25's worsted.

4. Now use the graph to work the following exercises, checking your results by working as shown in Chapter I.

(*a*) Convert the following into worsted counts, 18's Yorkshire skeins ; 2/60's cotton ; 35's West of England ; 18's Continental yarn.

(*b*) Convert the following into West of England counts, 20's worsted ; 60's Yorkshire skeins ; 90's linen.

(*c*) Convert into Gala Cuts and worsted hanks, 16's Yorkshire skeins.

(*d*) Convert into American "Runs" and "Cuts," 2/40's cotton ; 60's Yorkshire skeins ; 2/30's worsted.

Example 4.

To construct a graph to enable you to convert number of threads per inch (Huddersfield "sett" system) into (1) Bradford, (2) Leeds, (3) Dewsbury, (4) Scotch tweed, (5) metric, (6) Glasgow cotton sett systems.

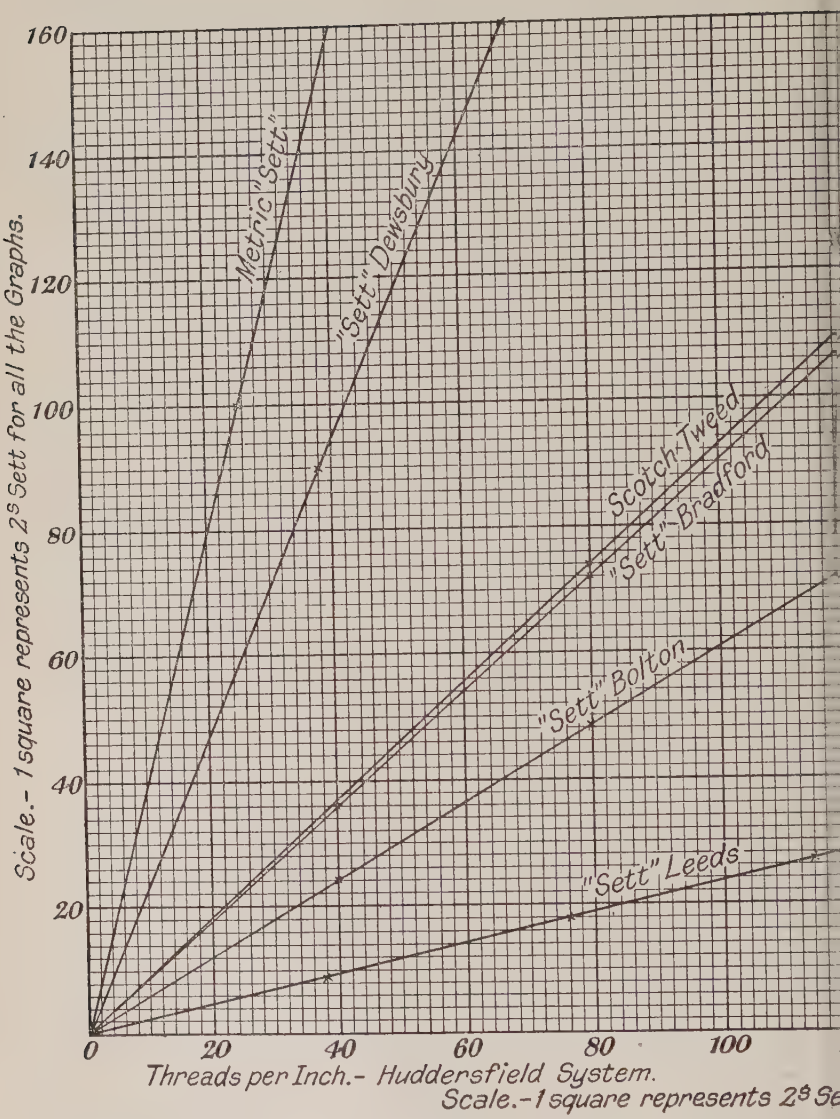


FIG. 58.—To convert the "Sett" in Various Systems to "Threads per Inch" and *vice versa*.

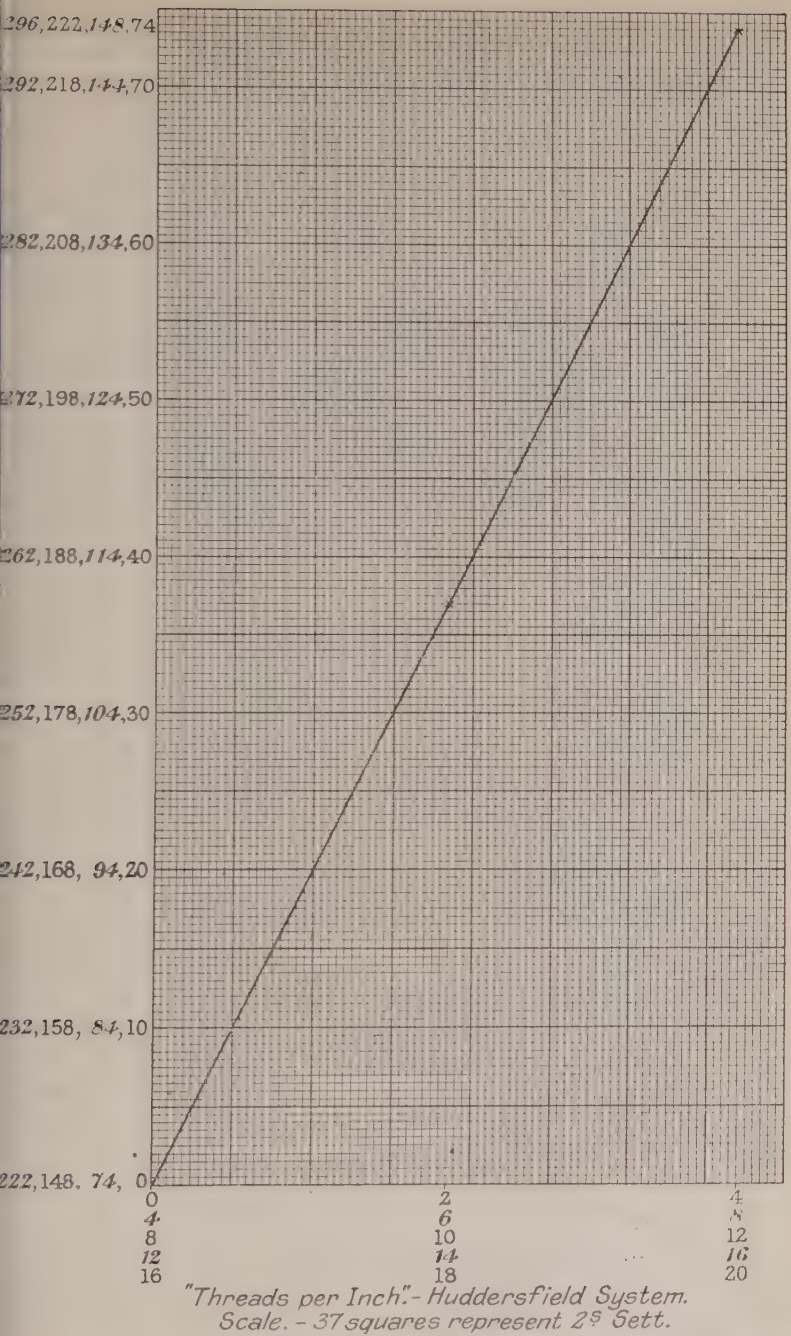


FIG. 59.—To convert "Threads per Inch" to "Setts Glasgow" and *vice versa*.

1. The Bradford sett system is based on a "sett" of 40 ends, and the number of these "setts" in a width of 36 inches gives the sett.

Suppose the "sett" be x in Huddersfield system.

" " " " y in Bradford "

Then x's sett Huddersfield means x ends per inch, or $36x$ ends in 36 inches.

Since a "sett" = 40 ends.

$$\therefore \quad \text{the "sett" Bradford} = \frac{36x}{40} = \frac{9x}{10}.$$

But we supposed the "sett" Bradford = y.

$$\therefore \quad y = \tfrac{9}{10}x \quad (1).$$

The graph is therefore a straight line through the origin.

2. Referring to the various "sett systems" given in Chapter II, the student should verify the following equations for the other graphs.

(2) $y = \dfrac{9x}{38}$ Huddersfield into Leeds system.

(3) $y = \dfrac{45x}{19}$ " " Dewsbury system.

(4) $y = \dfrac{37x}{40}$ " " Scotch Tweed system.

(5) $y = \dfrac{10x}{2{\cdot}54}$ " " Metric system.

(6) $y = \dfrac{37x}{2}$ " " Glasgow system.

The first five graphs are drawn as in Fig. 58. The graph for No. 6 is drawn as in Fig. 59.

Use the graph to work the following as correctly as possible, and check results as shown in Chapter II.

(*a*) If a cloth must contain 60 threads per inch, what sett equivalent to this in Bradford, Leeds, Dewsbury, Scotch Tweed and Glasgow Cotton?

(*b*) Given 80's sett Bradford, 20's sett Leeds, 150's sett Dewsbury. Find the number of ends per inch.

(*c*) Reduce 80's sett Bradford to Scotch Tweed and Leeds system.

92. *Simultaneous Equations.*

Introduction.—Suppose the following problem presents itself: "A warp is required to be made of two threads of 2/60's worsted to one thread of 2/40's worsted. Length of warp is to be 300 yards and weight of warp 50 lbs. What number of ends of each sort will be required?"

Let us suppose these are x ends of the 2/60's worsted, and y ends of the 2/40's worsted.

Then length of the 2/60's thread = $300\,x$ yards, and length of the 2/40's thread = $300\,y$ yards.

And since 2/60's worsted means that 30 hanks of 560 yards weigh 1 lb.

$$\therefore \text{ Weight of 2/60's worsted used } = \frac{300\,x}{30 \times 560} = \frac{x}{56} \text{ lbs.}$$

Also 2/40's worsted means that 20 hanks of 560 yards weigh 1 lb.

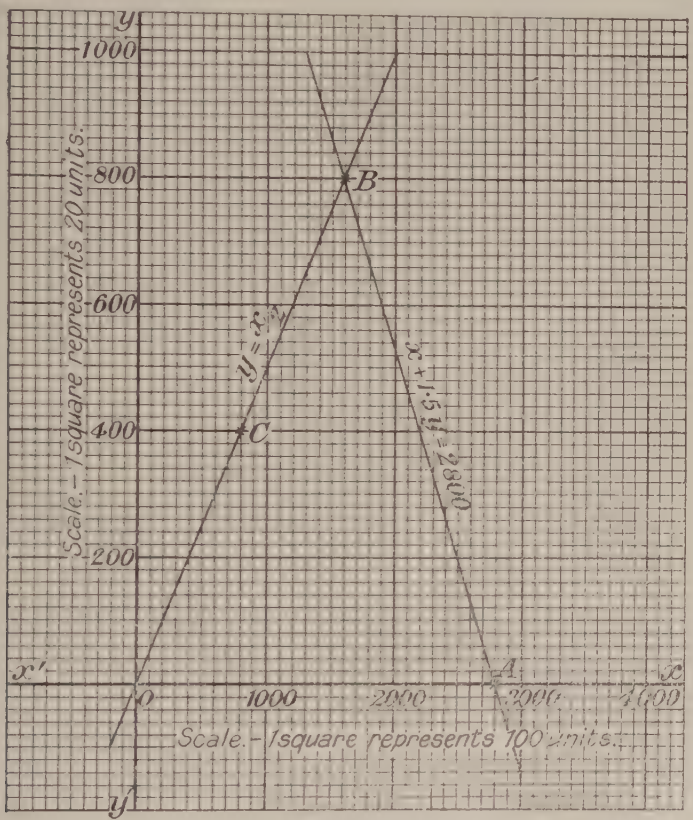


Fig. 60.

$$\therefore \text{ Weight of 2/40's worsted used } = \frac{300\,y}{20 \times 560} = \frac{1{\cdot}5\,y}{56} \text{ lbs.}$$

Hence Weight of threads in warp $= \left(\frac{x}{56} + \frac{1{\cdot}5\,y}{56}\right)$ lbs.

But Weight of warp $= 50$ lbs.

Hence $$\frac{x}{56} + \frac{1{\cdot}5\,y}{56} = 50.$$

Multiply by 56 and $x + 1{\cdot}5\,y = 2800$ (1).

We are also told that twice as many threads of 2/60's as threads of 2/40's are used or

$$x = 2y \quad (2).$$

And the values for x and y which we are seeking must satisfy both conditions (1) and (2).

Equations (1) and (2) must be used together if x and y are both to be found, and consequently are called SIMULTANEOUS EQUATIONS.

93. *Graphic Solution of Simultaneous Equations.*—Let us take the foregoing equations as an example.

$$x + 1{\cdot}5\,y = 2800 \quad (1).$$
$$x = 2y \quad (2).$$

Equation (1) may be written

$$\tfrac{3}{2}y = - \; x + 2800$$

or

$$3y = - \; 2x + 5600$$

or

$$y = - \tfrac{2}{3}x + \frac{5600}{3} \quad (3).$$

Also equation (2) may be written

$$y = \frac{x}{2} \quad (4).$$

The graph of each equation is a straight line.

In equation (1) when $x = 1600$; $y = 800$.

,, $x = 2800$; $y = 0$.

Plot these points A and B (see Fig. 60) and draw the graph.

In equation (2) when $x = 0$; $y = 0$.

,, $x = 800$; $y = 400$.

Plot these point O and C (see Fig. 60) and draw the graph.

Examining these graphs we have :—

(*a*) The coordinates of all points on the line AB satisfy the equation $x + 1{\cdot}5y = 2800$ (1),
and thus we have any number of solutions to equation (1).

(*b*) The coordinates of all points on the line OC satisfy the equation $x = 2y$ (2),
and again we have an infinite number of solutions to equation(2).

(*c*) *The lines cross at the point* B. Then since B is a point on both lines, its coordinates must satisfy equations (1) and (2).

The coordinates of B are $x = 1600$; $y = 800$.

Then $\left.\begin{matrix} x = 1600 \\ y = 800 \end{matrix}\right\}$ is the solution of the equations,

or in the warp there must be 1600 ends of 2/60's worsted, and 800 ends of 2/40's worsted.

. *Algebraical Solution of Simultaneous Equations.*—As fore. Solve $x + 1{\cdot}5y = 2800$ (1).

$x = 2y$ (2).

Since we are seeking values of x and y which satisfy both uations, we have

From (1) $x = -1{\cdot}5\,y + 2800$

From (2) $x = 2y$,

d these values for x must be the same.

Hence $2y = -1{\cdot}5y + 2800$

$2y + 1{\cdot}5y = 2800$

$3{\cdot}5y = 2800$

$$y = \frac{2800}{3{\cdot}5} = \frac{400}{0{\cdot}5} = 800.$$

Substitute this value for y in either equation (1) or (2), d from (2) $x = 2y = 2 \times 800 = 1600$.

Hence $x = 1600$ } is the solution as obtained by the
$y = 800$ } graphical method.

Note.—This is the only solution. The two straight lines AB and OB n only cut at one point so that the coordinates of one point only tisfy both equations.

EXAMPLE 1.

Solve the equations :

$y - 2x = -3$ (1).

$2y + x = 14$ (2).

A. ALGEBRAICAL SOLUTION.

From (1) $y - 2x = -3$

$y = 2x - 3$ (3).

From (2) $2y = 14 - x$

$$y = 7 - \frac{x}{2} \quad (4).$$

Since we are seeking values of x and y which satisfy both uations (1) and (2) or their equivalents (3) and (4),

$$2x - 3 = 7 - \frac{x}{2} \quad (5).$$

Transposing terms $2x + \frac{x}{2} = 7 + 3$

$$\frac{5x}{2} = 10.$$

Divide by $\frac{5}{2}$ and $x = 4$.

Substitute this value for x in either (3) or (4), and

from (3) $y = 2x - 3$

$= 8 - 3$

$= 5$.

Then $\left.\begin{matrix} x = 4 \\ y = 5 \end{matrix}\right\}$ is the solution.

VERIFICATION.—Substitute $x = 4$; $y = 5$ in equations (1) and (2). And

From (1) $y - 2x = 5 - 8 = -3$.

From (2) $2y + x = 10 + 4 = 14$.

Hence the values $\left.\begin{matrix} x = 4 \\ y = 5 \end{matrix}\right\}$ satisfy both equations.

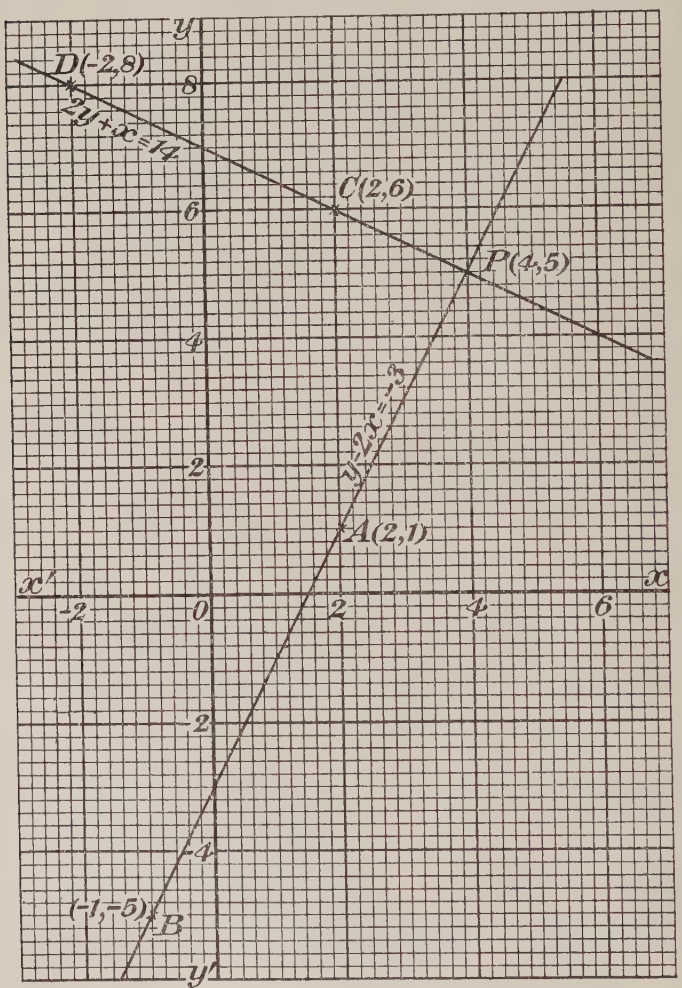


FIG. 61.

Note.—By equating the values of y given in equations (3) and (4), we get a new equation (5) containing only one unknown quantity (x). This process of getting rid of an unknown quantity is called ELIMINATION.

We might have effected the above solution by eliminating x and obtaining the value of y first. We should then obtain the value of x by substituting this value of y in one of the original equations.

B. GRAPHICAL SOLUTION.—Expressing the equations in the forms

$$y = 2x - 3 \quad (3)$$

$$y = -\frac{x}{2} + 7 \quad (4),$$

we see that the graph of each is a straight line.

In equation (3) when $x = 2$; $y = 1$.

,, $x = -1$; $y = -5$.

Plot these points A and B (Fig. 61) and draw the graph.

In equation (4) when $x = 2$; $y = 6$.

,, $x = -2$; $y = 8$.

Plot these points C and D (Fig. 61) and draw the graph. Notice that the lines BA and DC cut one another at the point P.

Then the coordinates of P will satisfy both equations, and these coordinates are $x = 4$; $y = 5$.

Hence $\left.\begin{matrix} x = 4 \\ y = 5 \end{matrix}\right\}$ is the solution.

EXAMPLE 2.

Solve the equations

$$4x - 3y = 14 \quad (1).$$
$$3x - 4y = 0 \quad (2).$$

Suppose we wish to eliminate x, we may as in Example 1 find the value of x in each equation, and then equate these values, or we may proceed as follows :—

Re-write each equation so that the coefficient of x is the same in each, thus :

Multiply equation (1) by 3 and equation (2) by 4,

and $12x - 9y = 42 \quad (3)$.

$12x - 16y = 0 \quad (4)$.

Now if equals be subtracted from equals the remainders are equal.

Subtract (4) from (3) and $7y = 42$.

Divide by 7 and $y = 6$.

Substitute this value for y in equation (1) or (2), and from (1)

$$4x - 18 = 14,$$

or $4x = 32$.

$\therefore$ $x = 8$.

Hence $\left.\begin{matrix} x = 8 \\ y = 6 \end{matrix}\right\}$ is the solution.

VERIFICATION.—Substitute $x = 8$; $y = 6$ in each of the original equations.

From (1) $4x - 3y = 32 - 18 = 14.$

From (2) $3x - 4y = 24 - 24 = 0.$

Hence $\left.\begin{matrix} x = 8 \\ y = 6 \end{matrix}\right\}$ satisfies both equations.

EXAMPLE 3.

A straight line graph passes through the points (2, 1); (− 3, 4). Find its equation.

The equation for a straight line is of the form

$$y = ax + b,$$

and the coordinates of every point on the line must satisfy this equation.

Since the point (2, 1) is on the line

then $1 = 2a + b \quad (1).$

And since the point (− 3, 4) is also on the line

then $4 = -3a + b \quad (2).$

We thus get two equations containing the unknowns, a and b.

To eliminate b subtract equation (2) from equation (1),

and $-3 = 5a,$

or $a = -\frac{3}{5}.$

Substitute this value for a in (1),

and $1 = -\frac{6}{5} + b,$

or $b = 1 + \frac{6}{5} = 1\frac{1}{5}.$

Then the equation for the line is

$$y = -\tfrac{3}{5}x + 1\tfrac{1}{5}.$$

Multiply by 5 and $5y = -3x + 11,$

or $5y + 3x = 11.$

EXAMPLE 4.

A factory produces per week L yards of material at a total cost of C pounds, including wages, interest on capital, rates, etc.

L .	7200	5500	4240	2000	yards.
C .	190	163	143	107	£

Plot these observed values of L and C.

If C and L are connected by an equation

$$C = aL + b.$$

Find the most *probable values of* a *and* b.

The points are plotted as in Fig. 62.

These are seen to lie on a straight line.

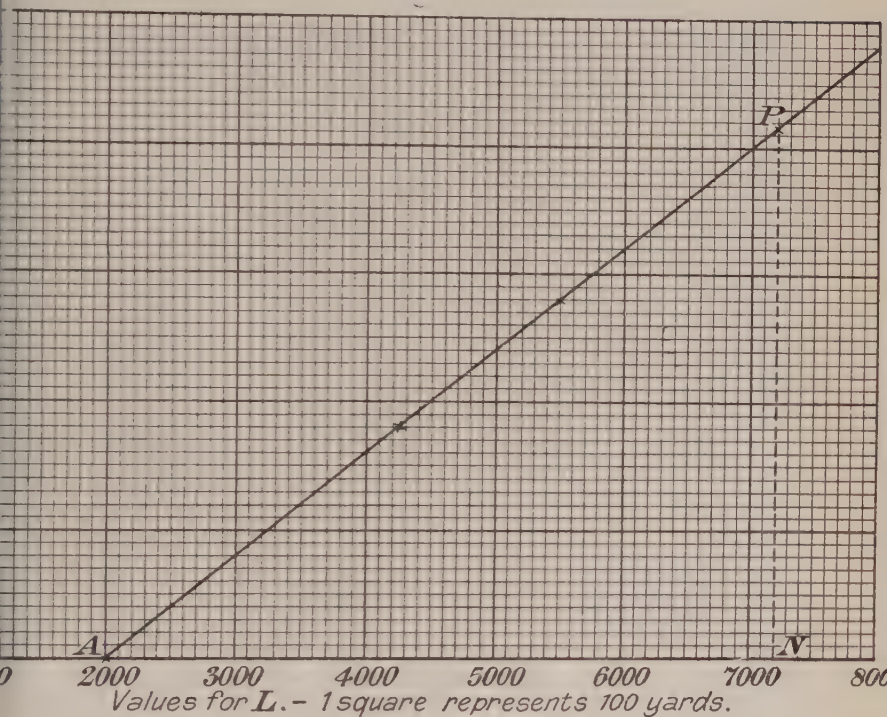


FIG. 62.

Its equation is of the form

$$C = aL + b.$$

[$y = ax + b$, and we plot values of C along y axis and values for L along x axis.]

Since the numbers are large, we cannot begin at zero on both axes, or we could use the method explained in Examples 1 and 2 (§ 90) to find values of a and b.

(*a*) We can, however, determine a thus :—

$$a = \text{Gradient of Line} = \frac{PN}{AN}$$

$$= \frac{83}{5200}$$

Since PN is $41\frac{1}{2}$ squares representing £83.
AN is 52 ,, ,, 5200 yards.

$= 0{\cdot}016$ correct to nearest $\frac{1}{1000}$.

Since the point (2000, 107) is on the line,
then $107 = a \times 2000 + b$.

Substitute the value for a already found,

and $$107 = \frac{83 \times 2000}{5200} + b = \frac{1660}{52} + b.$$

$$\therefore \quad b = 107 - \frac{1660}{52} = 75{\cdot}1 \text{ to nearest } \tfrac{1}{10}.$$

Then the law connecting C and L is

$$C = 0{\cdot}016\,L + 75{\cdot}1.$$

(*b*) We may obtain the values for a and b as shown in Example 3.

Now when $C = 107$; $L = 2000$
then $107 = 2000a + b$ (1),
and when $C = 163$; $L = 5500$
then $163 = 5500a + b$ (2).

Subtract (1) from (2) and

$$56 = 3500a,$$

or $$a = \frac{56}{3500} = \frac{8}{500} = \frac{1{\cdot}6}{100} = 0{\cdot}016.$$

Substitute this value for a in (1),
and $107 = 2000 \times 0{\cdot}016 + b$,
or $b = 107 - 32$
$= 75.$

Then the law connecting C and L is

$$C = 0{\cdot}016L + 75.$$

Note.—When $L = 0$; $C = 75$, or, according to the law, when the factory is standing, *i.e.*, no material is being manufactured, the cost per week to the firm is £75 in interest on capital, rates, etc.

CHAPTER VIII

Ratio and Proportion

95.—Units and Measurements. 96.—Ratio. 97.—Mutual Ratios. 98.—Some Important Ratios—Ratio of Circumference of a Circle to its Diameter. 99.—Ratio of Area of a Circle to the Area of the Square on its Radius. 100.—Specific Gravity. 101.—Percentages. 102.—Proportion. 103.—A Straight Line drawn Parallel to one Side of a Triangle cuts the other two Sides Proportionally. 104.—To Divide a given straight Line into any Number of equal Parts. 105.—To Divide a given straight Line in a given Ratio. 106.—To find a Fourth Proportional to Three given Quantities. 107.—To find a Third Proportional to Two given Quantities. 108.—Equiangular Triangles have their corresponding Sides in Proportion. 109.—The Construction and Use of a Diagonal Scale. 110.—The Angle in a Semi-circle is a Right Angle. 111.—A Right-angled Triangle divided by the Perpendicular from the Right Angle to the Hypotenuse. 112.—To find a Mean Proportional between Two given Straight Lines. 113.—To find the Square Root of a Number Graphically. 114.—To describe a Square equal in Area to a given Rectangle. 115.—In any two Circles the *Arcs* or the *Chords* which subtend equal Angles at the Centres are proportional to the Radii of the Circles. 116.—When a Lever, straight or bent, moves about its Fulcrum, the Distances moved by the Ends of the Arms are Proportional to the Lengths of the Arms. 117.—Depth of Shed—Bradford Tappet Shedding Mechanism.

95. *Units and Measurements.*—In testing a given sample of yarn for its counts we need to know its length and its weight. To specify its length we fix upon some particular distance or length for our unit, and say how many times this unit is contained in our particular length of yarn.

Thus we may say its length is 10 yards (unit of measurement = 1 yard), or its length is 30 feet (unit of measurement = 1 foot), or its length is 360 inches (unit of measurement = 1 inch), or its length is 914·4 cms. (unit of measurement = 1 cm.), or its length is 9·144 metres (unit of measurement = 1 metre). Also its weight may be expressed in lbs. or ounces or grammes or kilogrammes.

If we wish to state the size of an angle between two directions, we must choose a unit (right angle, or degree) and we must

say how many times (using a number) the angle measured contains the unit.

Thus the angle $ABC = 60°$ or $\frac{2}{3}$ right angle.

Again :—Areas of pieces of cloth are stated in terms of some unit of area (square inch, square foot, square yard, square metre, square centimetre).

Note.—The choice of the unit depends upon the size of the quantity to be measured. For small quantities we choose small units, using larger units for larger quantities. Thus :

The length of a sheet of paper is expressed in inches or cms., whilst its area is expressed in square inches or square cms.

The length of a weaving shed will be expressed in feet, and the length of a piece of cloth or warp in yards or metres. The distance from Huddersfield to Leeds is expressed in miles or kilometres ; the distance from the earth to the sun in millions of miles.

96. *Ratio.*—It is frequently required to compare two magnitudes. Thus we may wish to compare (1) the weights of two pieces, (2) the lengths of two warps, (3) the costs of two sorts of wool, (4) the speeds of two pulleys, (5) the strengths of two yarns, etc.

To compare any two quantities we find HOW MANY TIMES THE FIRST QUANTITY CONTAINS THE SECOND. The number expressing this, whether it is a whole number or a fraction, is called A RATIO.

If one kind of wool costs 2*s.* 2*d.* per lb. and another kind 2*s.* per lb., the ratio of the cost of the first wool to the cost of the second wool is $26 \div 24$ or as 26 is to 24.

The ratio is written 26 : 24 or $\frac{26}{24}$ (fractional form).

Again 1 yard of union cloth weighs 16 ozs., and a yard of a certain cotton fabric weighs 10 ozs., then the ratio of the two weights is as 16 : 10 or $\frac{16}{10}$ or $\frac{1{\cdot}6}{1}$.

Note. 1. *A ratio* is a pure number ; it is an answer to the question, "How many times?"

2. We have not introduced any new idea ; the pupil has been familiar with a ratio from the time he divided 15 by 3, or found the number of articles at 2*s.* 6*d.* each which can be bought for 30 shillings.

He then found that $15 : 3 = 5 : 1$ or $\frac{15}{3} = 5$,

and $30 : 2\frac{1}{2} = 12 : 1$ or $\frac{30}{2\frac{1}{2}} = 12$.

3. The problems (*a*) Divide 448 by 6 ; (*b*) How many yards of cloth at 2*s.* per yard can be bought for £7 9 4 ? (*c*) How many hanks of yarn each 24 ozs weigh 1 cwt. ? are all equivalent to the question "What is the ratio of 448 to 6 ?"

In each case the numerical answer is $\frac{224}{3} = 74\frac{2}{3}$.

If, as we said, we are to find the number of times that one quantity contains another, it is evident that

(1) the magnitudes to be compared must be of the same kind (*i.e.*, both lengths, or both weights, or both sums of money, or both angles). In other words, the quantities to be compared must be capable of being measured with the same unit.

(2) Before finding the ratio between them we must express the quantities in terms of the same unit. Thus we cannot compare lbs. with inches, for they are not quantities of the same kind (one is a weight; the other, a length). And, again, we cannot compare a length in inches with a length in yards without first making the units alike, *i.e.*, we must either reduce the yards to inches or the inches to yards.

Example 1.

Find the ratio of 1 yard to 50 cms., being given that 1 inch = 2·54 cms.

Reducing these lengths to the same denomination, we have

$$1 \text{ yard} = 36 \text{ inches.}$$

$$50 \text{ cms.} = 50 \div 2{\cdot}54 = \frac{50}{2{\cdot}54} = 19{\cdot}69 \text{ inches.}$$

$$\therefore \quad \frac{1 \text{ yard}}{50 \text{ cms.}} = \frac{36}{19{\cdot}69} = \frac{1{\cdot}83}{1}$$

$$= 1{\cdot}83 \text{ to nearest } \tfrac{1}{100}.$$

Example 2.

Find the ratios of the surfaces of two cubes whose edges are 1 yard and 1 foot respectively.

Suppose we speak of these cubes as A and B respectively. Each face of A is a square with each side 1 yard long.

Then Area of each face of A = 1 square yard = 9 square ft.
And Total surface area of A = 6 × 9 = 54 square feet.

Again Area of each face of B = 1 square foot,
and Total surface area of B = 6 square feet.

Then $\dfrac{\text{Surface area of } A}{\text{Surface area of } B} = \frac{54}{6} = \frac{9}{1}$ or 9 : 1.

Example 3.

Find a length which is to 17 yards 2 feet 4 inches in the ratio 5 : 32.

Now 17 yards 2 feet 4 inches = (17 × 36 + 28) inches
= 612 + 28
= 640 inches.

Suppose required length is x inches.

Then $x : 640 = 5 : 32$

or $$\frac{x}{640} = \frac{5}{32}.$$

Multiply both sides by 640 and

$$x = \frac{5 \times 640}{32} = 5 \times 20 = 100.$$

$\therefore$ Required length = 100 inches = 8 feet 4 inches
= 2 yards 2 feet 4 inches.

97. *Mutual Ratios.*—Given several quantities of the same kind, the ratio of any pair can be found; for example, take the three quantities £1, 12/6, 3/6.

Then (*a*) £1 : 12/6 = 240 : 150 = 8 : 5.
(*b*) 12/6 : 3/6 = 25 : 7.
(*c*) £1 : 3/6 = 240 : 42 = 40 : 7.

These results show the comparisons of any pair out of the three, but do not at once show the comparative sizes of all the three. In (*a*) we used 2/6 as the unit for comparison; in (*b*) and (*c*) we used 6*d*. as the unit for comparison.

Now *sixpence* is the largest unit in terms of which all three quantities may be expressed so as to get ratios between whole numbers.

£1 = 40 sixpences.
12/6 = 25 ,,
3/6 = 7 ,,

Then we may state that

£1 : 12/6 : 3/6 = 40 : 25 : 7,

and the relative sizes of the three quantities may be seen at once.

Example 4.

The sides AB, BC, CA of a triangle are 5 feet 9 inches, 7 feet 8 inches, 9 feet 7 inches respectively. Express their mutual ratios in the simplest form.

Now AB = 5 feet 9 inches = 69 inches = (3 × 23).
BC = 7 feet 8 inches = 92 ,, = (4 × 23).
CA = 9 feet 7 inches = 115 ,, = (5 × 23).

From the factors of 69, 92, and 115 shown above, we see that the greatest unit we may use in the measurement of these sides to give whole numbers is a length of 23 inches.

Then AB has a length of 3 units (23 inches being the unit).
BC ,, ,, 4 ,, (,, ,, ,,).
CA ,, ,, 5 ,, (,, ,, ,,).

$\therefore$ $AB : BC : CA = 3 : 4 : 5.$

Example 5.

Squares X, Y, Z have sides 1 cm., 2 cms., 3 cms. If x, and z are the perimeters, complete the statement $x : y : z$.

Perimeter of $X = (4 \times 1) = 4$ cms.
,, $Y = (4 \times 2) = 8$ cms.
,, $Z = (4 \times 3) = 12$ cms.

If we use a unit of 4 cms. to measure each perimeter,

$x = 1$ unit long.
$y = 2$ units ,,
$z = 3$,, ,,

Hence $x : y : z = 1 : 2 : 3$.

3. *Ratio of the Circumference of a Circle to its Diameter.*—y experiment and measurement it was found (§60, p. 114) hat the length of the circumference of a circle is roughly $\frac{1}{7}$ times the length of its diameter; that is to say :—

$$\frac{\text{Circumference}}{\text{Diameter}} = 3\tfrac{1}{7} \text{ nearly,}$$

nd it can be proved that this ratio is the same for all circles. The ratio which the circumference of any circle bears to its iameter is denoted by the Greek letter π, so that :

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi,$$

r

$$\text{Circumference} = \pi \times \text{Diameter.}$$

9. *Ratio of the Area of a Circle to the Area of the Square on s Radius.*—Referring to his results of the experiment described in § 68, p. 122, the student will notice that the numbers n the last column are approximately the same. Hence for ll circles, great or small, we conclude :

$$\frac{\text{Area of a Circle}}{\text{Area of the Square on its Radius}} = \frac{3{\cdot}14}{1} = \pi.$$

00. *Specific Gravity.*—It is frequently required to compare he weights of different substances. It is evident that in this omparison we must deal with equal bulks or volumes of the ubstances.

For practical purposes we compare the weights of various ubstances with the weight of equal volumes of water.

This *ratio of the weight of any volume of a substance to the eight of an equal volume of water* is termed the Specific Gravity f the substance.

Example 1.

If 1 cubic foot of copper weighs 555·36 lbs. and 1 cubic foot of water weighs 62·4 lbs.

Then

$$\text{Specific gravity of copper} = \frac{\text{Weight of 1 cubic foot of copper}}{\text{Weight of 1 cubic foot of water}} = \frac{555{\cdot}36}{62{\cdot}4} = \frac{8{\cdot}9}{1} = 8{\cdot}9.$$

Example 2.

If the specific gravity of iron is 7·4, find the weight of 3·72 cubic feet of iron.

Since the specific gravity of iron = 7·4, we know that :—

$$\frac{\text{Weight of 1 cubic inch of iron}}{\text{Weight of 1 cubic inch of water}} = \frac{7{\cdot}4}{1}$$

or $$\frac{\text{Weight of 1 cubic cm. of iron}}{\text{Weight of 1 cubic cm. of water}} = \frac{7{\cdot}4}{1}$$

or $$\frac{\text{Weight of 10 cubic feet of iron}}{\text{Weight of 10 cubic feet of water}} = \frac{7{\cdot}4}{1}, \text{ etc.}$$

$$\therefore \quad \frac{\text{Weight of 3·72 cubic feet of iron}}{\text{Weight of 3·72 cubic feet of water}} = \frac{7{\cdot}4}{1},$$

or Weight of 3·72 cubic feet of iron = 7·4 × (weight of 3·72 cubic feet of water)

$$= 7{\cdot}4 \times 3{\cdot}72 \times 62{\cdot}3 \text{ lbs.}$$

Since Weight of 1 cubic foot of water = 62·3 lbs.

Suppose W lbs. is the required weight of the iron.

Then $$W = 7{\cdot}4 \times 3{\cdot}72 \times 62{\cdot}3 = [1736 \text{ lbs. approx.}]$$

$$\log W = \log 7{\cdot}4 + \log 3{\cdot}72 + \log 62{\cdot}3$$

$$= 0{\cdot}8692 + 0{\cdot}5705 + 1{\cdot}7945 = 3{\cdot}2342.$$

$$\therefore \quad W = \text{antilog } 3{\cdot}2342 = 1715 \text{ lbs.}$$

101. *Percentages.*—We have said that the ratio of 4 to 5 may be expressed in fractional form as $\frac{4}{5}$, and from our knowledge of fractions the value of a fraction is unaltered if the numerator and denominator be multiplied or divided by the same number.

Thus, $\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{0{\cdot}4}{0{\cdot}5}, \frac{0{\cdot}8}{1}, \frac{80}{100}$ are *equivalent ratios.*

In the ratio $a : b$ or $\frac{a}{b}$,

a the numerator of the fraction is known as the *antecedent* and b the denominator of the fraction is known as the *consequent* of the ratio.

To determine which is the larger of two fractions as $\frac{2}{3}$ and $\frac{3}{5}$, we express these as equivalent fractions having the same denominator, thus $\frac{2}{3} = \frac{10}{15}$ and $\frac{3}{5} = \frac{9}{15}$, and it at once appears that $\frac{2}{3}$ is greater than $\frac{3}{5}$.

Similarly, we often require to compare two or more ratios, and in like manner we express them as equivalent ratios with a common *consequent*.

For example: Three "lots" of "greasy" wool are washed and scoured, each "lot" yielding a certain weight of "clean" wool, and we wish to compare the yields to see which is the best. Suppose the weights are as follows:—

	Weight of greasy wool.	Weight of clean wool.	Wt. of clean wool. / Wt. of greasy wool.
1st Lot . .	750 lbs.	420 lbs.	$\frac{420}{750} = \frac{42}{75}$
2nd Lot . .	850 ,,	493 ,,	$\frac{493}{850}$
3rd Lot . .	1050 ,,	$593\frac{1}{4}$,,	$\frac{593\frac{1}{4}}{1050} = \frac{2373}{4200}$

It is not easy from this table to say from which "lot" the best yield is obtained.

We proceed to express each ratio with a consequent of 100. Thus:

From 1st Lot,

$$\frac{\text{Weight of clean wool}}{\text{Weight of greasy wool}} = \frac{42}{75} = \frac{14}{25} = \frac{56}{100}.$$

From 2nd Lot,

$$\frac{\text{Weight of clean wool}}{\text{Weight of greasy wool}} = \frac{493}{850} = \frac{49{\cdot}3}{85} = \frac{2{\cdot}9}{5} = \frac{58}{100}.$$

From 3rd Lot,

$$\frac{\text{Weight of clean wool}}{\text{Weight of greasy wool}} = \frac{2373}{4200} = \frac{339}{600} = \frac{56{\cdot}5}{100}.$$

It is evident now that the 2nd Lot gives the best yield.

These ratios with 100 as the consequent are known as PERCENTAGES.

In the above example we say:

From 1st Lot,

Weight of clean wool $= \frac{56}{100} \times$ weight of greasy wool

$= 56\%$ of weight of greasy wool

(*read* 56 per cent. of weight of greasy wool).

From 2nd and 3rd Lots we get 58% and 56·5% of the weigh of greasy wool in the form of clean wool.

As another example, a cloth merchant wishes to compar his profits on several kinds of cloth he is selling. He expresse his profit on each kind of cloth as a *percentage* of the cost pric of that cloth, thus :

	Cost price per yard.	Selling price per yard.	Profit per yard.	$\frac{\text{Profit}}{\text{Cost price.}}$
1st Cloth	2/–	3/6	1/6	$\frac{1/6}{2/-} = \frac{18}{24} = \frac{1{\cdot}5}{2} = \frac{75}{100} = 75\%$
2nd Cloth	3/6	5/–	1/6	$\frac{1/6}{3/6} = \frac{3}{7} = \frac{3\times14\frac{2}{7}}{100} = 42\frac{6}{7}\%$
3rd Cloth	5/–	7/6	2/6	$\frac{2/6}{5/-} = \frac{1}{2} = \frac{50}{100} = 50\%$
4th Cloth	7/6	10/–	2/6	$\frac{2/6}{7/6} = \frac{1}{3} = \frac{33\frac{1}{3}}{100} = 33\frac{1}{3}\%$

Note.—(i) In dealing with profit and loss, take care to expres the profit or the loss as a *Percentage of the Cost Price of th Article.*

(ii) A percentage of a quantity is a fraction of that quantit with 100 as denominator. In many cases it is best to dea with this fraction when reduced to its lowest terms.

For example : 25% of £3 4 8 $= \frac{25}{100}$ of £3 4 8
$= \frac{1}{4}$ of £3 4 8
$= 16/2.$

Similarly : 5% of $x = \frac{x}{20}$
75% of $x = \frac{3}{4}x$
$33\frac{1}{3}\%$ of $x = \frac{x}{3}$, etc .

Example 1.

If a sized warp weighs 130 lbs. and contains $7\frac{1}{2}\%$ of size what was the original weight ?

1st Method of Solution.

100 lbs. of warp took in $7\frac{1}{2}$ lbs. of size.

So that $107\frac{1}{2}$ lbs. of sized warp originally weighed 100 lbs.

and 1 lb. ,, ,, ,, ,, $\frac{100}{107\frac{1}{2}}$ lbs.

$\therefore$ 130 lbs. of sized warp originally weighed $\frac{100 \times 130}{107\frac{1}{2}}$ lbs.

$\therefore$ Original weight $= \frac{100 \times 130}{107\frac{1}{2}} = 120\frac{40}{43}$ lbs.

2nd Method.—Suppose the original weight of warp $= x$ lbs.

Increase in weight due to size $= 7\frac{1}{2}\%$ of $x = \frac{7\frac{1}{2}}{100}x$

$= \frac{15}{200}x = \frac{3}{40}x$ lbs.

Then Weight of warp after sizing $= (x + \frac{3}{40}x)$ lbs.

$= \frac{43}{40}x$ lbs.

But this weight is given as 130 lbs.

$\therefore \quad \frac{43}{40}x = 130.$

Divide by $\frac{43}{40}$ and $x = \frac{130 \times 40}{43} = \frac{5200}{43} = 120\frac{40}{43}$.

$\therefore$ Original weight $= 120\frac{40}{43}$ lbs.

Example 2.

A 40's twist cotton warp with 2000 ends and 150 yards long is increased 5% by sizing. What is its actual weight ?

A 40's cotton means (40 × 840) yards weigh 1 lb.

Length of yarn in the warp = (2000 × 150) yards.

Hence Weight of yarn before sizing $= \frac{2000 \times 150}{40 \times 840} = \frac{250}{28}$ lbs.

Increase in weight due to size $= 5\%$ of $\frac{250}{28}$ lbs.

$= (\frac{1}{20}$ of $\frac{250}{28})$ lbs.

Hence Weight after sizing $= \frac{250}{28} + (\frac{1}{20} \times \frac{250}{28})$ lbs.

$= \frac{250}{28}(1 + \frac{1}{20})$ lbs.

$[ab + ac = a(b + c)$
or $a + ac = a(1 + c)]$

$= \frac{250}{28} \times \frac{21}{20} = 9\frac{3}{8}$ lbs.

Aliter.—After obtaining weight of warp before sizing $= \frac{250}{28}$ lbs., we may proceed as follows :—

100 lbs. after sizing become 105 lbs.

1 lb. ,, ,, becomes $\frac{105}{100}$ lbs.

$\therefore \quad \frac{250}{28}$ lbs. ,, ,, become $\frac{105}{100} \times \frac{250}{28}$ lbs.

$\therefore$ Weight after sizing $= \frac{105}{100} \times \frac{250}{28} = 9\frac{3}{8}$ lbs.

Example 3.

The following table gives the quantity of wool unshipped at the various ports during 1913.

	Bales.
London	1,603,177
Liverpool	590,606
Hull	128,785
Goole	2,643
Grimsby	19,030
Bristol	73
Leith and Glasgow	5,882
Total	2,350,196

Express the quantity received at each port as a percentage of the total correct to 3 significant figures.

(*a*) *London.*—

Out of 2,350,196 London imports 1,603,177 bales.

" 1 " " $\frac{1,603,177}{2,350,196}$ "

" 100 " " $\frac{1603177 \times 100}{2,350,196}$ bales.

$\therefore$ Percentage of wool

Imported at London $= \frac{160317700}{2350196} = \frac{1600}{24} = 66\%$ approx

$= 68{\cdot}2\%$.

Contracted division.

$$2350)16032(68{\cdot}22$$
$$1932$$
$$52$$
$$6$$

(*b*) *Liverpool.*—In this solution we will vary the method. Suppose $x\%$ of the total is imported at Liverpool.

Then $\frac{x}{100} \times 2,350,196 = 590,606$

or $x = \frac{590606 \times 100}{2,350,196}$

$= \frac{59060600}{2350196} = \frac{600}{24} = 25\%$ approx

$= 25{\cdot}1\%$.

By Contracted Division.

$$2350\dot{2})59061(25{\cdot}13$$
$$12057$$
$$306$$
$$71$$

(*c*) *Bristol.*— Suppose $x\%$ of the total is imported at Bristol.

Then $\frac{x}{100} \times 2350196 = 73$

$$x = \frac{73 \times 100}{2350196}$$

$$= \frac{7300}{2350196} = \frac{0{\cdot}073}{24} = 0{\cdot}003\% \text{ approx.}$$

$$= 0{\cdot}00311\%.$$

By Contracted Division.

$$2350\dot{2})7300{\cdot}0(0{\cdot}003106$$
$$2494$$
$$144$$
$$3$$

The rest are left for the student to work out in a similar manner.

Example 4.

A piece is wanted to deliver 55 yards in length, $20\frac{1}{2}$ ounces per 36-inch yard. What length and weight must the piece be out of the loom on an estimated loss of $12\frac{1}{2}\%$ in weight and 8% in length ?

(*a*) *To calculate the length out of the loom.*—The 8% loss in length must be reckoned on the length out of the loom, *i.e.*, a piece 100 yards long (grey length) becomes 92 yards when finished.

So that 92 yards finished was 100 yards out of the loom.

And 1 yard ,, ,, $\frac{100}{92}$,, ,, ,,

And 55 yards ,, ,, $\frac{100 \times 55}{92}$ yards out of the loom.

$\therefore$ Length of piece out of the loom $= \frac{100 \times 55}{92}$

$= 59{\cdot}78$ yards.

$= 59\frac{3}{4}$ yards nearly.

(*b*) *To calculate the weight out of the loom.*—In this case we will vary the method.

Suppose the weight required $= x$ lbs.

Loss in finishing $= 12\frac{1}{2}\% = \frac{1}{8}$.

$\therefore$ Weight of piece finished $= x - \frac{1}{8}x$.

$= \frac{7}{8}x$ lbs.

But the weight finished $= (20\frac{1}{2} \times 55)$ ozs. $= \frac{20\frac{1}{2} \times 55}{16}$ lbs.

Hence $\frac{7}{8}x = \frac{20\frac{1}{2} \times 55}{16}$

or $x = \frac{41 \times 55 \times 8}{2 \times 16 \times 7} = 80{\cdot}53.$

$\therefore$ Weight of cloth out of loom $= 80\frac{1}{2}$ lbs. nearly.

Example 5.

Calculate the percentage stoppage of a carding engine delivering 300 lbs. in 10 hours, with 100 good threads, 9-inch surface drums running at 16 revolutions per minute. Material condensed to 8 yards per dram.

Diameter of surface drum $= 9$ inches.

Circumf. ,, ,, ,, $= \frac{22 \times 9}{7}$ inches = length of each thread delivered at each revolution.

Since there are 100 good threads,

Length delivered per revolution $= \frac{22 \times 9 \times 100}{7}$ inches.

Number of revolutions made in 10 hours $= 10 \times 60 \times 16$.

$\therefore$ Length delivered in 10 hours

$$= \frac{22 \times 9 \times 100 \times 10 \times 60 \times 16}{7 \times 36} \text{ yards}$$

And 8 yards of sliver weigh 1 dram.

$\therefore$ Weight of sliver delivered if running continuously

$$= \frac{22 \times 9 \times 100 \times 10 \times 60 \times 16}{7 \times 36 \times 8 \times 16 \times 16} \text{ lbs.} \quad \text{(A)}.$$

Weight actually delivered $= 300$ lbs.

$\therefore \frac{\text{Weight actually delivered}}{\text{Weight possible}} = 300 \div \text{(A)}$

$$= \frac{300 \times 7 \times 36 \times 8 \times 16 \times 1}{22 \times 9 \times 100 \times 10 \times 60 \times 1}$$

$$= \frac{896}{1100} = \frac{81{\cdot}5}{100} \text{ approx.}$$

$$= 81{\cdot}5\%.$$

$\therefore$ Percentage of stoppages $= 18{\cdot}5\%$ nearly.

102. *Proportion.*—Proportion deals with the equality of ratios. The ratio of £1 to 2/6 is 8 : 1 and the ratio of 1 lb. to 2 ounces is 8 : 1.

Hence $$\frac{\text{£1}}{2/6} = \frac{1 \text{ lb.}}{2 \text{ oz.}}$$

Here there are four quantities, £1, 2/6, 1 lb., 2 ozs., and the ratio between the first two is equal to the ratio between the second two.

These four quantities are said to be in proportion.

Definition of Proportion.

"*Four quantities are said to be in proportion when the ratio of the first to the second is equal to the ratio of the third to the fourth.*"

From this it is evident that the first pair of quantities must be of the same kind, and also the second pair. The second pair need not be of the same kind as the first pair. Many quantities are obviously in proportion.

For example, if greasy wool be sold at a fixed rate of 9*d.* per lb., then the weights of two quantities of wool and the prices paid for them are in proportion.

Thus 360 lbs. sell for £13 10*s.*
480 lbs. ,, ,, £18.

Then $$\frac{360 \text{ lbs.}}{480 \text{ lbs.}} = \frac{36}{48} = \frac{3}{4}$$

and $$\frac{\text{£13 10}s.}{\text{£18}} = \frac{27 \text{ half sovereigns}}{36 \text{ half sovereigns}} = \frac{3}{4}.$$

Hence $$\frac{360 \text{ lbs.}}{480 \text{ lbs.}} = \frac{\text{£13 10}s.}{\text{£18}}$$

and the four quantities are in proportion.

Putting this in general terms:—if w_1 and w_2 represent the weights of any two quantities of wool and C_1 and C_2 represent the number of units of the cost of each at a fixed price of 9*d.* per lb.,

$$w_1 : w_2 = C_1 : C_2 \text{ or } \frac{w_1}{w_2} = \frac{C_1}{C_2}.$$

As another example :—

If all looms of the same kind and pattern turn out on an average the same length of cloth per week, then the numbers of looms in different sheds and the lengths of cloth turned out by them respectively are in proportion.

If n_1 and n_2 represent the numbers of the looms, and l_1 and l_2

represent the numbers of yards of cloth woven by them respectively, then in mathematical language we say

$$\frac{n_1}{n_2} = \frac{l_1}{l_2} \text{ or } n_1 : n_2 = l_1 : l_2.$$

The simplest case of proportion is when all the four quantities are of the same kind.

Then four *numbers*, say a, b, c, and d, are in proportion if

$$a : b = c : d \text{ or } \frac{a}{b} = \frac{c}{d}.$$

From our knowledge of fractions, if $\frac{a}{b} = \frac{c}{d}$,

inverting each we have $\frac{b}{a} = \frac{d}{c}$.

Note carefully.

1. The *fourth proportional* to three numbers a, b, c is a number x such that $a : b = c : x$.
2. The *third proportional* to two numbers a, b is a number x such that $a : b = b : x$.
3. The *mean proportional* between two numbers a, b is a number x such that $a : x = x : b$.

Example 1.

Find the fourth proportional to the numbers 5·6, 46, 0·63.

If x is the number required,

then $$5{\cdot}6 : 46 = 0{\cdot}63 : x$$

or $$\frac{5{\cdot}6}{46} = \frac{0{\cdot}63}{x}.$$

Inverting each ratio

$$\frac{x}{0{\cdot}63} = \frac{46}{5{\cdot}6}.$$

Multiply by 0·63 and

$$x = \frac{46 \times 0{\cdot}63}{5{\cdot}6} = \frac{46 \times 0{\cdot}9}{8} = \frac{41{\cdot}4}{8} = 5{\cdot}175.$$

Example 2.

Find a third proportional to $5\frac{1}{4}$ and 7. Let x be the third proportional required.

Then $$5\tfrac{1}{4} : 7 = 7 : x$$

or $$\frac{5\frac{1}{4}}{7} = \frac{7}{x}.$$

Inverting both ratios,

$$\frac{x}{7} = \frac{7}{5\frac{1}{4}}.$$

Multiply by 7 and

$$x = \frac{49}{5\frac{1}{4}} = \frac{49 \times 4}{21} = \frac{28}{3} = 9\frac{1}{3}.$$

Example 3.

Find the *mean proportional* to 1·3 and 0·637.

Let y be the mean proportional.

Then $1{\cdot}3 : y = y : 0{\cdot}637$

or $$\frac{1{\cdot}3}{y} = \frac{y}{0{\cdot}637}.$$

Multiply by $(0{\cdot}637 \times y)$ and $1{\cdot}3 \times 0{\cdot}637 = y^2$,

or $$y = \sqrt{1{\cdot}3 \times 0{\cdot}637} = \sqrt{\tfrac{13}{10} \times \tfrac{637}{1000}}$$

$$= \sqrt{\frac{13 \times 13 \times 49}{10000}} = \frac{13 \times 7}{100} = 0{\cdot}91.$$

103. *Proportion treated Geometrically.*

Fact to learn.—"*A straight line drawn parallel to one side of a triangle cuts the other two sides proportionally.*"

Exercise 1.

Draw a triangle ABC making $AB = 40$ units, $AC = 32$ units. Angle $BAC = 71°$. Take a point P in AB so that $AP = 12$ units, and draw PQ parallel to BC meeting AC at Q. Measure AQ and QC and compare the ratios $\frac{AP}{PB}, \frac{AQ}{QC}$ and also the ratios $\frac{AB}{AP}, \frac{AC}{AQ}$. (Fig. 62*a*.)

(Use $\frac{1}{10}$″ or 1 mm. as unit.)

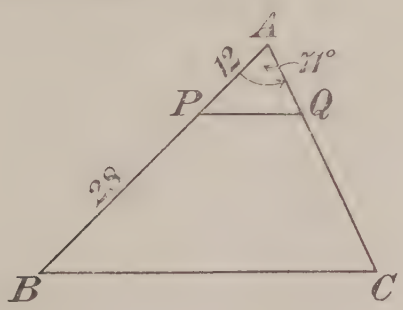


Fig. 62a.

Exercise 2.

On a base AB, 3·5″ in length, draw any triangle CAB; and from AB cut off AX 2·1″ long. From X draw XY parallel to BC to meet AC at Y. (Fig. 62*b*.) Measure AY, YC and hence compare the ratios

(i) $\frac{AX}{XB}, \frac{AY}{YC}$; (ii) $\frac{AB}{AX}, \frac{AC}{AY}$; (iii) $\frac{AB}{XB}, \frac{AC}{YC}$.

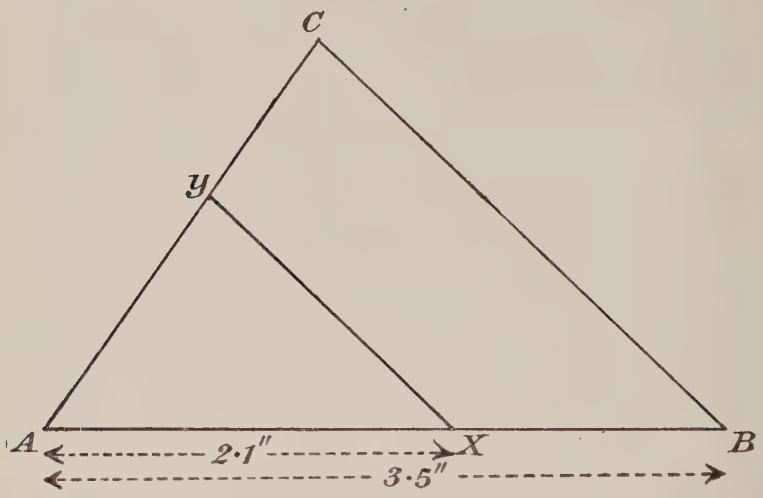


Fig. 62b.

Exercise 3.

Draw any triangle ABC making $AB = 6$ cms. (Fig. 62*c*.)

In AB take points P and R so that $AP = 1$ cm., $AR = 3$ cms., and draw PQ and RS parallel to BC meeting the side AC at Q and S re ectively.

Now $AP : PR : RB = 1 : 2 : 3$.

Measure AQ, QS, and SC.

Express $AQ : QS : SC$ in the simplest form.

If the drawings are carefully executed it is found that :—

In Exercise 1 $\quad \frac{AP}{PB} = \frac{AQ}{QC}$.

In Exercise 2 $\quad \frac{AX}{XB} = \frac{AY}{YC}$.

In Exercise 3 $\quad AP : PR : RB = AQ : QS : SC$.

It follows therefore,

If P (Exercise 1) is the middle point of AB,

$$\frac{AP}{PB} = \frac{1}{1} \text{ and therefore } \frac{AQ}{QC} = \frac{1}{1}$$

or $AQ = QC$.

That is, Q is the middle point of AC.

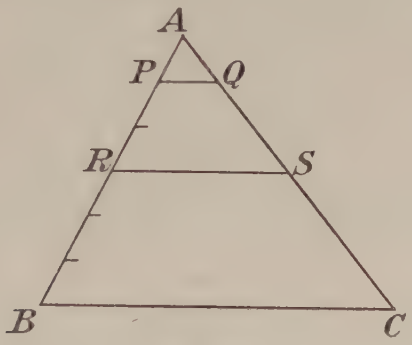


FIG. 62c.

Also, if X (Exercise 2) is the mid-point of AB, then Y is the mid-point of AC.

And in Exercise 3, if $AP = PR = RB$,

$$AP : PR : RB = 1 : 1 : 1$$

therefore $AQ : QS : SC = 1 : 1 : 1$

or $AQ = QS = SC$.

The student should now readily understand the methods employed in the following *important constructions.*

104. *To divide a given straight line into any number of equal parts.*

Example.—Divide a line AB, 3·5″ long, into 6 equal parts.

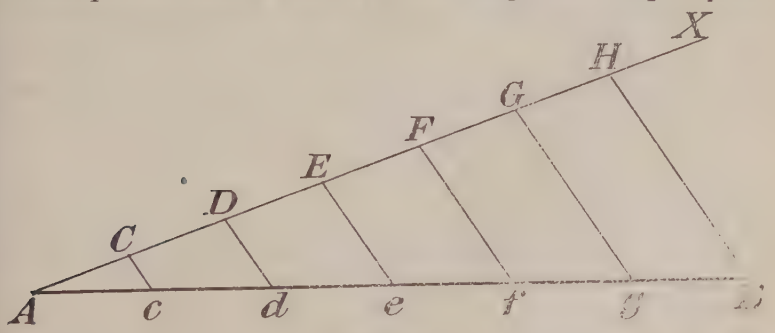


FIG. 63.

Construction.—Draw the line $AB = 3{\cdot}5''$ (Fig. 63). From A draw AX making a convenient angle with AB. Take a convenient distance (say $\frac{1}{2}''$) in your compasses and mark off six equal distances AC, CD, DE, EF, FG, GH along AX.

Join the points H and B, and through the points of division draw lines Gg, Ff, Ee, Dd, Cc, parallel to HB.

Then AB is divided into six equal parts at the points c, d, e, f, and g. Test this with your dividers.

105. *To divide a given straight line in a given ratio.*

EXAMPLE 1.

Draw a line 5·5 inches long and divide in the ratio of 3 to 8. Measure the parts and verify your drawing by calculation.

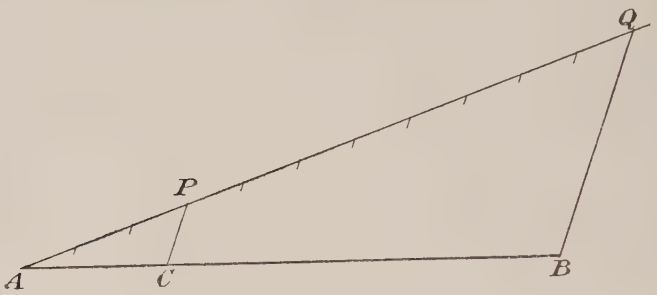


FIG. 64. (Scale $\frac{1}{2}$ size.)

Construction.—Draw $AB = 5{\cdot}5''$ (Fig. 64). From A draw AX making a convenient angle with AB. Take a convenient distance in your compasses as the unit and along AX measure $AP = 3$ units and $PQ = 8$ units. Join QB and with set squares draw PC parallel to QB, meeting AB at the point C.

Then $$\frac{AC}{CB} = \frac{AP}{PQ} = \frac{3}{8}.$$

AC measures $1{\cdot}5''$. CB measures $4''$.

Then $$\frac{AC}{CB} = \frac{1{\cdot}5}{4} = \frac{15}{40} = \frac{3}{8}.$$

Calculation.

Method 1.—If AB were divided into 11 equal parts $(3 + 8 = 11)$, and we made $AC = 3$ of these parts, and $CB = 8$ of these parts,

Then $$\frac{AC}{CB} = \frac{3}{8}.$$

Now $AB = 5{\cdot}5$ inches.

Hence $AC = \frac{3}{11} \times 5{\cdot}5 = 3 \times 0{\cdot}5 = 1{\cdot}5$ inches

and $CB = \frac{8}{11} \times 5{\cdot}5 = 8 \times 0{\cdot}5 = 4{\cdot}0$,,

Method 2.—Using an x,

Suppose $AC = x$ inches.

Then since $AB = 5{\cdot}5$ inches, $CB = (5{\cdot}5 - x)$ inches.

Now $\frac{AC}{CB} = \frac{3}{8}$. $\therefore \frac{x}{5{\cdot}5 - x} = \frac{3}{8}$.

Multiply both sides by $8\ (5{\cdot}5 - x)$ and

$$8x = 3\ (5{\cdot}5 - x)$$

or $$8x = 16{\cdot}5 - 3x.$$

Add $3x$ to both sides and $11x = 16{\cdot}5$.

Divide by 11 and $x = 1{\cdot}5$.

$\therefore$ $AC = 1{\cdot}5$ inches.

and $CB = (5{\cdot}5 - 1{\cdot}5) = 4{\cdot}0$ inches.

Example 2.

Divide a line 4·8 inches long into three parts proportional to 1, 3, and 4. Measure the parts and check your measurements by calculation.

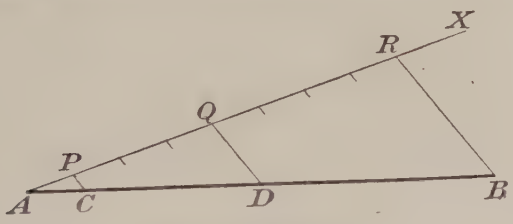


Fig. 65. (Scale ½ size.)

Construction.—Draw $AB = 4{\cdot}8''$ (Fig. 65). Draw AX making a convenient angle with AB. Take some convenient distance in your compasses as unit and along AX mark off $AP = 1$ unit; $PQ = 3$ units; $QR = 4$ units. Join RB and from Q and P draw QD, PC parallel to RB.

AC measures 0·6 inch or $1 \times 0{\cdot}6''$,

CD ,, 1·8 inches or $3 \times 0{\cdot}6''$,

DB ,, 2·4 ,, or $4 \times 0{\cdot}6''$,

using $0{\cdot}6''$ as unit.

Then AC measures 1 unit.

CD ,, 3 units.

DB ,, 4 ,,

or $AC : CD : DB = 1 : 3 : 4$.

Calculation.—If AB were divided into 8 equal parts ($1 + 3 + 4 = 8$) and length of AC be 1 of these eighths, length of CD be 3 of these eighths, length of DB be 4 of these eighths,

Then $AC : CD : DB = 1 : 3 : 4.$

Hence $AC = \frac{1}{8} \times 4{\cdot}8'' = 0{\cdot}6'',$

$CD = \frac{3}{8} \times 4{\cdot}8'' = 3 \times 0{\cdot}6'' = 1{\cdot}8'',$

and $DB = \frac{4}{8} \times 4{\cdot}8'' = 4 \times 0{\cdot}6'' = 2{\cdot}4.''$

106. *To find a fourth proportional to three given quantities.*

Example.—Find the fourth proportional to, 2, 2·5 and 3, and verify result by calculation.

Construction.—Take 1 inch as the unit. Draw any line AX (Fig. 66) and a line AK making a convenient angle with AX.

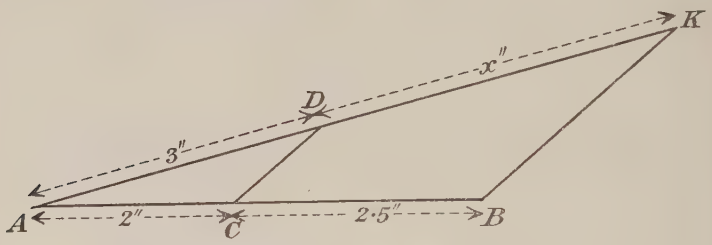


Fig. 66. (Scale ½ size.)

Along AX mark off $AC = 2''$ and $CB = 2{\cdot}5''$.

Along AK mark off $AD = 3''$. Join CD and from B draw BK parallel to CD, meeting AK at the point K.

Then since ABK is a triangle and CD is parallel to BK. CD cuts the sides AB and AK proportionately.

Hence $$\frac{AC}{CB} = \frac{AD}{DK} \text{ or } \frac{2}{2{\cdot}5} = \frac{3}{DK}.$$

Therefore DK is a 4th proportional to 2, 2·5 and 3. DK measures 3·75″ approx.

Calculation.—Suppose x is the fourth proportional to 2, 2·5 and 3. Then $$\frac{2}{2{\cdot}5} = \frac{3}{x}.$$

Inverting each ratio,

$$\frac{x}{3} = \frac{2{\cdot}5}{2}.$$

Multiply by 3 and

$$x = \frac{2{\cdot}5 \times 3}{2} = \frac{7{\cdot}5}{2} = 3{\cdot}75.$$

107. *To find a third proportional to two given quantities.*

Example.—Find a third proportional to 1·8 and 2·3 and verify by calculation.

Construction.—Take 1 inch as the unit.

Draw two lines AX, AY (Fig. 67) making any convenient angle with one another. Along AX cut off $AB = 1{\cdot}8''$ and $BC = 2{\cdot}3''$. Along AY cut off $AD = 2{\cdot}3''$. Join BD and from C draw CK parallel to BD meeting AY at the point K.

Then since BD is parallel to CK one side of a triangle ACK, therefore BD cuts AC and AK proportionally.

Hence $$\frac{AB}{BC} = \frac{AD}{DK} \text{ or } \frac{1{\cdot}8}{2{\cdot}3} = \frac{2{\cdot}3}{DK}.$$

Therefore DK is the third proportional to 1·8, and 2·3.

DK measures 2·95″.

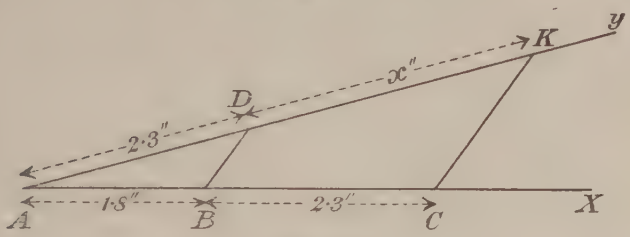


FIG. 67.

Calculation.—Suppose x is the third proportional to 1·8 and 2·3. Then $$\frac{1{\cdot}8}{2{\cdot}3} = \frac{2{\cdot}3}{x}.$$

Inverting each ratio,

$$\frac{x}{2{\cdot}3} = \frac{2{\cdot}3}{1{\cdot}8}$$

Multiply both sides by 2·3 and

$$x = \frac{2{\cdot}3 \times 2{\cdot}3}{1{\cdot}8} = \frac{5{\cdot}29}{1{\cdot}8}$$

$= 2{\cdot}94$ correct to 3 significant figures.

Note.—The student should work through these examples using different units, *e.g.*, $\frac{1}{2}$ inch ; 1 cm. ; 2 cms., etc.

108. Fact to learn.—*If two triangles are equiangular to one another, their corresponding sides are proportional.*

Let ABC, DEF be two triangles (Fig. 67*a*) having

Angle ABC = Angle DEF.

,, BCA = ,, EFD.

,, CAB = ,, FDE.

The sides in each triangle which are opposite equal angles are said to be corresponding sides.

Thus AB and DE, BC and EF, CA and FD are pairs of corresponding sides.

We wish to know what relation exists between the ratios

$$\frac{AB}{DE}, \frac{BC}{EF}, \frac{CA}{FD}.$$

Exercise 1.—Draw two triangles ABC, DEF, having their bases BC and EF, 4″ and 3″ respectively. Make the angles at the base of each triangle 50° and 70°, *i.e.*, $\hat{ABC} = \hat{DEF} = 50°$ and $\hat{ACB} = \hat{DFE} = 70°$. Since the three angles of every triangle together equal two right angles, then the third angle BAC is equal to the third angle EDF, each being 60°.

Now measure the sides of each triangle in inches and compare the ratios

$$\frac{AB}{DE}, \frac{BC}{EF}, \frac{CA}{FD}.$$

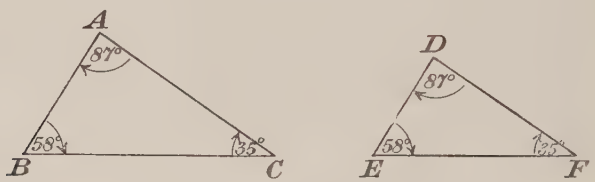


Fig. 67A.

Exercise 2.—Repeat Exercise 1 with two triangles on bases 2·3″, 3·2″ each with base angles 30° and 100°.

Exercise 3.—Draw a triangle with sides 4, 5, and 6 cms., and another triangle with sides 6, 7·5 and 9 cms. Are the sides of the two triangles proportional? Measure their angles.

On working these exercises carefully the student will find that the ratio between any pair of corresponding sides in two equiangular triangles is equal to the ratio between any other pair, or we say the corresponding sides are proportional. The triangles are said to be *similar triangles*.

We shall now be able to understand

109. *The Construction and Use of a Diagonal Scale.*

(*a*) ABC is any triangle. The side AB is divided into *ten* equal parts, and lines are drawn through these points of division parallel to the base BC.

Now in Fig. 68, ADE and ABC are equiangular triangles.

Hence $$\frac{DE}{BC} = \frac{AD}{AB}.$$

But $\frac{AD}{AB} = \frac{1}{10}$.

Therefore $\frac{DE}{BC} = \frac{1}{10}$.

$DE = \frac{1}{10}BC$.

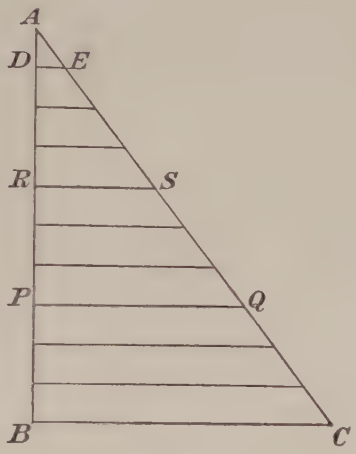


FIG. 68.

Similarly, APQ and ABC are equiangular triangles.

Then $\frac{PQ}{BC} = \frac{AP}{AB}$.

But $\frac{AP}{AB} = \frac{7}{10}$.

Therefore $\frac{PQ}{BC} = \frac{7}{10}$ or $PQ = \frac{7}{10}BC$.

In this way we may express each of the lines parallel to BC a definite fraction of BC.

Suppose BC represented 1 yard.

Then DE would represent $\frac{1}{10}$ yard
and PQ ,, ,, $\frac{7}{10}$,,
,, RS ,, ,, $\frac{4}{10}$,, etc.

Also, if BC were made $\frac{1}{10}$ inch,

Then DE would be $\frac{1}{100}$ inch
PQ ,, $\frac{7}{100}$,,
RS ,, $\frac{4}{100}$,, etc.

(b) *To construct a Diagonal Scale to show Inches, Tenths and Hundredths.*—A straight line AB (Fig. 69) is divided (from A) to inches and the points of division marked 0, 1, 2 . . .

The primary division OA is subdivided into *tenths*, the secondar divisions being numbered (from O) 1, 2, 3, . . . , 9. We ma now read on AB *inches* and *tenths* of an inch.

Hundredths.—In order to read *hundredths* ten lines are taken : any equal intervals parallel to AB ; and perpendiculars a drawn through 0, 1, 2 . . . The primary (or inch) division corr sponding to OA on the tenth parallel is now subdivided in *ten* equal parts ; and diagonal lines are drawn as in the diagra joining 0 to the *first* point of subdivision on the 10th paralll

,, 1 ,, *second* ,, ,, ,, ,, ,,
,, 2 ,, *third* ,, ,, ,, ,, ,,

and so on.

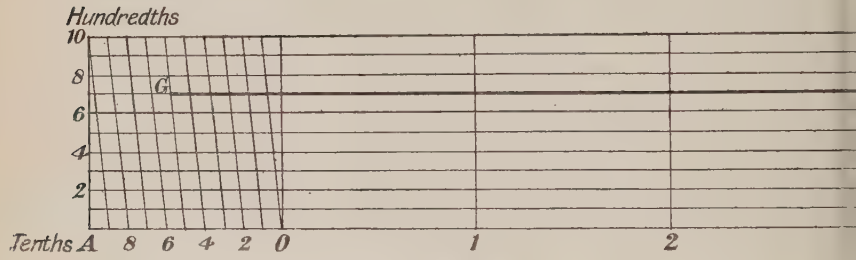


Fig. 69.

The scale is now complete, and its use is shown in the followi example.

Example.—To take from the scale a length of 3·57 *inches.*

(i) Place one point of the dividers on the inch mark 3 on A and extend the arms till the other point reaches 5 in the su divided inch OA. We have now 3·5 inches in the dividers.

(ii) To get the remaining 7 *hundredths*, move the right-han point up the perpendicular through the 3-inch mark till reaches the seventh parallel (point F in Fig. 69). Th extend the dividers till the left point reaches the diagon through the $\frac{5}{10}$-inch mark also on the seventh parallel (point G We have now 3·57 inches in the dividers (Line FG).

Similarly, by means of this scale, the length of a given straight li may be measured to the nearest hundredth of an inch.

Again, if one inch division on the scale represents 10 feet, then 3· inches represent 35·7 feet. And if one inch division on the sca represents 1 metre, then 3·57 inches represent 357 cms. Thus a diagon scale is useful in preparing plans of enclosures, buildings, parts machinery, where it is necessary that every dimension of the act object must be represented by a line of proportional length on the pl

Exercise.—Draw a diagonal scale, 2 centimetres to represent 1 ya showing yards, feet, and inches.

). Fact to learn.—*The angle in a semicircle is a right angle.*

Exercise 1.

Take two pins and fix them in your paper at points A and B g. 70).

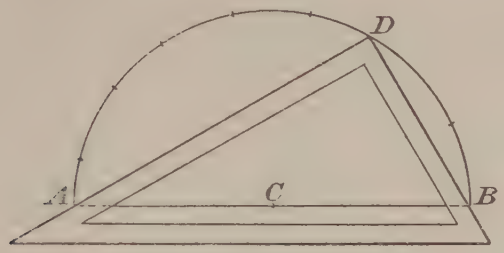


Fig. 70.

Take your set square and place it flat on the paper—with the right gle as in the figure—and mark the position of D the right angle, pping the sides of the set square touching the pins.
Now turn the set square round, keeping the edges against the pins, d mark the various positions of D as shown in the figure.
Bisect AB at C and with C as centre and CA as radius describe a micircle and notice that the curve passes through the points plotted.

Exercise 2.

Repeat Experiment 9, § 59, with another semicircle of any radius d again record your results (Fig. 71).

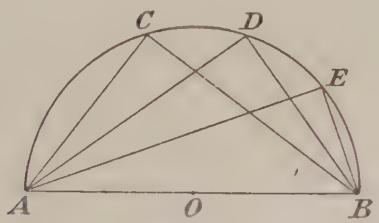


Fig. 71.

These exercises show that if you join any point on the semi-circumference of a circle to the ends of the diameter the angle so formed is *right angle.*

This result we express by saying that *the angle in a semicircle a right angle.*

The knowledge of this fact enables us "*to describe a right-gled triangle having given the length of the hypotenuse and one her dimension.*"

Example.—Draw a right-angled triangle (Fig. 72), given that th hypotenuse is 8·6 cms. and one side is 5·6 cms. Measure the thir side and verify result by calculation.

The hypotenuse in a right-angled triangle is the side opposite th right angle. From what we have just learnt we must make the hypo tenuse the diameter of a semicircle.

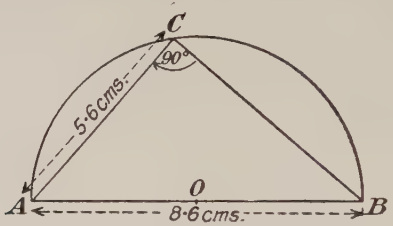


FIG. 72. (Scale ½ size.)

Thus we draw $AB = 8{\cdot}6$ cms. and bisect it at the point O, and wit centre O and radius OA describe a semicircle. Wherever we take a poin C on the circumference the triangle ACB will be right-angled, and i we only had two dimensions given, (*a*) the hypotenuse AB; (*b*) th angle $ACB = 90°$, there would be an infinite number of solutions But we have learnt before (§ 59, § 83) that three dimensions must b known to fix a triangle.

In the present exercise, the third dimension is the length of anothe side = 5·6 cms. Hence, with centre A (or B) and radius 5·6 cms describe an arc cutting the semicircumference at C.

Join CA and CB and ABC is the triangle required. CB measure 6·5 cms. to nearest $\frac{1}{10}$.

Calculation.—In any right-angled triangle the area of the squar described on the hypotenuse is equal to the sum of the areas of th squares on the sides containing the right angle (§ 67).

Hence in Fig. 72

$$CB^2 + AC^2 = AB^2$$

or

$$\begin{aligned} CB^2 &= AB^2 - AC^2 \\ &= (8{\cdot}6)^2 - (5{\cdot}6)^2 \text{ sq. cms.} \\ &= (8{\cdot}6 + 5{\cdot}6)\ (8{\cdot}6 - 5{\cdot}6) \text{ sq. cms.} \\ &= 14{\cdot}2 \times 3 = 42{\cdot}6 \text{ sq. cms.} \end{aligned}$$

$\therefore CB = \sqrt{42{\cdot}6} = 6{\cdot}5$ cms. to nearest $\frac{1}{10}$th.

111. Fact to learn.—"*In a right-angled triangle, if a per pendicular is drawn from the right angle to the hypotenuse, th triangles on each side of it are similar to the whole triangle an to one another.*"

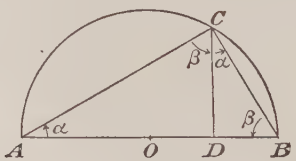


FIG. 73.

Exercise.—On $AB = 4''$ describe a semicircle (Fig. 73). aw AC inclined at 35° to AB and meeting the arc at C. in CB and draw the altitude CD.

(*a*) Write in the size of each angle of the triangles CDA, ABC. Fill in the following table.

Triangle ABC.	Triangle ADC.	Triangle CDB.
(i) $\frac{AC}{CB} =$	(iv) $\frac{AD}{DC} =$	(vii) $\frac{CD}{DB} =$
(ii) $\frac{AB}{AC} =$	(v) $\frac{AC}{AD} =$	(viii) $\frac{CB}{CD} =$
(iii) $\frac{AB}{BC} =$	(vi) $\frac{AC}{CD} =$	(ix) $\frac{BC}{BD} =$

(*b*) You will notice that you get three sets of equal ratios. t us see why ratios (i) and (iv) are equal.

The triangles ABC, ACD are equiangular, for

Angle ACB = Angle ADC (each is a right angle).

„ CAB = „ CAD (same angle).

∴ Third angle CBA = third angle ACD (the three angles any triangle together equal two right angles).

Then sides AC, CB in triangle ACB are corresponding sides AD, DC in triangle ADC, and we have learnt that in equi-gular triangles the corresponding sides are in proportion.

Hence $$\frac{AC}{CB} = \frac{AD}{DC}.$$

w try and reason why any other pair of those ratios in (*a*) equal.

(*c*) Consider ratios (iv) and (vii).

Triangles ADC, CDB are equiangular, for

ngle ACD + angle BCD = angle $ACB = 90°$,

„ ACD + „ $CAD = 90°$ (since angle $ACD = 90°$),

n angle CAD = angle BCD.

„ „ CDA = „ CDB (each is a right angle).

Third angle ACD = third angle CBD.

Now sides AD, DC in triangle ADC correspond to sides DC, in triangle DCB.

$$\therefore \quad \frac{AD}{DC} = \frac{DC}{DB}.$$

ote the side DC is in both triangles. So that we have *three s in proportion*, and 1st : 2nd = 2nd : 3rd.

Then the 2nd is a mean proportional between the other two (§ 102), or *CD is a mean proportional between AD and DB.*

Similarly from ratios (ii) and (v) we see *AC is a mean proportional between AB and AD.*

Also from ratios (iii) and (ix) *BC is a mean proportional between AB and BD.*

A knowledge of the above fact concerning a right-angled triangle enables us to see the reason for the following constructions :—

112. *To find the mean proportional between two given straight lines.*

Example.—Find geometrically the mean proportional between 7·2 cms. and 5·0 cms. and check your result by calculation.

Method 1.—Draw $AB = 12{\cdot}2$ cms. (Fig. 74), and from A cut off $AD = 7{\cdot}2$ cms. Bisect AB at O and with centre O and radius OA describe a semicircle. From D draw DC perpendicular to AB meeting the arc at the point C.

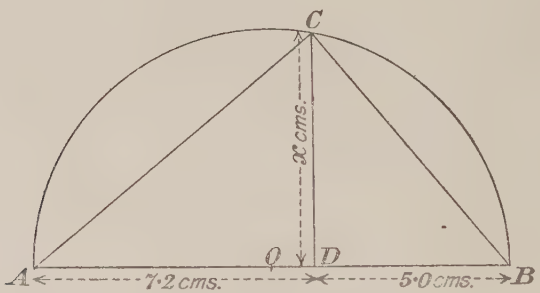


Fig. 74. (Scale $\frac{1}{2}$ size.)

Then CD is the *mean proportional between AD and DB* or between 7·2 and 5·0.

Proof.—The angle $ACB = 90°$ being angle in a semicircle, CD is the perpendicular drawn from the right angle to the hypotenuse of a right-angled triangle ABC. Then the triangles ADC, CDB are similar, and

$$\frac{AD}{DC} = \frac{DC}{DB}.$$

CD measures 6 cms.

Method 2.—Draw $AB = 7{\cdot}2$ cms. (Fig. 75). From A cut off $AD = 5$ cms. Bisect AB at O and with centre O and radius OA describe a semicircle. From D draw DC perpendicular to AB meeting the arc at the point C. Join AC.

Then AC is the *mean proportional* between AB and AD or tween 7·2 and 5·0.

Proof.—As before ACB is a right-angled triangle and CD the perpendicular drawn from the right angle to the hypo-nuse. Then the triangles ABC, ADC are similar.

Hence
$$\frac{AB}{AC} = \frac{AC}{DA}.$$

AC measures 6 cms.

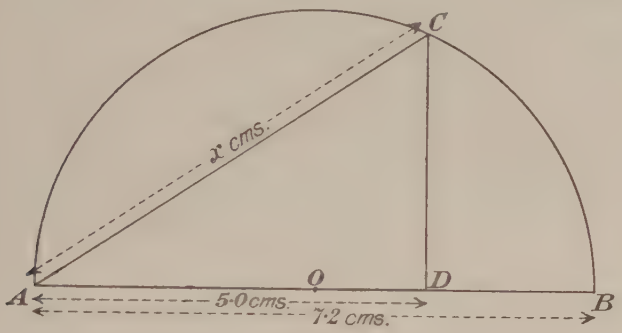


Fig. 75.

Calculation.—Let the mean proportional be x cms.

Then
$$\frac{7{\cdot}2}{x} = \frac{x}{5{\cdot}0}.$$

Multiply both sides of this equation by $5x$ and
$$36 = x^2$$
or $$x = \sqrt{36} = 6.$$

Then 6·0 cms. is the mean proportional between 7·2 cms. nd 5·0 cms.

3. *To find the Square Root of a number graphically.*

Example.—By a geometrical construction find $\sqrt{12}$ and verify our result by calculation.

Let us try and analyse the problem first.

Suppose x is the required square root.

Then
$$\sqrt{12} = x$$
or $$12 = x^2$$
[The number we are seeking, when squared, equals 12.] (§ 29.)
or $$3 \times 4 = x \times x.$$

Divide both sides by $3x$ and

$$\frac{4}{x} = \frac{x}{3}.$$

Thus 4, x, and 3 are three quantities in proportion.

In other words x is a *mean proportional* between 4 and 3.

Hence we get the following construction :—

Draw $AB = 7$ cms. (Fig. 76). From AB cut off $AD = 4$ cms. On AB describe a semicircle and from D draw DC perpendicular to AB. Then DC is the mean proportional between AD and DB; that is

$$\frac{AD}{DC} = \frac{DC}{DB}$$

$$\text{or } DC^2 = AD \times DB = 4 \times 3 = 12.$$

$$\therefore \quad DC = \sqrt{12}.$$

CD measures 3·47 cms. $\quad \therefore \quad \sqrt{12} = 3{\cdot}47.$

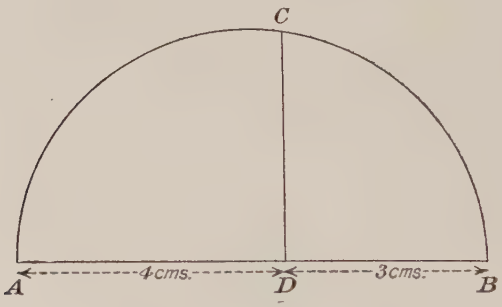


Fig. 76.

Calculation.

Let $\quad x = \sqrt{12}$

then $\quad \text{Log } x = \frac{1}{2} \log 12$

$= \frac{1}{2} (1{\cdot}0792) = 0{\cdot}5396.$

$\therefore \quad x = \text{antilo} \quad {\cdot}5396 = 3{\cdot}464.$

$\therefore \quad \sqrt{12} = 3{\cdot}46$ c rrect to 3 significant figures.

Note.—$12 = 4 \times 3$ (i) $= 6 \times 2$ ii) $= 12 \times 1$ (iii).

If $x^2 = 12$,

then $\quad x \times x = 4 \times 3$ (i) or $x \times x = 6 \times 2$ (ii)

or $x \times x = 12 \times 1$ (iii).

Dividing equation (i) by $3x$ we have

$$\frac{4}{x} = \frac{x}{3}$$

or x is the *mean proportional* between 4 and 3.

Dividing equation (ii) by $2x$ we have

$$\frac{6}{x} = \frac{x}{2}$$

or x is the *mean proportional* between 6 and 2.

Dividing equation (iii) by x we have

$$\frac{12}{x} = \frac{x}{1}$$

or x is the *mean proportional* between 12 and 1.

In each case $x = \sqrt{12}$.

Thus in the drawing we may make :

(i) $AD = 4$ cms. ; $DB = 3$ cms.
or (ii) $AD = 6$ cms. ; $DB = 2$ cms.
or (iii) $AD = 12$ cms. ; $DB = 1$ cm.

and still obtain the same length for the perpendicular DC. The student should try methods (ii) and (iii).

114. *To describe a Square equal in Area to a given Rectangle.*

Example.—Construct a square equal in area to a rectangle whose length and breadth are 3·0″ and 1·5″. Test your work by measurement and calculation.

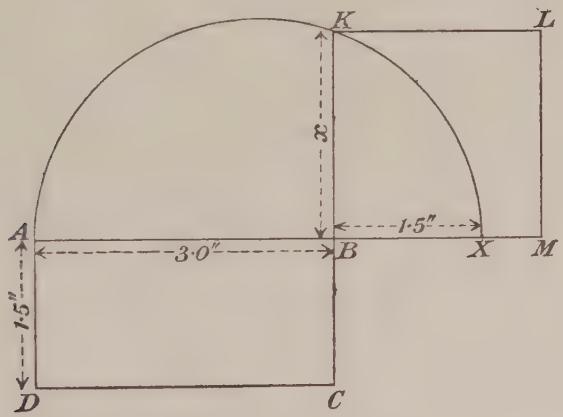


Fig. 77. (Scale ½ size.)

If we suppose that the length of one side of the required square $= x$ inches, then area of square $= x^2$ square inches. But area of rectangle $= 3{\cdot}0 \times 1{\cdot}5$ square inches.

Hence $3{\cdot}0 \times 1{\cdot}5 = x^2$.

Dividing each side by $(1{\cdot}5 \times x)$ we have

$$\frac{3{\cdot}0}{x} = \frac{x}{1{\cdot}5}$$

and x is the mean proportional between 3·0″ and 1·5″. An inspection of the figure (Fig. 77) will show the construction.

ABCD is the given rectangle.
BKLM is the square equal in area to the rectangle *ABCD*.
BK measures 2·11″ (using diagonal scale).

Calculation.

Let $BK = x''$.

Then $x^2 = 3{\cdot}0 \times 1{\cdot}5$,

or $x = \sqrt{4{\cdot}5} = 2{\cdot}121$ (by logs.).

Then $BK = 2{\cdot}12''$ correct to nearest $\frac{1}{100}''$.

115. Fact to learn.—*In any two circles, the arcs or the chords which subtend equal angles at the centre are proportional to the radii of the circles.*

Exercise to show this,

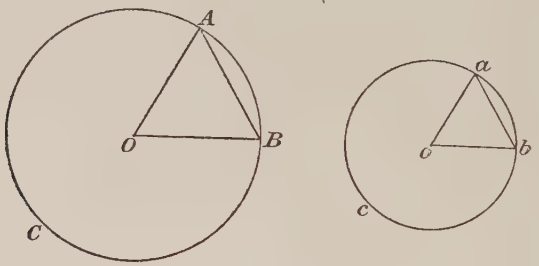


Fig. 78.

Draw two circles *ABC*, *abc*, as in Fig. 78, with radii 3″ and 2″. From the centre of each circle draw radii inclined at 40°.

(1) Measure *arcs AB*, *ab* by means of thread.

(2) Measure *chords AB*, *ab* by means of a scale.

Note.—*Arc AB* is the distance along the curve from *A* to *B*. *Chord AB* is the straight line distance from *A* to *B*.

(i) Show (*a*) $\dfrac{\text{Arc } AB}{\text{Arc } ab} = \dfrac{\text{Radius } OA}{\text{Radius } oa} = \frac{3}{2}$.

(*b*) $\dfrac{\text{Chord } AB}{\text{Chord } ab} = \dfrac{\text{Radius } OA}{\text{Radius } oa} = \frac{3}{2}$.

(ii) Repeat (i) by varying the angles at the centre.

(iii) Repeat (i) and (ii) by drawing circles of radii 4″ and 3″; 3 cms. and 4 cms., etc.

A knowledge of the foregoing fact helps us to understand the results of the following examples.

116. *When a lever, straight or bent, moves about its fulcrum the distances moved by the ends of the arms are proportional to the lengths of the arms.*

Example 1.

AOB represents a straight lever (Fig. 79) capable of turning about a point O. $AO = 3$ feet, $OB = 5$ feet. (Scale 1″ represents 1 foot.)

Suppose it turns about the fulcrum O through various angles 30°, 40°, 45°, 60°, etc.

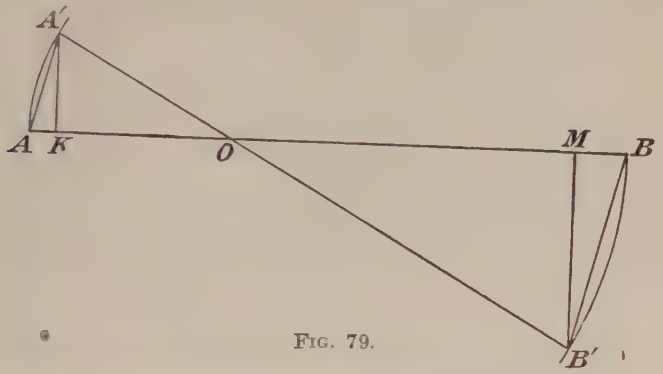


Fig. 79.

Show by measurement that in each case :

(i) $\dfrac{AA'}{BB'} = \dfrac{OA}{OB}$.

(ii) If $A'K$ and $B'M$ be perpendiculars from A' and B' to the horizontal position AOB,

$$\frac{A'K}{B'M} = \frac{OA}{OB}$$

[*Note.*—Triangles $OA'K$, $OB'M$ are similar.]

Example 2.

A straight lever OAB (Fig. 80) turns about a fulcrum O. $OB = 7$ feet. A is a point in OB so that $OA = 4$ feet. (Scale 1″ represents 1 foot.)

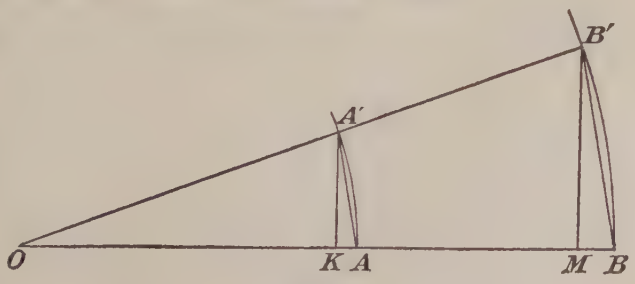


Fig. 80.

Suppose the lever turns about the point O through various angles 30°, 40°, 50°, 60°, etc.

Show by measurement that in each case

(i) $$\frac{AA'}{BB'} = \frac{OA}{OB}.$$ [*Note.*—Triangles OAA', OBB' are equiangular.]

(ii) If $A'K$, $B'M$ be perpendiculars from A' and B' to the original position OAB,

$$\frac{A'K}{B'M} = \frac{OA'}{OB'} = \frac{OA}{OB}.$$

[Triangles $OA'K$, $OB'M$ are equiangular and therefore similar.]

Example 3.

AOB represents a bent lever, the arms OA and OB being at right angles (Fig. 81). The arm OA turns through an angle of 40° and the lever now occupies the position marked $A'OB'$.

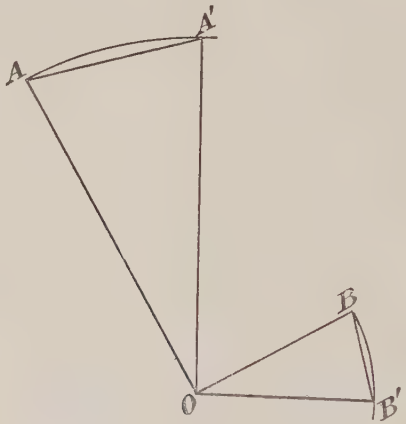


Fig. 81.

Make arms $OA = 4''$; $OB = 2{\cdot}5''$, and show by measurement that

$$\frac{AA'}{BB'} = \frac{OA}{OB}.$$

If the arms OA and OB were 4 feet and 2·5 feet respectively, and $BB' = 6$ inches, find AA'.

Now $$\frac{AA'}{BB'} = \frac{OA}{OB} \text{ or } \frac{AA'}{6''} = \frac{4 \text{ feet}}{2{\cdot}5 \text{ feet}}$$

or $$AA' = \frac{6 \times 4}{2{\cdot}5} = \frac{6 \times 4 \times 2}{5} = 9{\cdot}6''.$$

17. *Depth of Shed—Bradford Tappet Shedding Mechanism.* This is a very common type of shedding mechanism.

The tappets (for construction of tappets see § 158) depress the treadles, by making contact with the anti-friction bowls B_1, B_2 (Fig. 82). The treadles T_1, T_2 are placed at right angles to the low shaft of the loom and are pivoted at F.

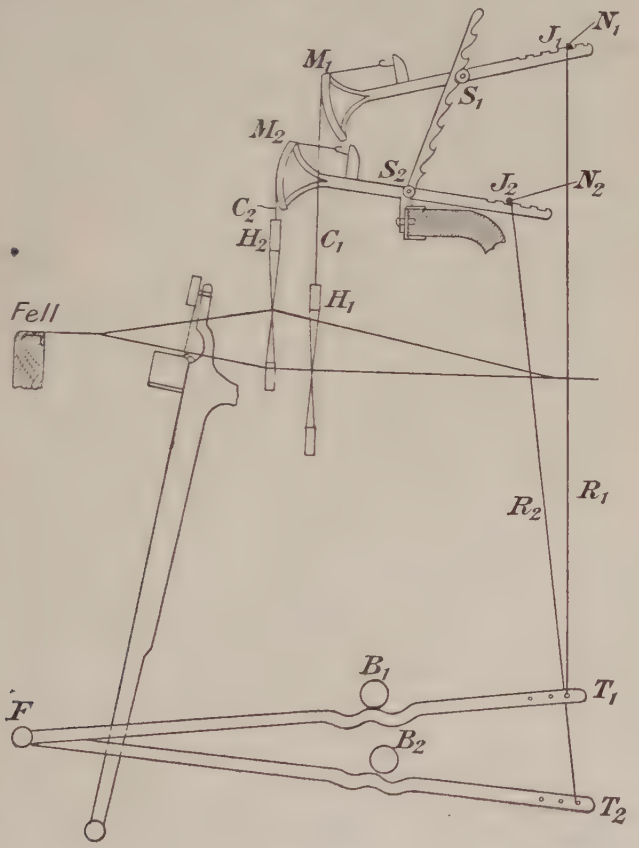


FIG. 82.

Connecting rods R_1, R_2 connect the treadles with the jack levers J_1, J_2. These are attached to the square shafts S_1, S_2. These square shafts are carried in brackets and are of such a length as to allow two half-moon levers M_1, M_2 being mounted on them, one at each end of the heald shaft H_1, H_2, to which they are connected by straps and cords C_1, C_2.

The jack levers J_1, J_2 are provided with a series of notches about $\frac{5}{8}''$ apart. Any alteration of the nut N_1 from one notch to another increases or decreases the distance through which the heald shaft is raised.

(*a*) *Thus*, whilst the jack lever is moved downwards as the treadle is forced down by the tappet, the half-moon lever moves upwards, raising the heald shaft and the warp threads.

In Fig. 83, if PA represent the jack lever and PB the half-moon lever, and if the lowering of the treadle causes A to move to A' and consequently B to B', then from similar triangles (§ 116),

$$\frac{BB'}{AA'} = \frac{PB}{PA} \quad \text{or} \quad BB' = \frac{PB}{PA} \times AA'.$$

With the same throw of the tappet, AA' will keep the same. PB is always the same length. Then,

If we decrease PA we shall increase BB'.

If we increase PA we shall decrease BB'.

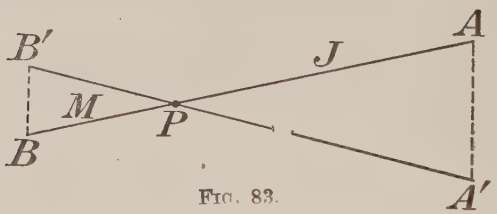


Fig. 83.

To give the same depth of shed at the same distance from the fell of the cloth, the back heald shafts must be raised more than the front shafts, since all the treadles are lowered the same distance by one particular kind of tappet. Hence the nuts N_1 (Fig. 82) on the jack lever connected with the back heald shaft will be nearer to the square shaft, thus decreasing the effective length of the jack lever.

(*b*) The size of shed required to produce the best results varies with the size of the shuttle and the quality of the warp yarns employed. If too deep a shed is formed, an excessive strain is put on the threads, and this results in frequent breakages. If the shed is too small, extra power will be required to drive the shuttle through the shed and the friction or rubbing of the shuttle will wear down the threads.

Example 1.

Suppose the throw of the tappet is 3 inches ; the treadle is 36 inches long, and the anti-friction pulley is 24 inches from

he fulcrum; the half-moon lever is 8 inches and the jack ever 9 inches long; the front heald shaft is 12 inches from he fell of the cloth and the shuttle crosses when the front f the race is 6 inches from the fell. Find the depth of the huttle required, allowing $\frac{1}{4}$ inch for clearance.

Fig. 84, not drawn to scale, shows, sufficiently for calculation urposes, the upper and lower positions of the front heald haft H, H'.

The throw of the tappet is 3 inches; then the vertical distance noved downwards by the anti-friction bowl B is 3″ (BB').

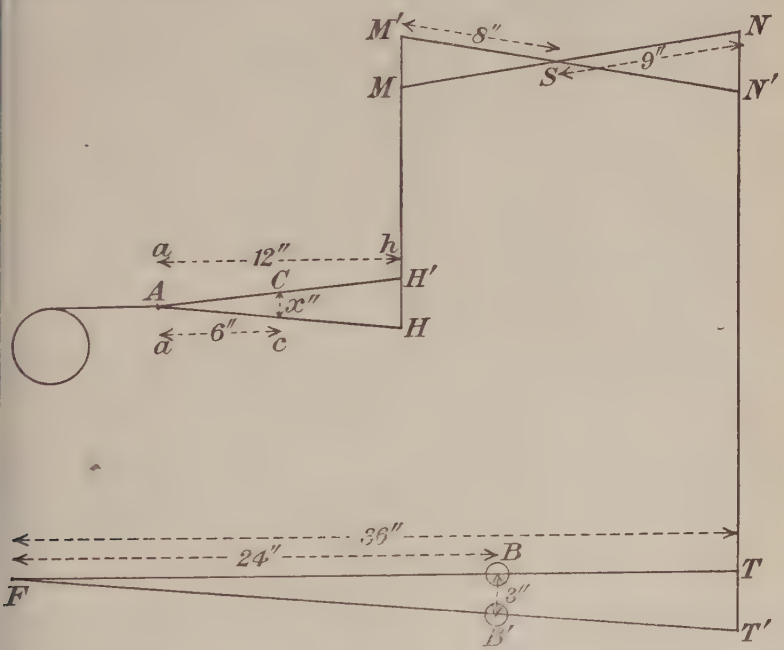


Fig. 84.

(Remember B moves on the arc of a circle with centre F and radius 24″.)

As B moves to B', the end of the treadle T moves to T', and at the same time the end of the jack lever N moves downwards to N', and $NN' = TT'$.

As N moves to N', the end of the half-moon lever M moves upwards to M', raising the heald shaft H to H', and $HH' = MM'$.

CC' measures the depth of the shed at a point 6″ from the

fell of the cloth A, and the distance CC' must be sufficient to allow the shuttle to cross at this point.

(*a*) *To find the length of* TT'.

Now from Example 2, § 116,

we notice $$\frac{TT'}{BB'} = \frac{36}{24} = \frac{FT}{FB}$$

or $$\frac{TT'}{3} = \frac{36}{24}$$

or $$TT' = \frac{36 \times 3}{24} = 4\tfrac{1}{2}'' = NN'.$$

(*b*) *To find the length of* MM'.

Referring to Example 1, § 116, we have :—

$$\frac{MM'}{NN'} = \frac{MS}{SN}$$

or $$\frac{MM'}{4\frac{1}{2}} = \frac{8}{9}$$

or $$MM' = \frac{8 \times 4\frac{1}{2}}{9} = 4'' = HH'.$$

(c) *To find the length of* CC'.

As in (*a*) $$\frac{CC'}{HH'} = \frac{6}{12} = \frac{ac}{ah}$$

or $$\frac{CC'}{4} = \frac{6}{12}$$

or $$CC' = \frac{6 \times 4}{12} = 2''.$$

Then *depth of shed* at the point 6″ from the fell of the cloth s 2 inches.

Allowing $\frac{1}{4}$ inch for clearance, this would allow a shuttle $1\frac{3}{4}''$ deep being used.

Example 2.

Find the throw of the tappet for a 4-inch shuttle, allowing $\frac{1}{4}$ inch for clearance. The shuttle enters 5 inches from the fell of the cloth ; the front heald is 12 inches from the fell ; the half-moon lever is 10 inches long, whilst the jack lever arm is 11 inches (effective length). The treadle is 36 inches long and the anti-friction pulley is 24 inches from the fulcrum.

We will use the diagram for Example 1 (Fig. 84) with the following dimensions.

$ac = 5''$; $CC' = 4\frac{1}{4}''$ at least

$ah = 12''$

(allowing $\frac{1}{4}''$ clearance for a 4″ shuttle).

$MS = 10''$; $SN = 11''$; $FT = 36''$; $FB = 24''$.

$BB' = x''$ = the throw of the tappet.

To find length of HH'.

$$\frac{HH'}{CC'} = \frac{ah}{ac} \text{ or } \frac{HH'}{4\frac{1}{4}} = \frac{12}{5}.$$

$$\therefore \quad HH' = \frac{12 \times 4\frac{1}{4}}{5} = 10{\cdot}2'' = MM'.$$

To find length of NN'.

$$\frac{NN'}{MM'} = \frac{SN}{SM} \text{ or } \frac{NN'}{10{\cdot}2} = \frac{11}{10}.$$

$$\therefore \quad NN' = \frac{11 \times 10{\cdot}2}{10} \text{ inches} = TT'.$$

To find length of BB'.

$$\frac{BB'}{TT'} = \frac{FB}{FT} \text{ or } \frac{BB'}{\frac{11 \times 10{\cdot}2}{10}} = \frac{24}{36}.$$

$$\therefore \quad BB' = \frac{24}{36} \times \frac{11 \times 10{\cdot}2}{10} = 7{\cdot}48''.$$

∴ *Throw of the tappet* $= 7\frac{1}{2}$ inches.

EXAMPLE 3.

The depth of the shed in an open shed loom is $4\frac{1}{2}$ inches the front shaft and 6 inches at the back shaft. The front

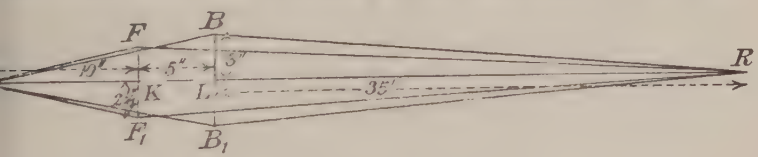


FIG. 85.

aft is 10 inches from the edge of the piece or fell of the cloth, d the back shaft is 35 inches from the back rest and 5 inches m the front shaft. Calculate the difference in length of o threads, one drawn on the front shaft and the other on back shaft when fully lifted.

In the diagram (Fig. 85) A represents the fell of the cloth ; F_1 the front shaft ; BB' the back shaft ; R the back rest. AK represents 10″, KL 5″, LR 35″.

Since depth of shed at the front shaft is $4\frac{1}{2}$″ represented by ″, then FK represents $2\frac{1}{4}$″. Similarly BL represents 3″.

a) To find length of a thread drawn on the front shaft.

This is represented by $(AF + FR)$.

Since AFK and FKR are right-angled triangles,

$$AF^2 = AK^2 + KF^2 \quad [\text{See } \S\ 67.]$$
$$= 10^2 + (2{\cdot}25)^2 = 105{\cdot}0625 \text{ square inches.}$$
$$\therefore \quad AF = \sqrt{105{\cdot}0625} = 10{\cdot}25 \text{ inches.}$$

Similarly $FR^2 = FK^2 + KR^2$

$$= (2{\cdot}25)^2 + 40^2 = 1605{\cdot}0625 \text{ square inch}$$

and $FR = \sqrt{1605{\cdot}0625} = 40{\cdot}06$ inches.

Then length of a thread drawn on the front shaft $= 10{\cdot}25 + 40{\cdot}06$

$= 50{\cdot}31$ inches.

(*b*) *To find the length of a thread drawn on the back shaft.*

This is represented by $(AB + BR)$.

Since ALB and BLR are right-angled triangles,

$$AB = \sqrt{AL^2 + BL^2} = \sqrt{15^2 + 3^2}.$$
$$= \sqrt{234} = 15{\cdot}30 \text{ inches.}$$

Also $BR = \sqrt{BL^2 + LR^2} = \sqrt{3^2 + 35^2}$

$$= \sqrt{1234} = 35{\cdot}13 \text{ inches.}$$

Then length of a thread drawn on the back shaft $= 15{\cdot}30 + 35{\cdot}13$

$= 50{\cdot}43$ inches

and Difference in length $= 50{\cdot}43 - 50{\cdot}31$

$= 0{\cdot}12$ inch to nearest $\frac{1}{100}''$.

Example 4.

What should be the size of shed in a power loom, wit particulars as follows: Shuttle $1\frac{3}{4}$ inches deep, 2 inches wid crank 3 inches, shafts 12 inches from the fell of the cloth ?

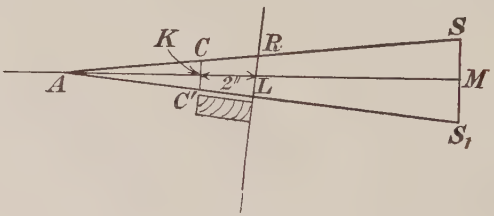


Fig. 86.

Since the crank is 3″ long and the reed is beating up th cloth when crank is at its inner dead centre, then the ree cannot recede from the cloth more than 6 inches.

Hence in Fig. 86 $AL = 6''$. The width of the shuttle is inches, then the front face of the shuttle is never more tha 4 inches from the fell of the cloth and $AK = 4''$.

ow at this point the distance (CC') between the warp
ads must be 2 inches, allowing $\frac{1}{4}$ inch for clearance.
SS' represents the *depth of the shed at the healds*, and
$= 12''$,
rom similar triangles ACC', ASS', we have:

$$\frac{SS'}{CC'} = \frac{AM}{AK} \text{ or } \frac{SS'}{2} = \frac{12}{4}$$

$$\text{or } SS' = \frac{12 \times 2}{4} = 6 \text{ inches.}$$

. Depth of shed must be 6 inches.

ıe following example is left as an exercise for the student.
ample 5.—What throw of tappet would be required for the first
d stave, assuming measurements as follows: Distance of heald
fell of cloth 9 inches: distance of race from fell of cloth when
shuttle enters the shed $3\frac{1}{2}$ inches; depth of shuttle $1\frac{3}{4}$ inches on
side of the same; other particulars as in Example 2?—(Ans. 3·8″.)

CHAPTER IX

Trigonometrical Ratios

118.—Introduction. 119.—Ratios connected with Angles. 120. Definitions of Trigonometrical Ratios. 121.—To find by Drawing t Approximate Values of the Trigonometrical Ratios of a given Ang 122.—Given one of the Trigonometrical Ratios of an Angle to constr the Angle. 123.—Approximate determination of the Sines of vari Acute Angles by Geometrical Construction. 124.—Graph of the Si 125.—Approximate Cosines of various Acute Angles determi Geometrically. 126.—Graph of the Cosine. 127.—Complement Angles. 128.—Tables of Trigonometrical Ratios. 129.—Solution Right-angled Triangles. 130.—Area of a Triangle. 131.—Proble involving the Use of Trigonometrical Ratios.

118. *Introduction.*

In the last chapter we dealt with the meaning of t term "ratio," and the values of certain important rati

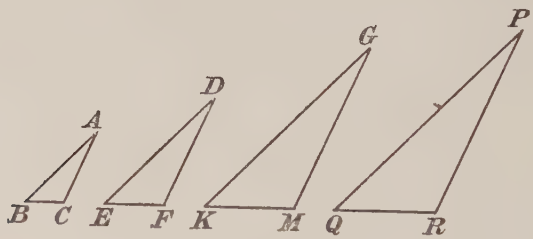


Fig. 86A.

Also one of the geometrical facts learnt was: "*If two more triangles are equiangular, the ratio between any pair sides in one triangle is equal to the ratio between the pair corresponding sides in any of the other triangles.*"

Thus, if ABC, DEF, GKM, PQR be equiangular triang having angles at A, B and C respectively equal to the ang at D, E and F or G, K and M, or at P Q and R (Fig. 86A),

Then $$\frac{AC}{AB} = \frac{DF}{DE} = \frac{GM}{GK} = \frac{PR}{PQ}$$

d $$\frac{AC}{CB} = \frac{DF}{FE} = \frac{GM}{MK} = \frac{PR}{RQ}$$

d $$\frac{BC}{BA} = \frac{EF}{ED} = \frac{KM}{KG} = \frac{QR}{QP}.$$

9. *Ratios connected with angles.*

In this chapter we shall introduce certain *ratios connected th angles* and shall explain their meaning and use when the gles involved are not greater than 90° or a right angle.

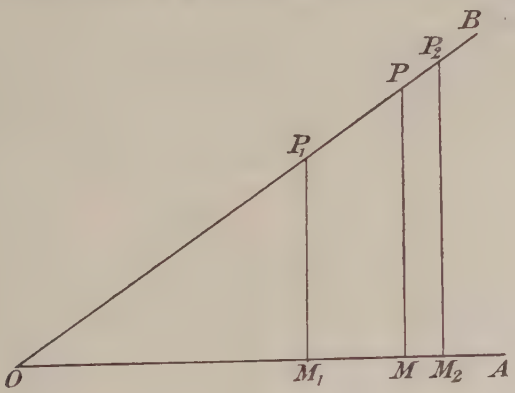


FIG. 87.

Draw two lines OA and OB (Fig. 87), any length, from the oint O, making the angle $AOB = 35°$.

Take any point P in one arm OB and draw PM perpendicular to the other arm OA.

(a) Measure MP and OP in cms. and work out $\frac{MP}{OP}$ to two ecimal places.

Thus in figure $MP = 3{\cdot}52$ cms. ; $OP = 6{\cdot}15$ cms. and

$$\frac{MP}{OP} = \frac{3{\cdot}52}{6{\cdot}15} = 0{\cdot}57 \text{ correct to 2 decimal places.}$$

Now take P in various other positions along OB as P_1, P_2. Draw perpendiculars P_1M_1 ; P_2M_2 as before and find value ratios $\frac{M_1P_1}{OP_1}$, $\frac{M_2P_2}{OP_2}$.

[Other members in the class will have taken P at random, so compare ur results with those of others.]

The lengths of MP and OP will be different every time, but good drawing should show that the value of the ratio $\frac{MP}{OP}$ is always the same.

That this result should be so is seen from the fact that the right-angled triangles POM, P_1OM_1, P_2OM_2 are equiangular, and the fact noted above in § 118 can be applied to them.

Definition.—The ratio $\frac{MP}{OP}$ is called the *Sine* of the angle AOB.

Notation.—The result obtained above is written shortly

$$\text{Sin } 35^\circ = 0{\cdot}57.$$

(*b*) Now measure OM and OP and work out $\frac{OM}{OP}$.

Thus in figure $OM = 5{\cdot}04$ cms.; $OP = 6{\cdot}15$ cms. and

$$\frac{OM}{OP} = \frac{5{\cdot}04}{6{\cdot}15} = 0{\cdot}82 \text{ correct to nearest } \tfrac{1}{100}.$$

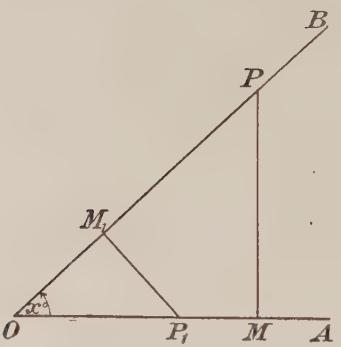


Fig. 88.

Also find values $\frac{OM_1}{OP_1}$, $\frac{OM_2}{OP_2}$ and compare with other members of the class. Here again the value of the ratio is constant.

Definition.—The ratio $\frac{OM}{OP}$ is called the *Cosine* of the angle AOB, and written shortly we have

$$\text{Cos } 35^\circ = 0{\cdot}82.$$

(*c*) Finally work out the ratio $\frac{MP}{OM}$.

Thus $MP = 3{\cdot}52$ cms.; $OM = 5{\cdot}04$ cms. and

$$\frac{MP}{OM} = \frac{3\,52}{5{\cdot}04} = 0{\cdot}70.$$

Again by careful drawing and measuring you will find that

$$\frac{MP}{OM} = \frac{M_1P_1}{OM_1} = \frac{M_2P_2}{OM_2} = \text{constant.}$$

Definition.—The ratio $\frac{MP}{OM}$ is called the *Tangent* of the angle

OB, or we say Tan 35° = 0·70.

(*d*) Now draw any other angle AOB (Fig. 88); choose your n size. Take any point P in OB and draw a perpendicular M to the other arm OA.

Also take a point P_1 in the arm OA and draw P_1M_1 perndicular to the other arm OB.

Compare the ratios

(i) $\frac{MP}{OP}$; $\frac{M_1P_1}{OP_1}$. (ii) $\frac{OM}{OP}$; $\frac{OM_1}{OP_1}$. (iii) $\frac{MP}{OM}$; $\frac{M_1P_1}{OM_1}$.

Can you explain why these are equal in pairs ?

20. *Definitions of Trigonometrical Ratios.*—Collecting the sults obtained in § 119 (*a*), (*b*), (*c*) and (*d*) we see that to etermine the trigonometrical ratios of an acute angle BOA e draw from any point on either arm a line perpendicular the other arm, thus forming a right-angled triangle.

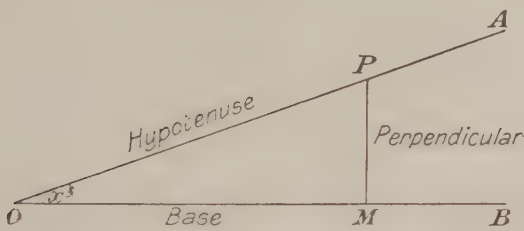


Fig. 89.

If the size of the angle AOB or POM (Fig. 89) be $x°$, and OM is the right-angled triangle, the side PM *opposite the ngle* POM is termed *the perpendicular*. The side OM *adjacent the angle* POM is termed *the base*. The side OP opposite he right angle PMO is termed the *hypotenuse*.

Then the trigonometrical ratios of the angle POM or $x°$ are defined as follows.

(1) *Sine* $x°$, written Sin $x°$ $= \dfrac{MP}{OP} = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$.

(2) *Cosine* $x°$, written Cos $x°$ $= \dfrac{OM}{OP} = \dfrac{\text{Base}}{\text{Hypotenuse}}$.

(3) *Tangent* $x°$, written Tan $x°$ $= \dfrac{MP}{OM} = \dfrac{\text{Perpendicular}}{\text{Base}}$.

Fig. 90.

There are also three other ratios which are the reciprocals of the first three :—

(4) *Cosecant* $x°$, written Cosec $x°$ $= \dfrac{1}{\text{Sin } x°}$

$= \dfrac{OP}{MP} = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

(5) *Secant* $x°$, written Sec $x°$ $= \dfrac{1}{\text{Cos } x°}$

$= \dfrac{OP}{OM} = \dfrac{\text{Hypotenuse}}{\text{Base}}$.

(6) *Cotangent* $x°$, written Cot $x°$ $= \dfrac{1}{\text{Tan } x°}$

$= \dfrac{OM}{MP} = \dfrac{\text{Base}}{\text{Perpendicular}}$.

As an example let us consider the right-angled triangle ABC (Fig. 90).

For conciseness it is usual to designate the sizes of the parts a triangle (three angles and three sides) by single letters us :—

Size of the angle $BAC = A°$;
" " " $ABC = B°$;
" " " $BCA = C°$.

Length of the side BC, opposite angle $BAC = a$ units.
" " " CA " " $ABC = b$ "
" " " AB " " $BCA = c$ "

Then if $a = 3$ cms.; $b = 4$ cms.; $C = 90°$, we know from 37

$$c^2 = a^2 + b^2 = (9 + 16) \text{ square cms.}$$
$$= 25 \text{ square cms.}$$
d $$c = 5 \text{ cms.}$$

Considering the angle ABC or $B°$ we have AC is the perndicular; BC is the base, and AB is the hypotenuse of the ght-angled triangle ACB.

$$\text{Sin } B° = \frac{AC}{AB} = \frac{b}{c} = \frac{4}{5}; \quad \text{Cosec } B° = \frac{1}{\text{Sin } B} = \frac{c}{b} = \frac{5}{4}.$$

$$\text{Cos } B = \frac{BC}{AB} = \frac{a}{c} = \frac{3}{5}; \quad \text{Sec } B = \frac{1}{\text{Cos } B} = \frac{c}{a} = \frac{5}{3}.$$

$$\text{Tan } B = \frac{CA}{BC} = \frac{b}{a} = \frac{4}{3}; \quad \text{Cot } B = \frac{1}{\text{Tan } B} = \frac{a}{b} = \frac{3}{4}.$$

Again, if we consider the angle BAC or $A°$ we must consider C as the perpendicular, CA as the base, and AB as the ypotenuse of the right-angled triangle ACB.
hen

$$\text{Sin } A° = \frac{BC}{BA} = \frac{a}{c} = \frac{3}{5}; \quad \text{Cosec } A° = \frac{1}{\text{Sin } A} = \frac{c}{a} = \frac{5}{3}.$$

$$\text{Cos } A = \frac{CA}{AB} = \frac{b}{c} = \frac{4}{5}; \quad \text{Sec } A = \frac{1}{\text{Cos } A} = \frac{c}{b} = \frac{5}{4}.$$

$$\text{Tan } A = \frac{BC}{CA} = \frac{a}{b} = \frac{3}{4}; \quad \text{Cot } A = \frac{1}{\text{Tan } A} = \frac{b}{a} = \frac{4}{3}.$$

1. *To find by drawing the approximate value of the Trigometrical Ratios of a given angle.*

(*a*) *To find Sin* 35° *and Cos* 35° (approximately).—Using the otractor, make the angle $AOB = 35°$ as accurately as you n (Fig. 91). Along the arm OA measure $OP = 10$ cms. sing ruler and compasses or set squares, draw PM perpencular to the arm OB.

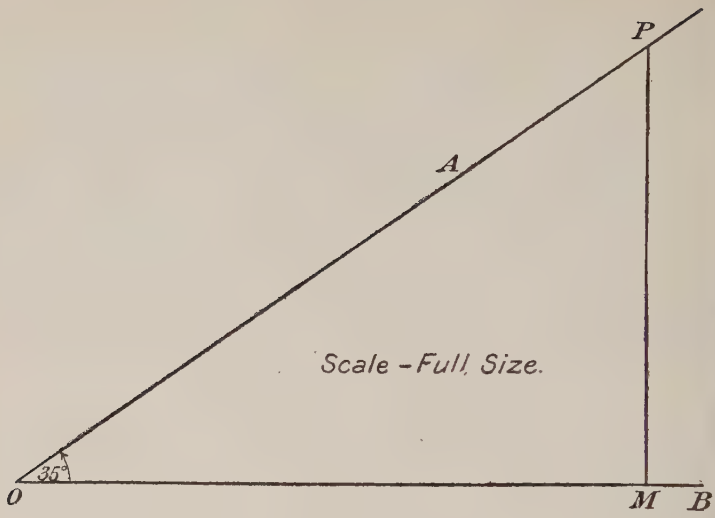


Fig. 91.

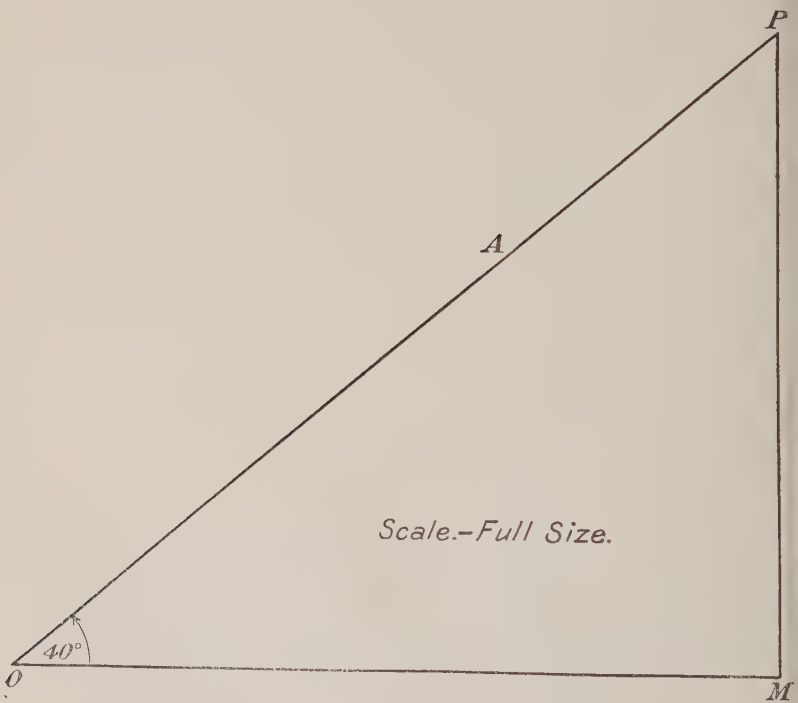


Fig. 92.

Then $OP = 10$ cms.; $PM = 5{\cdot}7$ cms.; $OM = 8{\cdot}2$ cms. and

$$\text{Sin } 35° = \frac{PM}{OP} = \frac{5{\cdot}7}{10} = 0{\cdot}57.$$

$$\text{Cos } 35° = \frac{OM}{OP} = \frac{8{\cdot}2}{10} = 0{\cdot}82.$$

It is possible to *calculate* the sine and cosine of 35° to a much greater accuracy than 0·57 and 0·82, and the results of such calculations are recorded in tables.

For instance a "four-figure" table gives

$$\text{Sin } 35° = 0{\cdot}5736 \text{ and Cos } 35° = 0{\cdot}8192.$$

It is impossible to express in figures the exact value of sin 35° or cos 35°, but they can be calculated to any required degree of accuracy.

(*b*) *To find the approximate value of Tan* 40°.—Use your protractor to make the angle $AOB = 40°$ (Fig. 92). Along the arm OB measure $OM = 10$ cms., and from M draw MP at right angles to OM meeting the arm OA at the point P. Measure the length of MP in cms.

PM measures 8·37 cms. and OM measures 10 cms. and

$$\text{Tan } 40° = \frac{PM}{OM} = \frac{8{\cdot}37}{10} = 0{\cdot}837.$$

The four-figure table of tangents gives Tan 40° = 0·8391.

2. *Given one of the Trigonometrical Ratios of an angle, to construct the angle.*

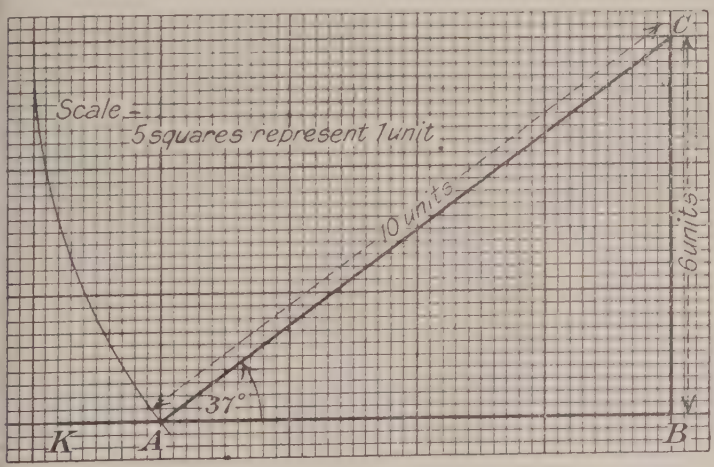


Fig. 93.

Example 1

To draw an angle whose Sine is 0·6.—We may use either squared or plain paper. Suppose we use squared paper as in Fig. 93 and let 10 squares represent 2 units of length.

Draw a line BC 30 squares long to represent 6 units.

Draw BK at right angles to BC. With centre C and radius 50 squares to represent 10 units, describe an arc of a circle cutting BK at the point A. Then $CA = 10$ units in length.

Since $\text{Sin } CAB = \dfrac{CB}{CA} = \dfrac{6}{10} = 0{\cdot}6$,

$\therefore$ CAB is the angle required.

If measured with a protractor, angle $CAB = 37°$ (nearly).

A "four-figure" table gives Sin $37° = 0{\cdot}6018$.

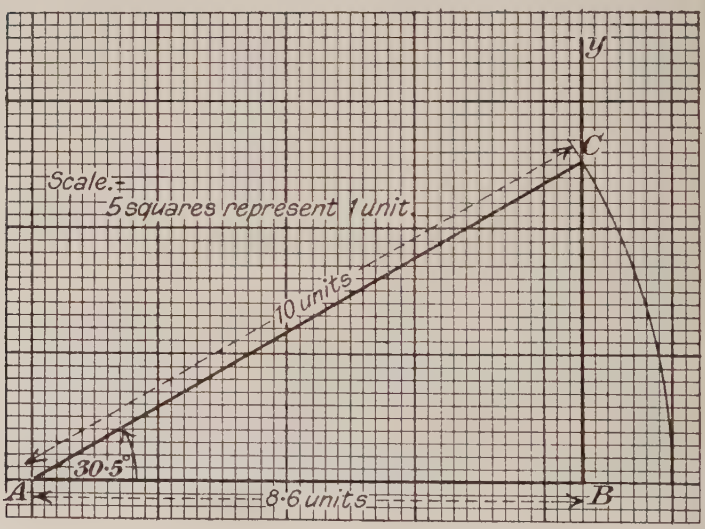


Fig. 94.

Example 2.

To construct an angle whose Cosine is 0·86.—Suppose [illegible] squares represent 2 units of length (Fig. 94). Draw AB [illegible] squares long to represent 8·6 units. From B draw BY [illegible] right angles to AB. With centre A and radius 50 square[illegible]

represent 10 units, describe an arc of a circle cutting BY the point C. Then $AC = 10$ units in length.

Since Cos $CAB = \frac{AB}{AC} = \frac{8{\cdot}6}{10} = 0{\cdot}86$,

Therefore CAB is the angle required.
If measured with a protractor, angle $CAB = 30{\cdot}5°$ (nearly).
From a "four-figure" table we get Cos $30{\cdot}5° = 0{\cdot}8616$.

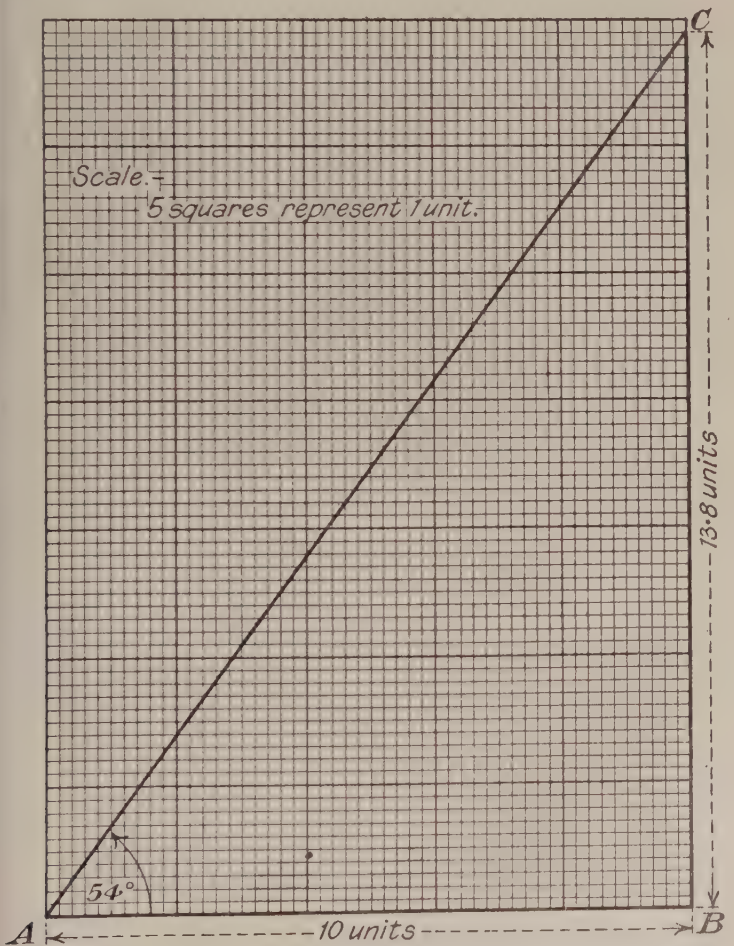


FIG. 95.

Example 3.

To construct an angle whose Tangent is 1·38.—As before, let 10 squares represent 2 units of length. Draw BC 69 squares long to represent 13·8 units of length (Fig. 95). From B draw BA at right angles to BC and make BA 50 squares long to represent 10 units.

$$\text{Since Tan } CAB = \frac{BC}{AB} = \frac{13{\cdot}8}{10} = 1{\cdot}38,$$

$\therefore$ CAB is the required angle and measures 54° (nearly).
From the tables :—Tan 54° = 1·3764.

123. *Approximate Determination of the Sines of various acute angles by Geometrical Construction.*

The figure (Fig. 96) shows approximately the sine of any angle between 0° and 90°. Some of the properties of the sine appear evident from this diagram.

(*a*) *To use the figure to find the sine of an angle say* 15°.—The length of the radius of the arc is taken as the unit. Join the point P marked 15° on the arc to 0. Then the angle POX = 15°. If PN be drawn perpendicular to OX,

$$\text{Sin } 15° = \frac{PN}{OP} = \frac{0{\cdot}259}{1} = 0{\cdot}259.$$

Similarly by joining P' (60°) to O and drawing $P'N'$ perpendicular to OX we have

$$\text{Sin } 60° = \frac{P'N'}{OP'} = \frac{0{\cdot}87}{1} = 0{\cdot}87.$$

The lengths of PN and $P'N'$ are obtained from the scale of lengths marked along OY.

(*b*) *The values of Sin* 0° *and Sin* 90°.—Referring to the figure again we notice that as the angle PON becomes less, the length of the perpendicular PN decreases, whilst the length of OP remains the same, and thus the sine of the angle PON decreases.

When the angle PON becomes 0°, then OP lies along the line OX, and the perpendicular PN is zero, and thus

$$\text{Sin } 0° = 0.$$

Again, as the angle PON increases, P taking positions on the arc marked 75°, 80°, 85°, etc., the length of the perpendicular PN increases and the sine of the angle increases.

When the angle PON becomes 90°, then OP coincides with OY and the perpendicular PN merges into the line PO, and thus

$$\text{Sin } 90° = \frac{PO}{OP} = 1.$$

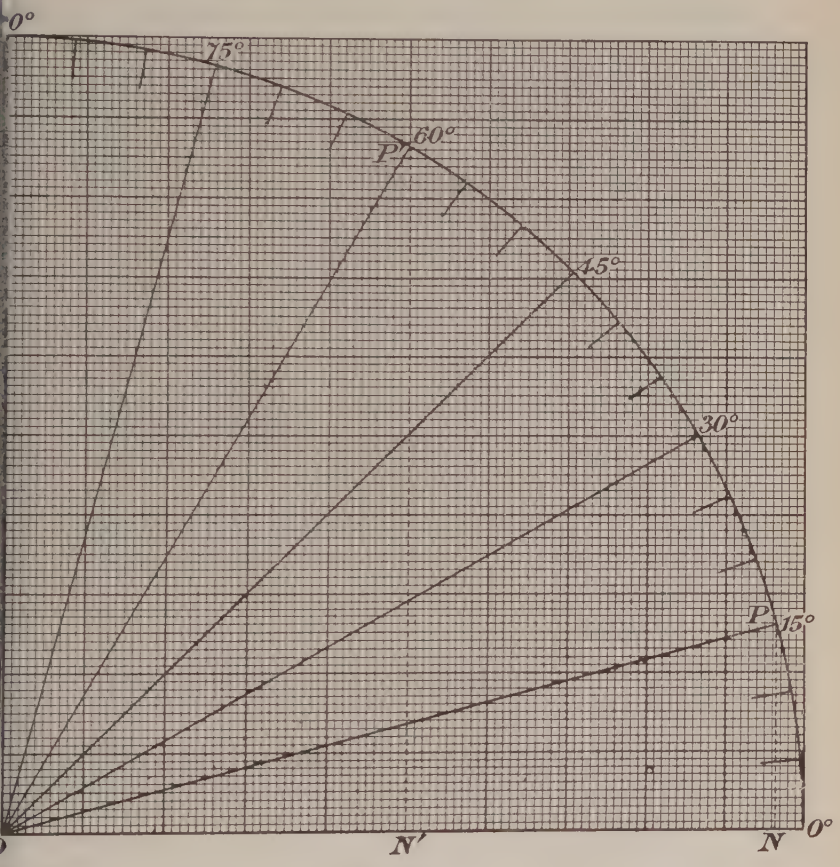


FIG. 96.

(c) *To draw the figure and write out a Table of Sines.*

Using millimetre squared paper let 100 squares represent the unit. If you use paper squared in $\frac{1}{10}$ths of an inch, take 50 squares for the unit.)

Draw OX and OY (Fig. 96) at right angles. With centre O and radius 1 unit describe an arc of a circle terminated by OX and OY.

Using a protractor, mark the positions of P along the arc every 5°. Read the values of PN, for various positions of P, from the scale along OY and tabulate your results as shown below.

Size of Angle *POX*.	Value of Sin *POX*.	Increase in Sine for each 5°.	Average. Increase per degree.
0°	0		
		0·087	0·018
5°	0·087		
		0·086	0·017
10°	0·173		
		0·086	0·017
15°	0·259		
		0·081	0·016
20°	0·34		
		0·08	0·016
25°	0·42		
		0·08	0·016
30°	0·50		
		0·07	0·014
35°	0·57		
		0·07	0·014
40°	0·64		
		0·067	0·013
45°	0·707		
		0·058	0·012
50°	0·765		
		0·055	0·011
55°	0·82		
		0·045	0·009
60°	0·865		
		0·041	0·008
65°	0·906		
		0·034	0·007
70°	0·940		
		0·025	0·005
75°	0·965		
		0·02	0·004
80°	0·985		
		0·010	0·002
85°	0·995		
		0·005	0·001
90°	1		

The table shows us that the sine appears to increase gradually from 0 to 1 as the angle grows from 0° to 90°. But does it increase *uniformly, i.e.* by the same amount for every degree ? No. At first the sine appears to increase rapidly from the value 0 and the rate of increase is fairly constant. Then as the angle grows the sine increases less rapidly and the increase is very slow as the sine approaches the value 1 or as the angle approaches 90°.

124. *Graph of the Sine.*—If we take the results tabulated in § 123 (*c*) and plot the values for the angles horizontally and values of the corresponding sines vertically we obtain a graph similar to the one shown in the accompanying figure (Fig. 97).

1·0
0·8
0·6
0·4
0·2
Values for Sines and Cosines.
Curve of Sines.
Curve of Cosines.
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
Values for Angles.

Fig. 97.

This graph shows approximately how the sine changes as the angle grows from 0° to 90° and also exhibits the properties of the sine noticed above. We may use the graph to determine approximately the sine of any acute angle, but for greater accuracy we refer to a "*Table of Sines*" as explained later.

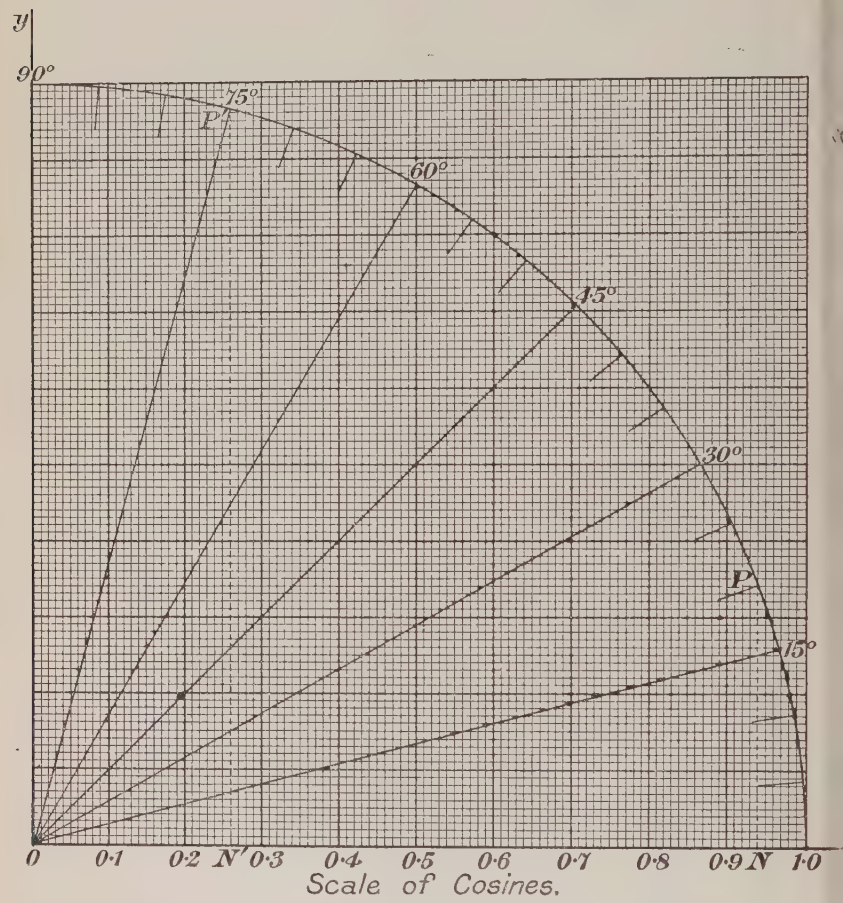


Fig. 98.

125. *Approximate Cosines of various acute angles determine geometrically.* — Fig. 98 shows approximately the cosin of any angle between 0° and 90°. The construction of thi figure is the same as that for Fig. 96, which gives the sine of acute angles.

(*a*) *To use the figure to find the Cosine of* 20°.—Mark the point P on the arc at the point marked 20°. Join P to O. Then the angle $POX = 20°$. If PN be drawn perpendicular to OX, then

$$\text{Cos } 20° = \frac{ON}{OP} = \frac{0{\cdot}94}{1} = 0{\cdot}94.$$

The radius of the arc is taken as the unit and the length of ON is obtained from the scale of lengths marked along OX.

Similarly by taking a point P' on the arc at 75°, joining P' to O, and drawing $P'N'$ perpendicular to OX we have

$$\text{Cos } 75° = \frac{ON'}{OP'} = \frac{0{\cdot}26}{1} = 0{\cdot}26.$$

(*b*) *The Values of Cos* 0° *and Cos* 90°.—Referring to the figure we notice that as the angle PON becomes less the length of ON increases, whilst the length of OP remains the same. Then the ratio $\frac{ON}{OP}$ increases or the cosine of the angle PON increases.

When the angle PON becomes 0°, OP lies along OX, and the points P and N coincide so that ON is the same length as OP.

Hence $$\text{Cos } 0° = \frac{OP}{OP} = 1.$$

Again, as the angle PON increases, the length of ON decreases and the cosine of the angle decreases.

As the angle PON approaches 90°, ON becomes very small, so that when the angle PON becomes 90°, OP coincides with the line OY and ON becomes zero.

Hence $$\text{Cos } 90° = \frac{O}{OP} = 0.$$

(*c*) Use your figure to write down the cosine of various angles from 0° to 90° and tabulate the results as shown below.

Value of angle .	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
Value of cosine .	1	0·995	0·98	0·97	0·94	0·90	0·87	0·82	0·766	0·707

Value of angle .	50°	55°	60°	65°	70°	75°	80°	85°	90°
Value of cosine .	0·642	0·572	0·50	0·42	0·342	0·26	0·174	0·09	0

126. *Graph of the Cosine.*—Take the results obtained in §125 (*c*) and plot values for the angle horizontally and corresponding values for the cosine vertically, and you will obtain a graph similar to the one shown in Fig. 97.

This graph shows us approximately how the cosine changes as the angle grows from 0° to 90°. At first the cosine appears to change very slowly from 1, but as the angle grows the cosine appears to decrease more and more rapidly. Finally, as the angle approaches 90° the cosine appears to decrease most rapidly. The graph enables us to obtain approximately the value of the cosine of any acute angle. For greater accuracy we refer to the tables as explained later.

127. *Complementary Angles.*—(*a*) When the sum of two angles is 90° or a right angle, the two angles are said to be "*complementary*," or each angle is called the "*complement*" of the other.

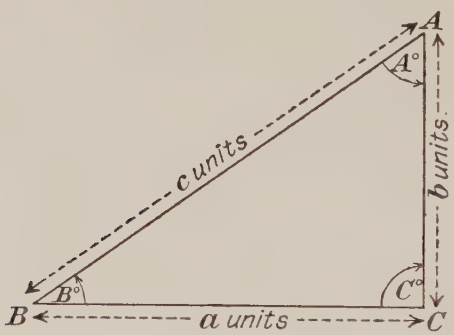


Fig. 99.

Thus 30° and 60° are complementary angles, or 30° is the complement of 60°, and *vice versa*.

Similarly	15°	is the complement of	75°.
	20°	„ „	70°.
	$x°$	„ „	$(90 - x)°$.
	$A°$	„ „	$(90 - A)°$.

(*b*) Now turn to your Graphs of Sines and Cosines (Fig. 97). Use them to find the values of :—

(i) Sin 30°, Cos 60° ; (ii) Sin 15°, Cos 75° ; (iii) Sin 38°, Cos 52° (iv) Cos 45°, Sin 45° ; (v) Sin 75°, Cos 15° ; (vi) Sin 83°, Cos 7°.

You will no doubt find each pair of values the same, *i.e.*,

Sin 30° = Cos 60° ; Sin 15° = Cos 75°, etc.

Then the graphs show you approximately that "*the sine f an angle is equal to the cosine of its complement,*" and "*the osine of an angle is equal to the sine of its complement.*"

(*c*) Again, consider the right-angled triangle ABC (Fig. 99), aving the angle ACB a right angle.

The angles ABC ($B°$) and BAC ($A°$) are complementary, or

$$B° = (90 - A)° \text{ and } A° = (90 - B)°.$$

To obtain the Trigonometrical ratios of the angle ABC we notice hat AC (the side opposite ABC) is the perpendicular, and BC he side adjacent to ABC) is the base.

Then $\quad \text{Sin } B° = \frac{b}{c}; \quad \text{Cos } B° = \frac{a}{c} \quad \text{Tan } B° = \frac{b}{a}.$

To obtain the Trigonometrical ratios of the angle BAC we must ake BC (the side opposite BAC) as the perpendicular, and AC he side adjacent to BAC) as the base.

Then $\quad \text{Sin } A° = \frac{a}{c}; \quad \text{Cos } A° = \frac{b}{c}; \quad \text{Tan } A° = \frac{a}{b}.$

Comparing these results, we get

$$\text{Sin } B° = \frac{b}{c} = \text{Cos } A°,$$

$$\text{Cos } B° = \frac{a}{c} = \text{Sin } A°,$$

$$\text{Tan } B° = \frac{b}{a} = \frac{1}{\text{Tan } A°} = \text{Cot } A°,$$

r, since $A°$ and $B°$ are complementary,

$$\begin{aligned} \text{Sin } B° &= \text{Cos } (90 - B)°, \\ \text{Cos } B° &= \text{Sin } (90 - B)°, \\ \text{Tan } B° &= \text{Cot } (90 - B)°. \end{aligned}$$

28. *Tables of Trigonometrical Ratios.*—This property of omplementary angles is made use of in the arrangement f the tables of the Trigonometrical ratios of angles from 0° o 90° which are given at the end of the book.

If the values of the sines and tangents of angles from 0° o 45° are found, these values will also be those of the cosines nd cotangents of angles from 90° to 45°.

To explain the method of reading the tables at the end an extract is shown below.

Angle		Chord	Sine	Tan	Cot	Cosine			
Degrees	Radians								
0°	0	0	0	0	∞	1	1·414	1·5708	90°
10°	0·1745	0·174	0·1736	0·1763	5·6713	0·9848	1·286	1·3963	80°
25°	0·4363	0·433	0·4226	0·4663	2·1445	0·9063	1·075	1·1345	65°
37°	0·6458	0·635	0·6018	0·7536	1·3270	0·7986	0·892	0·9250	53°
45°	0·7854	0·765	0·7071	1·0000	1·0000	0·7071	0·765	0·7854	45°
								Radians	Degrees
			Cosine	Cot	Tan	Sine	Chord	Angle	

Angles from 0° to 45° are placed in Column 1, their complements from 90° to 45° in Column 10, each pair of complementary angles being in the same horizontal line. Thus 37° in Column 1 is opposite its complement 53° in Column 10. At the top of Col. 4 we find Sine and at the bottom Cosine.

„ „ „ 5 „ Tangent „ „ Cotangent.
„ „ „ 6 „ Cotangent „ „ Tangent.
„ „ „ 7 „ Cosine „ „ Sine.

Suppose we require to know the value of Sin 37°. Look for 37° in Column 1, and in Column 4 with Sine *at the top* read in a horizontal line with 37°. Sin 37° = 0·6018.

If we require Cos 53°, look in Column 10 for 53°, and read in Column 4 which has Cosine *at the bottom* the value

$$\text{Cos } 53° = 0{\cdot}6018,$$

which value is the same as Sin 37°, since 37° and 53° are complementary.

If we require Cot 25° we look for 25° in Column 1, and in Column 6 with Cotangent *at the top* we read the value

$$\text{Cot } 25° = 2{\cdot}1445.$$

But this is equal to Tan 65°, which we find to be the case if we look for 65° in Column 10, and in Column 6 which has tangent *at the bottom* we read the value

$$\text{Tan } 65° = 2{\cdot}1445.$$

Thus in reading any ratio from this table, if the angle is between 0° and 45°, *i.e.*, shown in Column 1, we take the column with

ine, Cosine, etc., *at the top* agreeing with the particular ratio e require; whereas if the angle is between 45° and 90°, *i.e.*, nown in Column 10, we take the column with Sine, Cosine, etc., *t the bottom.*

29. *Solution of Right-angled Triangles.*—A triangle has three des and three angles. These we may call the six parts of ie triangle. We have seen (§ 83) that when any three of the x parts are given, one given part being a side, the triangle can e constructed and the remaining parts found. The three ngles of a triangle determine its shape, but not its size; ecordingly, at least one side must be given to fix it both in ze and shape.

We shall now show how to calculate the remaining parts of a ght-angled triangle having given two other parts besides the ct that one angle is a right angle, one of which must be a ide.

A. *Having given one of the angles and one of the sides containing he right angle.*

Example.—Let $b = 8{\cdot}30$ feet; $B = 32°$; $C = 90°$. Solve he triangle.

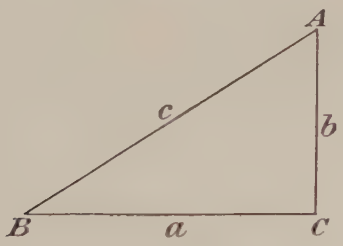


Fig. 100.

Solution by drawing.—Use squared paper and suppose 10 quares or 5 squares represent 1 foot.

Draw AC to represent 8·3 feet (Fig. 100). From C draw CB at right angles to CA.

Now $A + B + C = 180°$ and $C = 90°$

$\therefore \quad A + B = 90°$

nd since $B = 32°$, $A = (90° - 32°) = 58°$.

Hence, from A draw a line AB making an angle of 58° with AC and meeting CB at B.

Now measure the sides BC and BA.

Solution by calculation.—As shown above,

$$A = (90° - 32°) = 58°.$$

(1) To calculate the side BC (a units), we have

$$\frac{a}{b} = \text{Tan } A^\circ$$

$$\text{or } \frac{a}{8{\cdot}30} = \text{Tan } 58^\circ.$$

Multiply both sides of the equation by 8·30 and

$$a = 8{\cdot}30 \text{ Tan } 58^\circ = 8{\cdot}30 \times 1{\cdot}6003$$
$$\log a = \log 8{\cdot}30 + \log 1{\cdot}6003$$
$$= 0{\cdot}9191 + 0{\cdot}2041 = 1{\cdot}1232.$$
$$\therefore \quad a = \text{Antilog } 1{\cdot}1232 = 13{\cdot}28.$$

Hence side $BC = 13{\cdot}28$ feet.

(2) To calculate the side AB (c units), we have

$$\frac{b}{c} = \text{Sin } B^\circ$$

$$\text{or } \frac{8{\cdot}30}{c} = \text{Sin } 32^\circ,$$

$$\text{or } \frac{8{\cdot}30}{c} = \frac{0{\cdot}5299}{1}.$$

Inverting both ratios

$$\frac{c}{8{\cdot}30} = \frac{1}{0{\cdot}5299}.$$

Multiply by 8·30 and

$$c = \frac{8{\cdot}30}{0{\cdot}5299} \quad [\tfrac{83}{5} = 16 \text{ (approx.)}]$$

$$\log c = \log 8{\cdot}30 - \log 0{\cdot}5299$$
$$= 0{\cdot}9191 - \bar{1}{\cdot}7242 = 1{\cdot}1949,$$

then $c = \text{Antilog } 1{\cdot}1949 = 15{\cdot}66.$

Hence side $AB = 15{\cdot}66$ feet.

(3) We might have found the length of AB thus :—

Since ACB is a right-angled triangle,

$$AB^2 = BC^2 + CA^2 \text{ (Pythagoras' Theorem)}$$

or $$c^2 = a^2 + b^2$$

or $$c^2 = (13{\cdot}28)^2 + (8{\cdot}3)^2$$

and $$c = \sqrt{(13{\cdot}28)^2 + (8{\cdot}3)^2} \quad \text{(A)}.$$

We leave this calculation to be finished by the student, who will then see which method is to be preferred.

[(A) is unsuitable for calculation by logarithms.]

B. *Having given one of the angles and the Hypotenuse.*

Example.—Let $c = 12{\cdot}8$ inches ; $A = 29°$; $C = 90°$. Solve the triangle.

Solution by drawing.—Use squared paper and suppose 5 squares represent 1 inch.

Two methods are shown in Figs. 101 and 102, which are not drawn to scale.

Method 1. (Fig. 101.) Draw a vertical line AC any length. From A draw AB to represent $12{\cdot}8''$ and making an angle of $29°$ with AC. Through B draw a horizontal line BC.

Method 2. (Fig. 102.) This method depends on the fact that the angle in a semicircle is a right angle [§ 59 (ix)].

Draw AB to represent $12{\cdot}8''$. Bisect it and describe a semicircle ACB. From A draw AC making an angle of $29°$ with AB and meeting the circumference at C. Join CB.

In each case measure the sides BC and CA and the angle ABC.

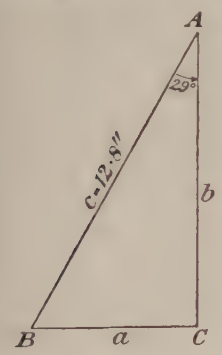


Fig. 101.

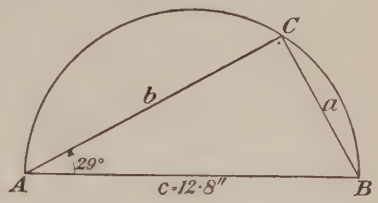


Fig. 102.

Solution by calculation.

Since $C = 90°$, $A + B = 90°$,

and since $A = 29°$, $B = 90° - 29° = 61°$.

(1) To calculate the side BC (a units), we have

$$\frac{a}{c} = \text{Sin } A°$$

$$\text{or } \frac{a}{12{\cdot}8} = \text{Sin } 29° = 0{\cdot}4848.$$

Multiply by 12·8 and

$$a = 0{\cdot}4848 \times 12{\cdot}8 \text{ [0·5 × 13 = 6·5 approx.]}$$

Then $\log a = \log 0{\cdot}4848 + \log 12{\cdot}8$

$= \bar{1}{\cdot}6855 + 1{\cdot}1072 = 0{\cdot}7927.$

$\therefore$ $a = \text{antilog } 0{\cdot}7927 = 6{\cdot}204$

or side $BC = 6{\cdot}20$ inches to nearest $\frac{1}{100}''$.

(2) To calculate the side CA (b units).

Here $\frac{b}{c} = \text{Sin } B°$

or $\frac{b}{12{\cdot}8} = \text{Sin } 61°$

or $b = 0{\cdot}8746 \times 12{\cdot}8 = 0{\cdot}9 \times 13 = 11{\cdot}7$ approx.

Log b = log $0{\cdot}8746$ + log $12{\cdot}8$

$= \bar{1}{\cdot}9418 + 1{\cdot}1072 = 1{\cdot}0490$

or b = antilog $1{\cdot}0490 = 11{\cdot}19$.

$\therefore$ Side $CA = 11{\cdot}19$ inches to nearest $\frac{1}{100}''$.

C. *Given the two sides containing the right angle.*

Example.—Let $a = 6{\cdot}8$ feet; $b = 5{\cdot}6$ feet. Solve the triangle.

Solution by drawing.—Using squared or plain paper, and choosing a convenient scale (5 squares represent 1 foot; 1 inch represents 1 foot),

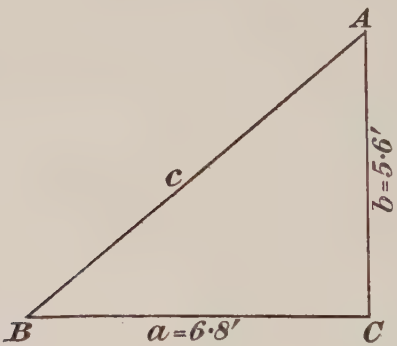


Fig. 103.

Draw BC to represent 6·8 feet (Fig. 103). Draw CA perpendicular to CB to represent 5·6 feet and join AB.

Measure the side BA and the angles CAB and ABC.

Solution by calculation.

(1) Tan $B° = \frac{b}{a} = \frac{5{\cdot}6}{6{\cdot}8} = 0{\cdot}8235$.

From the Table of Tangents we see

Tan $39° = 0{\cdot}8098$ and Tan $40° = 0{\cdot}8391$.

Thus B lies between 39° and 40°.

Here we assume that, for small changes in the angle, the increase or decrease in the trigonometrical ratio is proportional to the increase in the angle.

Thus $\text{Tan } 40° = 0{\cdot}8391.$
$\text{Tan } 39° = 0{\cdot}8098.$
Difference for $1° = 0{\cdot}0293.$

Suppose $B = (39 + x)°.$

Then $\text{Tan } (39 + x)° = 0{\cdot}8235.$
$\text{Tan } 39 = 0{\cdot}8098.$

nd Difference for $x° = 0{\cdot}0137.$

Hence $$\frac{x°}{1°} = \frac{0{\cdot}0137}{0{\cdot}0293} = 0{\cdot}4676$$

or $x = 0{\cdot}47°$ to 2 sig. figs.

nd $B = (39 + x)° = 39{\cdot}47°.$

(2) Also $A = 90° - B° = 90° - 39{\cdot}47° = 50{\cdot}53°.$

(3) To calculate the side AB (c units).

First method. $c^2 = a^2 + b^2$

or $c = \sqrt{6{\cdot}8^2 + 5{\cdot}6^2} = \sqrt{77{\cdot}60} = 8{\cdot}809.$

Then $AB = 8{\cdot}81$ feet to nearest $\frac{1}{100}$th.

Second method.

We have $$\frac{b}{c} = \text{Sin } B°$$

or $$c = \frac{b}{\text{Sin } B}$$

$$= \frac{5{\cdot}6}{\text{Sin } 39{\cdot}47°}$$

$$= \frac{5{\cdot}6}{0{\cdot}6356} \text{ (see note below)}$$

$$= 8{\cdot}810 \text{ (using logs.).}$$

N.B.—To find Sin 39·47°.

Sin 39·47° lies between sin 39° and sin 40°.

Now $\text{Sin } 40° = 0{\cdot}6428.$
$\text{Sin } 39° = 0{\cdot}6293.$
Difference for $1° = 0{\cdot}0135.$

Then Difference for $0{\cdot}47° = 0{\cdot}0135 \times 0{\cdot}47$
$= 0{\cdot}0063$ correct to 4 places.

Also the sine of an angle increases as the angle increases from 0° o 90°.

$\therefore$ $\text{Sin } 39{\cdot}47° = 0{\cdot}6293 + 0{\cdot}0063$
$= 0{\cdot}6356.$

D. *Given the lengths of the Hypotenuse and one other side.*

Example.—Suppose $c = 98$ inches, $a = 32$ inches.

Solution by drawing.

There are two methods as shown in Figs. 104 and 105 which are not drawn to scale.

First method. Draw BC (Fig. 104) (scale 5 squares represent 10 inches) representing 32 inches. Draw CA perpendicular to CB, and with centre B and radius representing 98 inches draw an arc of a circle cutting CA at A.

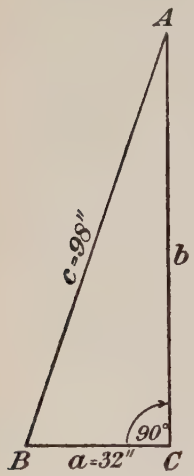


Fig. 104.

Second method. Draw AB (Fig. 105) representing 98 inches. On AB as diameter, describe a semicircle ACB. With centre B and radius representing 32 inches cut the semicircle at C. Join AC.

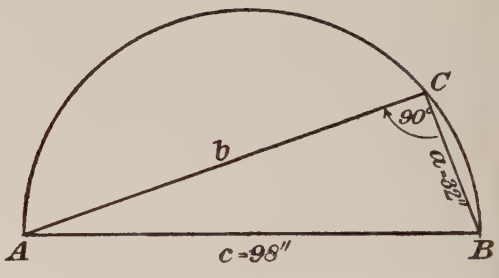


Fig. 105.

In each case measure AC ; Angle BAC ; Angle ABC.

Solution by calculation.

1. To calculate AC (b units) we have

$$c^2 = a^2 + b^2 \text{ or } b^2 = c^2 - a^2$$

$$\text{or } b = \sqrt{c^2 - a^2} = \sqrt{98^2 - 32^2}$$

$$= \sqrt{130 \times 66} \text{ since } c^2 - a^2 = (c+a)(c-a) \quad (\S\ 71)$$

$$= \sqrt{8580} = 92{\cdot}64 \text{ ins.}$$

or $AC = 92{\cdot}64$ inches to nearest $\frac{1}{100}$".

2. To calculate Angle BAC ($A°$).

$$\text{Sin } A° = \frac{a}{c} = \frac{32}{98} = 0{\cdot}3265.$$

From the tables we have

$$\text{Sin } 19° = 0{\cdot}3256 \text{ ; Sin } 20° = 0{\cdot}3420.$$

Then A lies between 19° and 20°.

Now
$$\text{Sin } 20^\circ = 0{\cdot}3420,$$
$$\text{Sin } 19^\circ = 0{\cdot}3256.$$
$$\text{Difference for } 1^\circ = 0{\cdot}0164.$$

If $A = (19 + x)^\circ$,
$$\text{Sin } (19 + x)^\circ = 0{\cdot}3265,$$
$$\text{Sin } 19^\circ = 0{\cdot}3256.$$
$$\text{Difference for } x^\circ = 0{\cdot}0009.$$

Then if the increase in the sine of an angle is proportional to the increase in the angle, we have

$$\frac{x^\circ}{1^\circ} = \frac{0{\cdot}0009}{0{\cdot}0164} = 0{\cdot}05 \text{ to nearest } \tfrac{1}{100}.$$

Then $B\hat{A}C = 19{\cdot}05^\circ$, and since $B + A = 90^\circ$,
$\therefore$ Angle $ABC = 70{\cdot}95^\circ$ and the triangle is solved.

30. *Area of a Triangle.*

We may use trigonometrical ratios to find the area of a triangle in terms of any two sides and the angles between them.

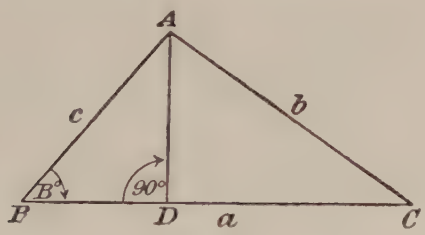


Fig. 106.

In the triangle ABC (Fig. 106) suppose we know the lengths of the side BC (a units) and the side AB (c units) and the angle ABC (B°).

Draw AD perpendicular to BC.

Then $\dfrac{AD}{c} = \text{Sin } B^\circ$ or $AD = c \text{ Sin } B^\circ$.

Now area of triangle ABC $= \frac{1}{2}$(Base $\times$ Height) (§ 65).
$= \frac{1}{2}\, a \times AD.$
$= \frac{1}{2}\, ac \text{ Sin } B.$

Similarly it can be proved that

Area of triangle $= \frac{1}{2}\, ab \text{ Sin } C^\circ = \frac{1}{2}\, bc \text{ Sin } A^\circ$
$$= \tfrac{1}{2}\left\{\begin{matrix}\text{(product of two}\\ \text{sides)}\end{matrix} \times \begin{matrix}\text{(sine of the angle}\\ \text{between them)}\end{matrix}\right\}.$$

Example.—In the triangle ABC if $a = 24{\cdot}5$ inches, $c = 10{\cdot}7$ inches. $B = 53°$. Find the area.

$$\text{Area} = \tfrac{1}{2}\, ac \operatorname{Sin} B°$$
$$= \tfrac{1}{2} \times 24{\cdot}5 \times 10{\cdot}7 \times \operatorname{Sin} 53°$$
$$= 12{\cdot}25 \times 10{\cdot}7 \times 0{\cdot}7986.$$

[$12 \times 11 \times 0{\cdot}8 = 105$ roughly.]

$$\text{Log (area)} = \log 12{\cdot}25 + \log 10{\cdot}7 + \log 0{\cdot}7986$$
$$= 1{\cdot}0882 + 1{\cdot}0294 + \bar{1}{\cdot}9023 = 2{\cdot}0199,$$
$$\text{and Area} = \text{Antilog } 2{\cdot}0199 = 104{\cdot}7 \text{ square inches.}$$

131. *Problems involving the use of Trigonometrical Ratios.*

In these problems the terms "Angle of *Elevation*," "Angle of *Depression*," are often used.

Suppose O and A to be two points (Fig. 107), A being at a higher level than O.

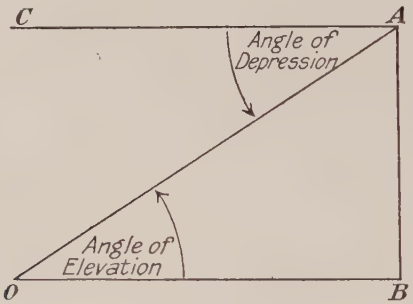


Fig. 107.

Let OB be a horizontal line drawn through O to meet the vertical line AB drawn through A at the point B.

The angle BOA is called the *Angle of Elevation* of the point A as seen from O.

Draw AC parallel to BO, so that AC is the horizontal line passing through A. Then the angle CAO is the *Angle of Depression* of the point O as seen from A.

Example 1.

Measurements are taken to determine the air space of a rectangular room in a warehouse. Length 60 feet, breadth 36 feet; inclination to the floor of the diagonal of the side wall 17°. Calculate the air space.

Here we must first determine the height of the room.
If $ABCD$ (Fig. 108 not drawn to scale) represents the side all, so that AB represents 60 feet, AC represents the diagonal, C represents the height. Then angle CAB is 17°.

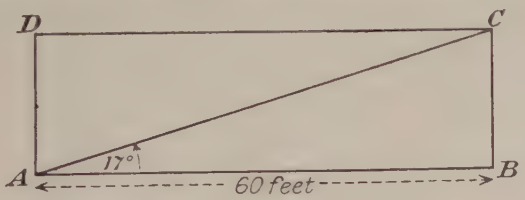


Fig. 108.

Since ABC is a right-angled triangle,

$$\frac{CB}{BA} = \text{Tan } 17°$$

$$\text{or } \frac{CB}{60} = 0{\cdot}3057.$$

Multiply by 60 and

$$CB = 0{\cdot}3057 \times 60 = 18{\cdot}342 \text{ feet,}$$

nd height of room = 18 feet to nearest foot.
Then air space = $60 \times 36 \times 18 = 38{,}880$ cubic feet.

Example 2.

To calculate the height of a chimney a man walks to a oint at the same level as the base of the chimney and 100 feet

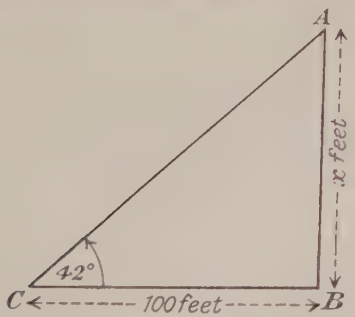


Fig. 109.

stant from the base. The angle of elevation of the top of the imney he finds to be 42°. What is the height of the imney ?

If AB (Fig. 109) represents the height of chimney, say x feet, nd BC represents 100 feet measured at right angles to BA

along the ground, angle ACB (42°) is the angle of elevation of A.

Then $$\frac{AB}{BC} = \text{Tan } 42°$$

or $$\frac{x}{100} = 0{\cdot}9004$$

or $$x = 0{\cdot}9004 \times 100 = 90{\cdot}04.$$

Then height of chimney = 90 feet to nearest foot.

EXAMPLE 3.

A ladder 20 feet long leans against a wall and makes with it an angle of 20°. Find the height of the top of the ladder above the ground, and also the distance of the foot of the ladder from the wall.

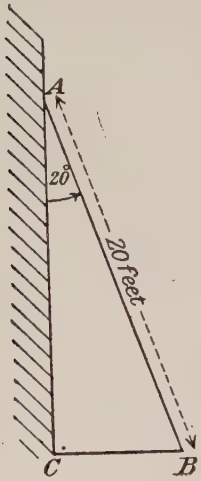


FIG. 110.

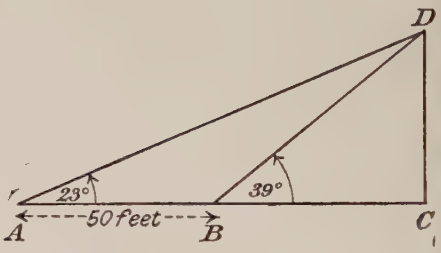


FIG. 111.

In the figure (110)

AB represents the ladder-length 20 feet.
AC ,, ,, wall.
CB ,, ,, ground.
Angle $BAC = 20°$.

Then (1) $$\frac{AC}{AB} = \text{Cos } 20° \text{ or } \frac{AC}{20} = 0{\cdot}9397$$

or $$AC = 0{\cdot}9397 \times 20 = 18{\cdot}794 \text{ feet.}$$

∴ Top of ladder is 18 feet 9 inches above the ground.

(2) $$\frac{BC}{BA} = \text{Sin } 20° \text{ or } \frac{BC}{20} = 0{\cdot}3420.$$

∴ $$BC = 0{\cdot}3420 \times 20 = 6{\cdot}840 \text{ feet.}$$

∴ Foot of ladder is 6 feet 10 inches from the wall.

Example 4.

A man observes the elevation of a building to be 23°. He walks 50 feet nearer and finds the elevation to be 39°. Find the height of the building.

CD (Fig. 111) represents height of building, A and B represent the points on the ground from which observations are made.

Angle $CAD = 23°$.
Angle $CBD = 39°$.
AB represents 50 feet.

In the right-angled triangle DAC,

$$\frac{AC}{DC} = \text{Cot } 23°$$

$$\text{or } AC = 2{\cdot}3559 \times DC.$$

In the triangle BCD,

$$\frac{BC}{DC} = \text{Cot } 39°$$

$$\text{or } BC = 1{\cdot}2349 \times DC.$$

But $$AC - BC = AB$$

$$\text{or } (2{\cdot}3559 \times DC) - (1{\cdot}2349 \times DC) = 50$$
$$\text{or } 1{\cdot}1210 \times DC = 50.$$

$$\therefore \quad DC = \frac{50}{1{\cdot}1210} = 44{\cdot}60 \text{ (by logs.)}.$$

$\therefore$ Height of building = 44·6 feet.

Example 5.

A plot of ground is in the form of a parallelogram. Its two adjacent sides measure 40 feet and 30 feet and the angle included by these sides is 35°. Find its area.

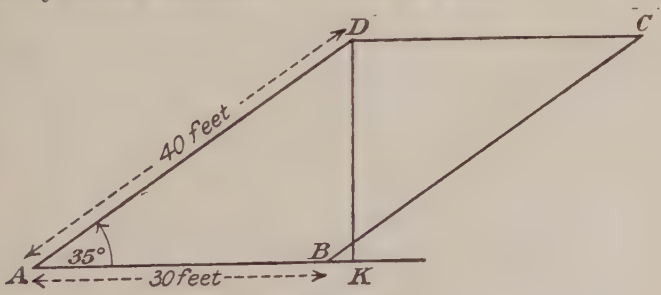


Fig. 112.

In the figure (Fig. 112)

AB represents 30 feet.

AD represents 40 feet.

Angle $BAD = 35°$.

(1) To calculate the altitude DK.

In the right-angled triangle DAK,

$$\frac{DK}{DA} = \text{Sin } 35°$$

$$\text{or } \frac{DK}{40} = 0{\cdot}5736$$

$$\text{or } DK = 0{\cdot}5736 \times 40$$
$$= 22{\cdot}944 \text{ feet.}$$

(2) Area of plot
$= AB \times DK$
$= 30 \times 22{\cdot}944$ square feet
$= 688$ square feet to nearest square foot.

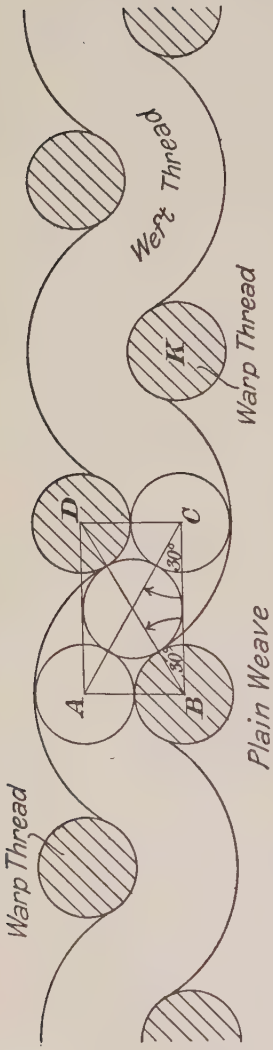


FIG. 113.

EXAMPLE 6.

Fig. 113 represents the cross section of a piece of cloth at right angles to the warp. Suppose each weft thread bends round the warp threads with the same curvature as that of a warp thread bending round the weft threads. Suppose the diameter of each warp and weft thread to be $\frac{1}{60}$ inch and that the straight line drawn from the centre of the weft thread in its lowest position to the centre of the thread in its highest position makes an angle of 30° with the horizontal. Since there is the same bending in the warp threads, the average slope of these will be 30°.

In the figure Angle $ACB = 30$ and Angle $CBD = 30°$.

Since diameters of warp and weft threads are the same ($\frac{1}{60}$ inch), BI represents $\frac{1}{30}$ inch (1 weft diameter + 1 warp diameter). If there were no bending in the threads, then one

rp thread together with one intersection of the weft would cupy $\frac{1}{30}$ inch in the woven piece.

But the figure shows that the distance measured across the th occupied by them is represented by BC.

Now $$\frac{BC}{BD} = \text{Cos } 30° = \frac{\sqrt{3}}{2}$$

$$\text{or } BC = \frac{\sqrt{3}}{2} \times \tfrac{1}{30} \text{ inch}$$

$$= 1{\cdot}732 \times \tfrac{1}{60} \text{ inch } (\sqrt{3} = 1{\cdot}732).$$

Now in a plain weave every repeat of the pattern (from B K in figure) contains 2 warp threads and 2 intersections the weft thread.

Then 2 warp threads and 2 intersections occupy $2 \times 1{\cdot}732 \times \frac{1}{60}$ ch.

Hence in "setting" the warp,

In $(2 \times 1{\cdot}732 \times \frac{1}{60})$ inch we must have 2 warp threads.

In 1 inch we must have $\dfrac{2}{2 \times 1{\cdot}732 \times \frac{1}{60}}$ warp threads.

$$\therefore \quad \textit{Ends per inch} = \frac{2 \times 60}{2 \times 1{\cdot}732} = 34{\cdot}64.$$

nd assuming the warp bends equally with the weft we must ave 34·64 *picks per inch.*

Example 7.

Suppose the two sets of threads in the last example undergo change so that they shall form *unequal* angles.

Suppose angle CBD formed by the warp has increased to 5°.

Then $$\frac{BC}{BD} = \text{Cos } 45° = \frac{1}{\sqrt{2}} = 0{\cdot}7071$$

r $BC = 0{\cdot}7071 \times \frac{1}{30}$ inch = { space in woven cloth taken up by 1 warp thread and 1 intersection of weft thread.

Supposing a plain weave again :—every repeat of the pattern contains 2 warp threads and 2 intersections. Then

2 warp threads + 2 intersections occupy $(2 \times 0{\cdot}7071 \times \frac{1}{30})$ inch

or

In $(2 \times 0{\cdot}7071 \times \frac{1}{30})$ inch we must have 2 warp threads,

In 1 inch we must have $\dfrac{2}{2 \times 0{\cdot}7071 \times \frac{1}{30}}$ warp threads,

or Sett required $= \dfrac{2}{2 \times 0{\cdot}7071 \times \frac{1}{30}}$

$= \dfrac{30}{0{\cdot}7071} = 42{\cdot}42$ ends per inch.

Note.—We shall refer to these last examples when dealing with the construction of woven fabrics (Chapter XIII).

CHAPTER X.

Textile Calculations involving Percentages and Proportional Division.

132.—Weight and Cost of Mixed Yarns. 133.—To find the Value of Mixture of Materials when the Quantity and Value of each Material known. 134.—To mix two or more Yarns together in order to produce Mixture of a Given Value. 135.—Conditioning—Moisture in Textile aterials. 136.—Weight and Cost of Woven Fabrics.

2. *Weight and Cost of Mixed Yarns.*

Wools of different qualities and values are frequently mixed produce yarns used in the construction of cloth. Sometimes different materials are mixed, such as cotton and wool; ungo, cotton and wool; flocks and wool, etc. The principal ject to be attained is that of reducing the cost of the yarn d hence of the woven fabric. There are cases, as in the ixture of silk with wool, where the object is to produce a harter and a more stylish thread and thus improve the ppearance of the cloth produced.

Blending also gives a combination of several colours or shades the same or different materials. A thread of this kind is luable to the designer in the production of new styles.

3. *To determine the value of a mixture of materials when e quantity and value of each material is known.*

Example 1.

Find the average cost per lb. of the following blend :—

110 lbs.	white Botany noils at	$15\frac{1}{2}d.$ per lb.
35 ,,	new Shetland drab at	$16d.$ per lb.
60 ,,	brown stockings at	$3\frac{3}{4}d.$ per lb.
33 ,,	new drab mixture serges at	$5\frac{1}{4}d.$ per lb.
10 ,,	dyed red stockings at	$4d.$ per lb.
44 ,,	old drab serges at	$4d.$ per lb.
48 ,,	best white flannels at	$8d.$ per lb.
100 ,,	cotton at	$4d.$ per lb.

Cost of 110 lbs. of white Botany noils at $15\frac{1}{2}d. = 15\frac{1}{2} \times 110 =$ 1705 pence.

Cost of 35 lbs. of new Shetland drab at $16d. = 16 \times 35 =$ 560 pence.

Cost of 60 lbs. of brown stockings at $3\frac{3}{4}d. = 3\frac{3}{4} \times 60 =$ 225 pence.

Cost of 33 lbs. of mixture serges at $5\frac{1}{4}d. = 5\frac{1}{4} \times 33 = 173\frac{1}{4}$ pence.

Cost of 10 lbs. of dyed red stockings at $4d. = 4 \times 10 = 40$ pence.

Cost of 44 lbs. of old drab serges at $4d. = 4 \times 44 = 176$ pence.

Cost of 48 lbs. of white flannels at $8d. = 8 \times 48 = 384$ pence.

Cost of 100 lbs. cotton at $4d. = 4 \times 100 = 400$ pence.

Then cost of 440 lbs. of materials mixed $= 3663\frac{1}{4}$ pence
$= 3663{\cdot}25$ pence,

and cost of 1 lb. of the blend $= \frac{3663{\cdot}25}{440} = 8{\cdot}3d.$ to nearest $\frac{1}{10}$.

Example 2.

Calculate the cost per lb. of a blend made up of the following wools :—

(*a*) 20 per cent. of a clean wool costing 8*d*. per lb. in the grease and losing 36 per cent. in washing ; (*b*) 50 per cent. of clean wool costing 14*d*. per lb. in the grease and losing 30 per cent. in the washing ; (*c*) 30 per cent. of wool at 15*d*. per lb.

If we consider 100 lbs. of the blend we shall have 20 lbs. of the first wool ; 50 lbs. of the second wool ; 30 lbs. of the third wool, all being weights of clean wool.

First of all find what weights of (*a*) and (*b*) were bought in the greasy state.

Remember the loss is reckoned as a percentage of the greasy weight.

(*a*) Suppose the weight of the first wool in the grease $= x$ lbs.

The loss $= 36\%$ of x lbs. $= \frac{36x}{100}$ lbs.

and the clean weight $= x - \frac{36x}{100} = \frac{64x}{100}$ lbs.

Then $\frac{64x}{100} = 20.$

Divide both sides of the equation by $\frac{64}{100}$ and

$$x = \frac{20 \times 100}{64}$$

nd the greasy weight $= \frac{20 \times 100}{64}$ lbs. [Don't work this out.]

The cost in the grease = 8*d*. per lb.

Then cost of first wool (20 lbs. when cleaned) $= \frac{20 \times 100 \times 8}{64} = 250$ pence.

(*b*) Now deal with the second wool in a similar manner.
If weight in the grease = y lbs.,

oss in weight = 30% of y lbs. $= \frac{3y}{10}$ lbs.

nd the weight when cleaned $= y - \frac{3y}{10} = \frac{7y}{10}$ lbs.

Hence $\frac{7y}{10} = 50$.

Divide both sides by $\frac{7}{10}$ and $y = \frac{50 \times 10}{7}$.

nd the greasy weight $= \frac{50 \times 10}{7}$ lbs.

The cost in the grease = 14*d*. per lb.

The cost of second wool (50 lbs. when cleaned) $= 50 \times 10 \times 2 = 1000$ pence.

Also cost of 30 lbs. of third wool at 15*d*. per lb. $= 15 \times 30$
$= 450$ pence.

Then Cost of 100 lbs. of Blend $= 250 + 1000 + 450$
$= 1700$ pence,

nd Cost of Blend per lb. $= \frac{1700}{100} = 17d. = 1s.\ 5d.$

Example 3.

(*a*) Calculate the cost of clean wool when the greasy wool ost $6\frac{1}{4}d$. per lb. and loses 35 per cent. in washing.

(*b*) Also calculate the cost of clean wool in a blend, made p (in the grease) as follows :—

75% New Zealand Crossbred at 9*d*. per lb.
15% Port Phillip ,, at 10*d*. ,, ,,
10% New Zealand Slipe at 8*d*. ,, ,,

ne blend losing 28% in washing

(*a*) If we take 100 lbs. of greasy wool, since the loss in washng is 35%, we shall obtain 65 lbs. of clean wool.

Now Cost of 100 lbs. of greasy wool $= 6\frac{1}{4} \times 100$
$= 625$ pence
and Value of 65 lbs. of clean wool $= 625$,,
,, ,, 1 lb ,, ,, $= \frac{625}{65}$,,

$\therefore$ Cost of clean wool $= \frac{625}{65} = 9\frac{3}{4}d.$ per lb. (practically).

(*b*) Suppose the blend weighs 100 lbs. in the grease. Then
Cost of 75 lbs. New Zealand Crossbred at $9d.$ $= 75 \times 9$
$= 675$ pence
Cost of 15 lbs. Port Phillip Crossbred at $10d.$ $= 150$,,
Cost of 10 lbs. New Zealand Slipe at $8d.$ $= 80$,,
and
Cost of 100 lbs. of Blend (in the grease) $= 905$,,
But in washing the loss in weight is 28 lbs. in every 100 lbs.
Then weight of blend after washing $= 72$ lbs.
$\therefore$ Cost of 72 lbs. of clean wool $= 905$ pence
and ,, ,, 1 lb. ,, ,, ,, $= \frac{905}{72} = 12{\cdot}57$ pence,

or cost of clean wool = 1*s.* 1*d.* per lb. (practically).

Example 4.

A blend is made up of two wools, one of which costs $12d.$ per lb. in the grease and loses 40 per cent. in washing, and the other loses 30 per cent. in washing. They are blended together (clean) in the proportion of three parts of the former to one of the latter and the scoured value of the blend is 21 pence per lb. What is the original cost in the grease of the second wool? Allow $1d.$ per lb. for the cost of washing.

Suppose we mix 30 lbs. of the first kind with 10 lbs. of the second. Then

(*a*) Value of 40 lbs. of clean wool $= 20 \times 40$
$= 800$ pence.

[*Note.*—Allowing $1d.$ per lb. for washing, value of blend is $20d.$ per lb.]

Now we must find the weights of the wools in the grease.

(*b*) First kind loses 40% of its greasy weight in washing.
Then 60 lbs. clean weighed 100 lbs. in the grease,
or 30 ,, ,, ,, 50 lbs. ,, ,, ,,
$\therefore$ Cost of 30 lbs. clean of first wool $= 50 \times 12 = 600$ pence.

(*c*) Second kind of wool loses 30% of its greasy weight in washing.

Then 70 lbs. clean weighed 100 lbs. in the grease.

$\therefore$ 10 ,, ,, ,, $\frac{100}{7}$,, ,, ,, ,,

If its original cost in the grease was x pence per lb.,

Cost of 10 lbs. clean of second wool = $\frac{100x}{7}$ pence.

Combining (b) and (c),

Cost of 40 lbs. of blend = $(600 + \frac{100x}{7})$ pence.

But from (a) this is 800 pence.

Hence $600 + \frac{100x}{7} = 800$.

Subtract 600 from both sides and

$$\frac{100x}{7} = 200.$$

Divide both sides by $\frac{100}{7}$ and

$$x = \frac{200 \times 7}{100} = 14.$$

$\therefore$ The original cost of the wool = 1*s.* 2*d.* per lb. (in the greasy state).

4. We will next consider *how to mix two or more yarns together in order to produce a mixture of given value.*

Example 1.

(*a*) A mixture consists of two wools; the value of one is 3*d.* per lb. and of the other 1*s.* 10*d.* per lb. In what proportion are the two kinds mixed if 1 lb. of the mixture is worth 1*s.* 6*d.*?

(*b*) What quantity of each will be required to produce a mixture of 100 lbs.?

(*a*) *Method* 1.

Suppose x lbs. of the first kind are mixed with y lbs. of the second.

The value of x lbs. of the first at 1*s.* 3*d.* per lb. = $15x$ pence.

,, ,, y ,, ,, ,, second at 1*s.* 10*d* ,, = $22y$ pence.

$\therefore$ Value of $(x + y)$ lbs. of mixture = $(15x + 22y)$ pence (A).

But the mixture is worth 1*s.* 6*d.* per lb.

$\therefore$ Value of $(x + y)$ lbs. of mixture = $18(x + y)$ pence (B),

and these two values (A) and (B) must be the same.

$\therefore$ $18(x + y) = 15x + 22y$

or $18x + 18y = 15x + 22y$.

Subtracting $15x$ and $18y$ from both sides of the equation we have $3x = 4y$.

Dividing both sides by $3y$ we have

$$\frac{3x}{3y} = \frac{4y}{3y} \text{ or } \frac{x}{y} = \frac{4}{3}.$$

Then the mixture is in the proportion of 4 lbs. of the first kind to 3 lbs. of the second kind.

[Check your result thus :—

Value of 4 lbs. at 1*s.* 3*d.* per lb.	=	60	pence
,, 3 lbs. at 1*s.* 10*d.* ,,	=	66	,,
,, 7 lbs. of mixture	=	126	,,
∴ ,, 1 lb. ,, ,,	=	18	,,]

(*a*) *Method* 2.

The value of every lb. of the mixture is 1*s.* 6*d.*, so that

On every lb. of the first kind in the mixture there is a *gain* of 3 pence.

On every lb. of the second kind in the mixture there is a *loss* of 4 pence.

Then *to make gains balance losses*, 4 lbs. of the first must be mixed with 3 lbs. of the second kind.

(*b*) Now in every 7 lbs. of the mixture we have 4 lbs. of the first kind and 3 lbs. of the second.

Since the total weight of mixture = 100 lbs.

Weight of first kind $= \frac{4}{7} \times 100 = 57\frac{1}{7}$ lbs.

Weight of second kind $= \frac{3}{7} \times 100 = 42\frac{6}{7}$ lbs.

Example 2.

Materials valued at 1*s.* 8*d.*, 1*s.* 4*d.*, and 10*d.* per lb. are blended and the mixture is worth 1*s.* per lb.

In what proportions may they be mixed and what quantities of each material will be required to make 240 lbs. of blend

We will use Method 2 of Example 1.

The value of every lb. of the mixture is 1*s.*, so that

On each lb. of the 1*s.* 8*d.* material in the blend there is a *loss* of 8*d.*

On each lb. of the 1*s.* 4*d.* material in the blend there is a *loss* of 4*d.*

On each lb. of the 10*d.* material in the blend there is a *gain* of 2*d.*

Then *to make gains balance losses* we may mix 1 lb. of the first with 1 lb. of the second and 6 lbs. of the third kind.

Note.—We may blend them in other proportions ; thus,

2 lbs. of 1*s.* 8*d.* material		3 lbs. of 1*s.* 8*d.* material	
3 lbs. of 1*s.* 4*d.* ,,	} : {	4 lbs. of 1*s.* 4*d.* ,,	}
14 lbs. of 10*d.* ,,		20 lbs. of 10*d.* ,,	

will give blends of different qualities but of the same value per lb.

Taking the first blend, it is seen that in every 8 lbs. of the blend there are 1 lb. of the first material
1 lb. ,, second ,,
6 lbs. ,, third ,,

Since the total weight of the blend = 240 lbs.,
Weight of first material required = $\frac{1}{8} \times 240 = 30$ lbs.
Weight of second material = $\frac{1}{8} \times 240 = 30$ lbs.
Weight of third material = $\frac{6}{8} \times 240 = 180$ lbs.

The student is left to calculate the weights of each material required if we choose the other blends shown above.

Example 3.

If we have a quantity of wool at 1*s.* 1½*d.* per lb., how much of it shall we require to use to make a blend at 9½*d.* per lb. if we have to mix it with 380 lbs. of cotton at 3½*d.* per lb. ?

As in Examples 1 and 2,
On each lb. of wool in the blend there is a *loss* of 4 pence.
On each lb. of cotton in the blend there is a *gain* of 6 pence.
To *make gains balance losses* we must mix 3 lbs. of wool with 2 lbs. of cotton.
Hence in every 5 lbs. of the blend we shall have
3 lbs. of wool and 2 lbs. of cotton,
or $\frac{3}{5}$ of the total weight of the blend is wool,
$\frac{2}{5}$,, ,, ,, ,, ,, ,, is cotton.
But the weight of cotton = 380 lbs.
∴ $\frac{2}{5}$ of the total weight = 380 lbs.
$\frac{1}{5}$,, ,, ,, = 190 lbs.
and $\frac{3}{5}$,, ,, ,, = 570 lbs.
∴ Weight of wool required = 570 lbs.

Example 4.

A parcel of wool at 8*d.* per lb. loses 50% in scouring ; another lot at 1*s.* per lb. loses 20%. How much of each will be required to mix with clean wool at 2*s.* per lb. to make a blend of 350 lbs. worth 1*s.* 6½*d.* scoured, allowing ½*d.* per lb. (clean weight) for scouring on the whole blend.

(*a*) Since ½*d.* per lb. is allowed for scouring on the whole blend, we must make the blend worth 1*s.* 6*d.* per lb.

(*b*) The wool at 8*d*. per lb. loses 50% or half its weight in scouring. Then 1 lb. in the grease yields $\frac{1}{2}$ lb. when scoured.

$\therefore$ $\frac{1}{2}$ lb. (clean weight) cost 8*d*.,

or 1 lb. ,, ,, ,, 1*s*. 4*d*.

(*c*) The wool at 1*s*. per lb. loses 20% or $\frac{1}{5}$ its weight. Then 1 lb. in the grease yields $\frac{4}{5}$ lb. when scoured, and

$\frac{4}{5}$ lb. (clean weight) cost 12 pence.

$\therefore$ 1 lb. ,, ,, ,, $\frac{12 \times 5}{4}$ or 15 pence.

(*d*) The third wool costs 2*s*. per lb. clean weight. Then

On each lb. of the first wool in the blend there is a *gain* of 2 pence.

On each lb. of the second wool in the blend there is a *gain* of 3 pence.

On each lb. of the third wool in the blend there is a *loss* of 6 pence.

To *make gains balance losses* we must mix

$\left\{\begin{matrix}3 \text{ lbs. of wool at } 8d. \\ 2 \text{ lbs. ,, ,, ,, } 1s. \\ 2 \text{ lbs. ,, ,, ,, } 2s.\end{matrix}\right\}$ or $\left\{\begin{matrix}6 \text{ lbs. of wool at } 8d. \\ 2 \text{ lbs. ,, ,, ,, } 1s. \\ 3 \text{ lbs. ,, ,, ,, } 2s.\end{matrix}\right\}$, etc.

Taking the first blend

In every 7 lbs. of the blend we shall have

3 lbs. of wool at 8*d*. per lb.
2 lbs. ,, ,, ,, 1*s*. ,,
2 lbs. ,, ,, ,, 2*s*. ,,

Hence in 350 lbs. of the blend :—

Weight of first wool $= \frac{3}{7} \times 350 = 150$ lbs.
,, ,, second wool $= \frac{2}{7} \times 350 = 100$ lbs.
,, ,, third wool $= \frac{2}{7} \times 350 = 100$ lbs

135. *Conditioning—Moisture in Textile Materials.*

Wool has a tendency to absorb moisture from the air, and it is an interesting fact that a wool top or skein varies in weight during the day. It will be heaviest at sunrise and lightest about 3 or 4 p.m. Exposed to dry air it loses moisture. Other fibres, such as cotton, flax, or silk, behave in a similar manner in varying degrees. They are said to be "hygroscopic." This varying weight in these materials is a source of inconvenience and trouble to dealers, spinners, and manufacturers. As a protection against vendors of tops, etc., damping their wares to an abnormal degree and thus selling water at the price of wool, etc., "*Conditioning Houses*" have been established in the leading centres of the textile industry. When testing, all lots received are weighed. Afterwards samples are taken from the top, centre, and bottom of the sheets (if wool) or bags

if tops). These are immediately weighed and brought to bsolute dryness in an oven which may be kept at a constant emperature a little above that of boiling water (212° F. or 00° C). The dried samples are then weighed, and from this he dry weight of the "lot" is found by proportion. To this s added the "Standard Regain" amounts for moisture, and this ives the "invoice weight" of the "lot," *i.e.*, the weight on hich the charge must be made. The certificate issued by the onditioning House is then accepted by all parties concerned.

The following table gives the standard allowances for moisture or Standard Regains" adopted by the Bradford Conditioning House.

Wool and waste	—Amount of regain	= 16	%	of absolute dry weight.		
Tops, oil combed	,, ,,	= 19	%	,,	,,	,,
Tops, dry combed	,, ,,	= 18¼	%	,,	,,	,,
Noils, ordinary	,, ,,	= 14	%	,,	,,	,,
Clean noils (scoured or carbonised)	,, ,,	= 16	%	,,	,,	,,
Worsted yarn	,, ,,	= 18¼	%	,,	,,	,,
Cloths, worsted or woollen	,, ,,	= 16	%	,,	,,	,,
Silk	,, ,,	= 11	%	,,	,,	,,
Cotton	,, ,,	= 8½	%	,,	,,	,,

Example 1.

Suppose 2 lbs. of combed top, in oil, when tested in the conitioning oven, lost 6 ozs. 2 drams in weight, this being due to he evaporation of the moisture, find (1) the direct loss per ent., (2) the absolute dry weight per cent., (3) the percentage egain to obtain the original weight, (4) the excess of moisture er cent. above that of the standard allowance of 19 per cent.

(1) Original weight = 2 lbs. = 512 drams.
Loss in weight = 6 ozs. 2 drams = 98 drams.

Then

Loss in weight on 512 drams = 98 drams.

,, ,, ,, ,, 1 dram $= \frac{98}{512}$

,, ,, ,, ,, 100 drams $= \frac{98 \times 100}{512}$

or *Direct Loss* $= \frac{9800}{512} = 19{\cdot}14\%$ of the original weight.

(2) Absolute dry weight = 512 − 98 = 414 drams.
If the loss in weight = 19·14% of the original weight,
Then the *Absolute dry weight* = (100 − 19·14)
= 80·86% of the original weight.

(3) Since the absolute dry weight = 414 drams, we must add 98 drams of moisture to obtain the original weight.

This regain must be expressed as a percentage of the absolute dry weight, thus :—

On 414 drams absolute dry weight the regain = 98 drams

On 1 dram ,, ,, ,, ,, ,, $= \frac{98}{414}$,,

On 100 drams ,, ,, ,, ,, ,, $= \frac{98 \times 100}{414}$ drams.

∴ Regain to obtain the original weight

$$= \frac{98 \times 100}{414}$$

$$= 23{\cdot}67\% \text{ of the absolute dry weight.}$$

(4) Now the "Standard Regain" for tops, oil combed, is 19% of the absolute dry weight.

Then 100 drams absolute dry weight should become 119 drams, and

119 drams original weight should give 19 drams of moisture

1 dram ,, ,, ,, ,, $\frac{19}{119}$,, ,, ,,

100 drams ,, ,, ,, ,, $\frac{19 \times 100}{119}$,, ,,

Hence, Direct loss allowed $= \frac{19 \times 100}{119}$

$$= 15{\cdot}97\% \text{ of the original weight.}$$

But the direct loss = 19·14% of the original weight (see (1)).

∴ Excess of moisture = (19·14 − 15·97)

= 3·17% of the original weight.

Note.—The student must remember that the "standard regain" is reckoned as a percentage of the absolute dry weight, and the loss in weight is expressed as a percentage of the weight of the material before drying.

Example 2.

The "Standard Regain" for wool is 16%. What will be the percentage loss in weight on drying wool which just contains the standard amount of moisture ?

Standard regain for wool = 16%.

This means that 100 lbs. absolute dry weight will absorb 16 lbs. of moisture and become 116 lbs., or 116 lbs. of wool which contains the standard amount of moisture will give 100 lbs. when fully dried.

Then 116 lbs. when dried will lose 16 lbs. of moisture.

1 lb. " " " " $\frac{16}{116}$ " " "

100 lbs. " " " " $\frac{16 \times 100}{116}$ lbs. of moisture.

Then Loss in weight $= \frac{16 \times 100}{116}$

$= 13{\cdot}79\%$ of the weight before drying.

If, then, on drying a sample of wool we find the direct loss to be $15{\cdot}79\%$ of the original weight before drying, there will be 2% excess of moisture.

Example 3.

Nine hanks taken from a bundle of worsted yarn weigh $\frac{3}{4}$ lb., and they lose 30 drams of moisture during testing. What is the direct loss per cent.. and how does this compare with the standard regain of moisture for worsted yarn ?

Weight before drying $= \frac{3}{4}$ lb. $= \frac{3 \times 256}{4} = 192$ drams.

Loss in weight $= 30$ drams.

Then $\frac{\text{Loss in weight}}{\text{Original weight}} = \frac{30}{192} = \frac{10}{64}$

$= \frac{1{\cdot}25}{8} = \frac{0{\cdot}15625}{1}$

$= \frac{15{\cdot}625}{100}$

or Direct loss $= 15{\cdot}625\%$ of original weight.

Now the standard regain for worsted yarn $= 18\frac{1}{4}\%$, that is 100 lbs. absolute dry weight absorb 18·25 lbs. of moisture and become 118·25 lbs.

Then on drying, 118·25 lbs. of yarn will lose 18·25 lbs. of moisture,

or $\frac{\text{Loss in weight}}{\text{Original weight}} = \frac{18{\cdot}25}{118{\cdot}25} = \frac{0{\cdot}1544}{1}$

$= \frac{15{\cdot}44}{100}$

or Loss in weight allowed $= 15{\cdot}44\%$ of the weight before drying.

Thus the given sample contains $(15{\cdot}625 - 15{\cdot}44)$ or $0{\cdot}185\%$ excess of moisture, a quantity so small that it can practically be ignored.

EXAMPLE 4.

A warp 70 yards long is ticketed as being made from 2/60's cotton, and 630 ends are taken from it and tested for moisture. When absolutely dry there is a loss of 36 drams. The original weight corresponds with the calculated weight. How does the material agree with the standard allowance of moisture of $8\frac{1}{2}\%$?

Length of the 630 ends $= 70 \times 630$ yards.

2/60's cotton means that (30×840) yards weigh 1 lb.

Then Weight of threads $= \frac{70 \times 630}{30 \times 840}$ lbs.

$$= \frac{70 \times 630 \times 16 \times 16}{30 \times 840} = 448 \text{ drams.}$$

Absolute dry weight $= 448 - 36 = 412$ drams.

$$\frac{\text{Loss in weight}}{\text{Original weight}} = \frac{36}{448} = \frac{9}{112}$$

$$= 0{\cdot}0803$$

$$= \frac{8{\cdot}03}{100}$$

$$= 8{\cdot}03\%.$$

$\therefore$ Direct loss $= 8{\cdot}03\%$ of original weight.

The standard regain for cotton $= 8\frac{1}{2}\%$, *i.e.*, 100 lbs. absolute dry weight becomes $108\frac{1}{2}$ lbs.

Then on drying $108\frac{1}{2}$ lbs. of yarn will lose 8·5 lbs.,

and $$\frac{\text{Loss in weight}}{\text{Original weight}} = \frac{8{\cdot}5}{108{\cdot}5} = \frac{0{\cdot}07834}{1}$$

$$= \frac{7{\cdot}83}{100} = 7{\cdot}83\%.$$

Thus the given sample contains $(8{\cdot}03 - 7{\cdot}83)$ or $0{\cdot}2\%$ excess of moisture.

136. *Weight and Cost of Woven Fabrics.*

To obtain the weight and cost per yard of a cloth, there is no new mathematical principle to learn. The calculation is simply a combination of several types of calculation which have been dealt with in former chapters of this book.

The following is a list of the steps in the calculation.

1. *To find the weight per yard.*

(*a*) Find the weight of warp and weft yarns used in the weaving (§ 22, Chapter II.).

(*b*) In calculating the weight of weft there are two methods use :—

(1) Take the length of the piece out of the loom for calcuting the length of the weft yarn used and then add a perntage (usually 5%) to cover loss due to waste in weaving.

(2) Take the warp length as the length of the piece and let is cover loss due to waste in weaving.

(*c*) From the total weight of warp and weft actually put into e cloth we must deduct a percentage for loss in scouring and ishing (*e.g.*, 10%–16%). This gives the total weight of e cloth.

(*d*) Divide this total weight by the *length of the cloth finished* d express the result in ounces.

2. *To find the cost per yard.*

(*a*) Find the cost of the warp and weft yarn required to roduce the cloth, allowing for waste in weaving.

(*b*) Add cost of weaving—this is generally reckoned per umber of picks in $\frac{1}{4}$, $\frac{1}{2}$, or 1 inch for some fixed length of warp. Thus if the piece is woven with 72 picks per inch and price weaving is 7*d*. per pick per quarter inch for 70 yards of arp (1 cut), then

$$\text{Cost of weaving 1 cut of warp} = 18 \times 7 = 126 \text{ pence} = 10s.\ 6d$$

The rate of payment for weaving is stated 7*d*. per $\frac{1}{4}$ pick r 70 yards of warp.

(*c*) Add other expenses for mending, knotting, burling, ouring, and finishing and dyeing.

(*d*) To the total of (*a*), (*b*) and (*c*) will be added a certain rcentage of that total for commission for travellers' expenses, c.

(*e*) The total cost (the sum of (*a*), (*b*), (*c*), and (*d*)) is divided the *length of the cloth finished* with one yard deducted as lowance for over-measure.

The following examples should be carefully studied :—

Example 1.

Find the cost of a piece made as follows :—

Warp—2/40's worsted at 2*s*. 9*d*. per lb., 64 Sett (Bradford), 2 inches wide in the reed, 70 yards long.

Weft—1/22's worsted at 2*s*. 1*d*. per lb., 62 picks per inch. Base the calculation on the warp length to allow for waste weaving. Weaving 7*s*. Other expenses 3*s*. 9*d*.

(*a*) *To find the weight of the warp.*

64 Sett Bradford means (64×40) ends in a width of 36 inches or $\frac{64 \times 40}{36}$ ends per inch.

Width of piece in reed = 62 inches.

Then Number of warp threads $= \frac{64 \times 40 \times 62}{36}$.

Each thread is 70 yards long.

Hence,

Length of yarn in the warp $= \frac{64 \times 40 \times 62 \times 70}{36}$ yards.

Now 2/40's worsted means that (20×560) yards weigh 1 lb.

$\therefore$ *Weight of warp* $= \frac{64 \times 40 \times 62 \times 70}{36 \times 20 \times 560}$ lbs. = 27 lbs. 9 ozs.

(*b*) *To find the weight of the weft.*

Length of weft in one yard of cloth $\Big\}$ = $\Big\{$ width in reed $\Big\}$ $\times$ $\Big\{$ No. of picks per inch $\Big\}$ [§ 22]

$= 62 \times 62$ yards.

Then Length of weft in whole piece $\Big\}$ $= 62 \times 62 \times 70$ yards,

and in a 1/22's worsted we have (22×560) yards weigh 1 lb.

$\therefore$ *Weight of weft in piece* $= \frac{62 \times 62 \times 70}{22 \times 560}$ lbs. = 21 lbs. $13\frac{1}{2}$ ozs.

(*c*) *To find the cost of the piece.*

		£	*s.*	*d.*	
Cost of warp	$= 2s.\ 9d. \times 27\frac{5}{11}$	3	15	9	to nearest penny
Cost of weft	$= 2s.\ 1d. \times 21\frac{3}{44}$	2	5	6	,, ,,
Cost of weaving		0	7	0	
Other expenses		0	3	9	
$\therefore$ Total cost of piece		£6	12	0	

Example 2.

Calculate the finished weight and cost per yard (56×37) of the following cloth :—

Face warp, 90 ends per inch, 2/48's worsted, at 3*s.* 6*d.* per lb.
Back warp, 45 ,, ,, ,, 2/30's ,, at 2*s.* 10*d.* ,,
Weft, 60 picks per inch, 32 skein, at 2*s.* 2*d.* per lb.

0 yards face warp, 70 yards back warp, to give 56 yards nished; width in loom, 66 inches; cost of manufacturing, e., £5 per piece; allow 5% waste on weft, 5% loss on worsted, nd 16% loss on woollen in finishing.

(*a*) *To find the weight of the warp.*

Width in loom = 66 inches; ends per inch of face warp = 90.
Length of face warp = 70 yards.

∴ Length of yarn in face warp = $90 \times 66 \times 70$ yards.

Since 2 48's worsted means (24×560) yards weigh 1 lb.,

$$\text{Weight of face warp} = \frac{90 \times 66 \times 70}{24 \times 560} \text{ lbs.} = 30 \text{ lbs. } 15 \text{ ozs.}$$

Similarly,

$$\text{Weight of back warp} = \frac{45 \times 66 \times 70}{15 \times 560} \text{ lbs.} = 24 \text{ lbs. } 12 \text{ ozs.}$$

∴ Total weight of warp = 55 lbs. 11 ozs.

(*b*) *To find the weight of the weft.*

Length of weft used for one yard of warp = 66×60 yards.

Length of weft used for 70 yards of warp = $66 \times 60 \times 70$ yards.

32's Yorkshire skeins means (32×256) yards weigh 1 lb.

$$\therefore \text{ Weight of weft required} = \frac{66 \times 60 \times 70}{32 \times 256} = 33{\cdot}84 \text{ lbs.}$$

This is the weight of the weft we must take when finding the ost of the cloth.

(*c*) *To find the weight per yard.*

(1) The warp loses 5% of its weight in finishing and $5\% = \frac{1}{20}$.

∴ Weight of warp (worsted yarn) in finished piece $= \frac{19}{20} \times 55$ lbs. 11 ozs.

$$= \frac{19}{20} \times 891 \text{ ozs.}$$

$$= 846{\cdot}45 \text{ ozs.}$$

(2) The weft loses 5% as waste in weaving.

∴ Weight of weft in piece out of loom $= \frac{19}{20} \times 33{\cdot}84$ lbs.

(3) There is a further loss of 16% of the weft yarn in the finishing.

Now $16\% = \frac{16}{100} = \frac{4}{25}$.

$\therefore$ Weight of weft (woollen yarn) in finished piece $= \frac{21}{25} \times \frac{19}{20} \times 33{\cdot}84$ lbs.

$= \frac{21 \times 19 \times 33{\cdot}84 \times 16}{25 \times 20}$ ozs.

$= 432{\cdot}07$ ozs.

Then Weight of piece finished $= 846{\cdot}45 + 432{\cdot}07$

$= 1278{\cdot}52$ ozs.

Now length finished $= 56$ yards.

$\therefore$ Weight per yard finished $= \frac{1278{\cdot}52}{56} \doteq 22$ to 23 ozs.

(*d*) *To find the cost per yard.*

Cost of face warp	$= 3s.\ 6d.$	$\times\ 30\frac{1}{1}\frac{5}{6}$	$= 108s.\ 3\frac{1}{2}d.$ nearly
Cost of back ,,	$= 2s.\ 10d.$	$\times\ 24\frac{3}{4}$	$= 70s.\ 1\frac{1}{2}d.$
Cost of weft	$= 2s.\ 2d.$	$\times\ 33{\cdot}84$	$= 73s.\ 4d.$ nearly
Cost of manufacturing, etc.			$= 100s.\ 0d.$
		Total cost of piece	$= 351s.\ 9d.$

When a piece is finished it is usually measured 37 inches to the yard. Here we have 56 yards finished (36 inches).

$\therefore$ Cost per yard $(37'') = \frac{351s.\ 9d. \times 37}{56 \times 36}$

$= 6s.\ 5\frac{1}{2}d.$ nearly.

Example 3.

Find the cost per yard of the following cloth :—

Warp—2/32's worsted at 1*s.* 6*d.* per lb., 32 Sett (Bradford), 64 inches wide in the reed, to finish 55 inches ; 50 yards long.

Weft—13 skeins woollen (Yorks) at 1*s.* 2*d.* per lb., 30 picks per inch.

Cost of weaving 4*s.* Dyeing and finishing $4\frac{1}{2}d.$ per lb. grey weight (to find grey weight, reckon the weft at full warp length and reed width deducting 5 per cent.). Other charges 2*s.* 6*d.* Add $7\frac{1}{2}$ per cent. to total cost. Finished length, 42 yards.

(*a*) *To find the weight of the warp.*

32 Sett Bradford means (32×40) ends in a width of 36 inches, or $\frac{32 \times 40}{36}$ ends per inch.

Since width in reed is 64 inches and length is 50 yards, o 2/32's worsted means (16 × 560) yards weigh 1 lb.
Then following the steps of Examples 1 and 2 we have

$$\text{Weight of warp} = \frac{32 \times 40 \times 64 \times 50}{36 \times 16 \times 560} = 12{\cdot}7 \text{ lbs.}$$

(*b*) *To find the weight of weft.*

Here we take the full length of the warp to cover waste of t in weaving.
As shown fully in Examples 1 and 2,

$$\left.\begin{array}{l}\text{Weight of weft required}\\ \text{to weave 50 yards of}\\ \text{warp}\end{array}\right\} = \frac{64 \times 30 \times 50}{13 \times 256} \text{ lbs.}$$
$$= 28{\cdot}85 \text{ lbs.}$$

This is the weight required to find the cost of the weft.]

Now for the grey weight we deduct 5% or $\frac{1}{20}$.

$$\text{en} \quad \left.\begin{array}{l}\text{Weight of yarn in the}\\ \text{cloth out of the loom}\\ (\textit{grey weight})\end{array}\right\} = (12{\cdot}7 - 28{\cdot}85) \times \frac{19}{20}$$
$$= (41{\cdot}55) \times \frac{19}{20} \text{ lbs.}$$
$$= 39{\cdot}48 \text{ lbs.}$$

[This is the weight required to find the cost of dyeing and ishing.]

(*c*) *To find the cost per yard.*

Cost of warp	= 1*s.* 6*d.* × 12·7	= 19*s.*	$0\frac{1}{2}d.$
Cost of weft	= 1*s.* 2*d.* × 28·85	= 33*s.*	8*d.*
Cost of weaving		= 4*s.*	0*d.*
Cost of dyeing and finishing	= $4\frac{1}{2}d.$ × 39·48	= 14*s.*	10*d.*
Other charges		= 2*s.*	6*d.*
∴ Total cost		= 74*s.*	$0\frac{1}{2}d.$
Add $7\frac{1}{2}$% or 3/40		= 5*s.*	$6\frac{1}{2}d.$
Final cost of piece		= 79*s.*	7*d.*

Since the finished length = 42 yards, then

$$\text{Cost per yard (36″)} = \frac{79s.\ 7d.}{42}$$
$$= 1s.\ 11d. \text{ nearly.}$$

Example 4.

Find the cost and weight per yard of the following :—

Warp—

Medium grey	; 2/28's worsted	at 2*s.* 10*d.* per lb.	8 4 – 8 2 2 – 8 6
Dark grey	; ,, ,,	at ,, ,,	8 – 8 8 – 2 2 8 –
Black	; ,, ,,	at 2*s.* 8*d.* ,,	8 – 8 8 – – 8 8 –
Black and white	; 4/84's ,,	at 3*s.* 10*d.* ,,	2 2 – – 4 4 – – –
Bright blue	; 40/2 spun silk	at 15*s.* ,,	– – – – – – – 4 –

Weft—Black, 25 yards per dram at 2*s.* 6*d.* per lb.

Loom particulars—80 ends and 48 picks per inch, 68 inches wide, 60 yards warped, 58 yards woven, 50 yards finished.

Cost of manufacturing—£3 15*s.*

Loss in finishing—5 per cent. for worsted, 16 per cent. for woollen.

(*a*) *To find the number of ends of each kind of warp yarn.*

Since warp is 68 inches wide and 80 ends per inch,

Total ends in warp $= 80 \times 68$.

Number of ends in one repeat of pattern $= 144$.

$$\therefore \text{ Number of repeats} = \frac{80 \times 68}{144}$$

$$= 37 \text{ repeats and } 112 \text{ threads over}$$

These 112 threads are shown bracketed in the warping plan above. In every repeat we have

38 ends of medium grey ; 12 ends of black and white ;
42 ,, ,, dark grey ; 4 ,, ,, spun silk.
48 ,, ,, black ;

Then *Total ends* of

(1) Medium grey $= (37 \times 38) + 32$ (additional threads) $= 1438$

(2) Dark grey $= (37 \times 42) + 36 = 1590$

(3) Black $= (37 \times 48) + 32 = 1808$

(4) Black and white $= (37 \times 12) + 12 = 456$

(5) Spun silk $= 37 \times 4 = 148$.

(*b*) *To find weight of the warp.*

Working step by step as in Examples 1 and 2, the student should check the following results.

(1) Weight of medium grey (worsted) $= \frac{1438 \times 60}{14 \times 560} = 11{\cdot}01$ lbs. (using logs.)

(2) Weight of dark grey (worsted) $= \frac{1590 \times 60}{14 \times 560} = 12{\cdot}17$ lbs.

(3) Weight of black $= \frac{1808 \times 60}{14 \times 560} = 13{\cdot}84$ lbs.

(4) Weight of black and white $= \frac{456 \times 60}{21 \times 560} = 2{\cdot}33$ lbs.

(5) Weight of spun silk $= \frac{148 \times 60}{40 \times 840} = 0{\cdot}26$ lbs.

$\therefore$ Total weight of warp $= 39{\cdot}61$ lbs.

[This is the weight used for finding the cost of the warp.]

(*c*) *To find the weight of the weft.*

Take the woven length 58 yards and add on 5% for waste.

Weight of weft used in weaving $= \frac{68 \times 48 \times 58 \times 21}{25 \times 256 \times 20} = 31{\cdot}06$ lbs.

[This is the weight used for finding the cost of the weft.]

(*d*) *To find the weight per yard finished.*

The warp loses 5% in finishing or $\frac{1}{20}$.

$\therefore$ Weight of worsted in finished cloth $= \frac{1}{20} \times 39{\cdot}61$

$= (39{\cdot}61 - 1{\cdot}98)$

$= 37{\cdot}63$ lbs.

The loss in weight of the weft due to finishing is 16% of the grey weight.

$\therefore$ Weight of woollen in finished cloth $= \frac{68 \times 48 \times 58}{25 \times 256} \times \frac{84}{100}$

$= 24{\cdot}85$ lbs.

Then

Total weight of cloth finished $= 62{\cdot}48$ lbs.

and there are 50 yards finished.

$\therefore$ Weight per yard $= \frac{62{\cdot}48 \times 16}{50} = 20$ ounces.

(*e*) *To find the cost per yard.*

Cost of 2 28's worsted at 2*s.* 10*d.* per lb. $= 2s.\ 10d. \times 23{\cdot}18 = 65s.\ 8d.$

Cost of 2 28's worsted (black) at 2*s.* 8*d.* per lb. $= 2s.\ 8d. \times 13{\cdot}84 = 36s.\ 11d.$

Cost of 4 84's worsted (black) at 3*s.* 10*d.* per lb. $= 3s.\ 10d. \times 2{\cdot}33 = 8s.\ 11d.$

Cost of 40/2's spun silk at 15*s.* per lb. $= 15s. \times 0{\cdot}26 = 3s.\ 11d.$

Cost of weft $= 2s.\ 6d. \times 31{\cdot}06 = 77s.\ 8d.$

Cost of manufacturing $= 75s.$

Total cost $= 268s.\ 1d.$

$\therefore$ Cost per yard (36″) $= \frac{268s.\ 1d.}{50} = 5s.\ 4d.$ nearly.

Example 5.

Four square inches of a finished dress fabric weigh 7·8 grains and count 70 threads and 50 picks per inch.

It was woven 64 inches wide in the reed, and has shrunk to 56 inches, 70 yards of warp giving 65 yards of grey and 61 yards of finished cloth. Find the weight per yard finished, the threads per inch in the reed, and picks per inch in the loom.

We will assume that the 61 yards finished are measured 37 inches to the yard.

(*a*) Now the weight varies directly as the product of the length and the breadth (§ 139).

Then $$\frac{\text{Weight per yard}}{\text{Weight of sample}} = \frac{37 \times 56}{4},$$

or $$\frac{\text{Weight per yard}}{7{\cdot}8} = \frac{37 \times 56}{4},$$

or $$\left.\begin{matrix}\text{Weight per yard} \\ (37'' \times 56'')\end{matrix}\right\} = \frac{37 \times 56 \times 7{\cdot}8}{4} \text{ grains}$$

$$= \frac{37 \times 56 \times 7{\cdot}8 \times 16}{4 \times 7000} \text{ ounces.}$$

$$= 9{\cdot}2 \text{ ounces.}$$

(*b*) *To find the threads per inch.*

There are 70 threads per inch in the finished width, 56 inches.

$\therefore$ Total ends $= 70 \times 56$.

But these were spread over 64 inches in the reed.

$$\therefore \text{ Ends per inch in the reed} = \frac{70 \times 56}{64} = 61\tfrac{1}{4}.$$

(*c*) *To find the picks per inch.*

There are 50 picks per inch in the finished length 61 yards.

$\therefore$ Total picks $= 50 \times 61 \times 36$.

But the length in the loom $= 65$ yards,

$= 65 \times 36$ inches.

$$\therefore \text{ Picks per inch in the loom} = \frac{50 \times 61 \times 36}{65 \times 36}$$

$$= 47.$$

Example 6.

A square inch of woollen double cloth weighs 6 grains; there are 66 ends and picks per inch finished; the weights of warp and weft are equal; take up in warp and weft 20 per cent.; loss in scouring and finishing 15 per cent. Find ends and picks in loom, counts of yarn and weight per yard for 54 inches by 37 inches finished.

(*a*) *To find the weight per yard.*

Now 1 square inch finished weighs 6 grains and in one yard we have (37 × 56) square inches.

$$\therefore \text{ Weight of one yard } (56'' \times 37'') = 6 \times 37 \times 56 \text{ grains}$$

$$= \frac{6 \times 37 \times 56 \times 16}{7000} \text{ ounces}$$

$$= 28{\cdot}4 \text{ ounces.}$$

(*b*) *To find ends per inch in loom.*

Number of ends in finished width of 56 inches = 66 per inch.

∴ Total ends in width of warp = 56 × 66.

Now the take-up in the width = 20% or $\frac{1}{5}$.

Hence if width in loom = x inches, then width finished = $x - \frac{1}{5}x = \frac{4}{5}x$ inches.

$$\therefore \quad \tfrac{4}{5}x = 56,$$

$$\text{or } x = \frac{56 \times 5}{4} = 70.$$

∴ Width in loom = 70 inches.

and Ends per inch in loom = $\frac{56 \times 66}{70} = 52{\cdot}8 = 53$ nearly.

(*c*) *To find picks per inch in loom.*

Since the take-up in the length is 20% or $\frac{1}{5}$,

If we consider 36 inches finished, then

$$\text{Length in loom} = \frac{36 \times 5}{4} = 45 \text{ inches. [See } (b).]$$

In 36 inches finished there are (66 × 36) picks.

$$\therefore \quad \text{Picks per inch in loom} = \frac{66 \times 36}{45} = 52{\cdot}8$$

$$= 53 \text{ nearly.}$$

(*d*) *To find the Counts of the Yarn used.*

$$\text{Weight of one yard } (56'' \times 36'') \text{ finished} = \frac{6 \times 36 \times 56}{7000} \text{ lbs.} \quad [\text{See } (a).]$$

Loss in scouring and finishing = 15% = $\frac{3}{20}$.

If grey weight of 1 yard = x lbs.,

hen finished weight = $x - \frac{3}{20}x$

$$= \tfrac{17}{20}x \text{ lbs.}$$

Hence,

$$\frac{17}{20}x = \frac{6 \times 36 \times 56}{7000}$$

$$\text{or} \quad x = \frac{6 \times 36 \times 56 \times 20}{7000 \times 17} = 2{\cdot}033 \text{ lbs.}$$

But this piece of cloth in the loom will be 70 inches wide and 45 inches long.

Since weight of warp and weft are the same,

then Weight of warp threads $= 1{\cdot}0165$ lbs.

and Length ,, ,, ,, $= 70 \times 53 \times 45$ inches

$$= \frac{70 \times 53 \times 45}{36} \text{ yards.}$$

$$\therefore \text{ Number of yards which weigh 1 lb.} = \frac{70 \times 53 \times 45}{36 \times 1{\cdot}0165}$$

The Yorkshire Skein hank $= 256$ yards.

$$\therefore \text{ Counts of yarn} = \frac{70 \times 53 \times 45}{36 \times 1{\cdot}0165 \times 256}$$

$$= 17{\cdot}82 = 18\text{'s Y. S.}$$

CHAPTER XI.

Variation and its Applications.

137.—Direct Variation. 138.—Inverse Variation. 139.—Joint Variation. 140. —Worked Examples. 140 (*a*).—Finding the Weight per Yard of a Cloth. 141.—Variation represented Graphically. 142. —The Diameters of Threads of the same Material vary inversely as the Square Roots of the Counts. 143.—For the same Material the Number of Diameters per Inch varies directly as the Square Root of the Counts. 144.—Speed and Surface Speed of a Pulley. 145.—For two pulleys connected by a belt—"Speeds vary inversely as the Diameters of the Pulleys." 146. —For two Spur Wheels—"the Speeds vary inversely as the Numbers of Teeth" on the Wheels. 147.—A Train of Wheels. 148. —Change Wheel and Gauge Point. 149.—The Driving and Speeding of Various Parts of the Woollen or Worsted Carding Engine. 150.—The Driving Motions of the Power Loom Calculations. 151.—Calculations on the "Take-up" Mechanism of the Loom. 152. —Speed of the Tappets. 153. —Calculations involved in the Drafting and Spinning of Yarns. 154. —Worked Examples.

137. *Direct Variation.*

(*a*) Consider the statement that "a worsted carding engine produces 25 lbs. of sliver per hour." Then if it continues to produce at the same rate, we shall obtain 50 lbs. in 2 hours; 75 lbs. in 3 hours; 125 lbs. in 5 hours, etc.

Tabulating these results we have :—

Weight of sliver produced . .	25	50	75	100	125	250	$12\frac{1}{2}$	lbs.
Time during which machine works .	1	2	3	4	5	10	$\frac{1}{2}$	hours

If we denote the weights by W_1, W_2, W_3 . . . W_7 lbs. and the corresponding times by T_1, T_2, T_3 . . . T_7, we see

$$\frac{W_1}{W_2} = \frac{25}{50} = \frac{1}{2} = \frac{T_1}{T_2}.$$

$$\frac{W_2}{W_3} = \frac{50}{75} = \frac{2}{3} = \frac{T_2}{T_3}.$$

$$\frac{W_6}{W_7} = \frac{250}{12\frac{1}{2}} = \frac{20}{1} = \frac{10}{\frac{1}{2}} = \frac{T_6}{T_7}.$$

Here we have chosen any two weights, *e.g.* W_2 and W_3, and found the ratio between them, which is 2 : 3, and we see that this is also the ratio between the corresponding times T_2 and T_3. Thus the weight delivered depends on the time the machine runs, and as the time alters the weight delivered changes in the same proportion. We describe this connection between the two quantities by saying that "the weight delivered *varies directly as* the time."

Definition.—"One magnitude is said to *vary directly as* another when the two are so related that the ratio of any two values of the one is equal to the ratio of the corresponding values of the other."

Since $$\frac{W_1}{W_2} = \frac{T_1}{T_2}.$$

Multiplying both ratios by $\frac{W_2}{T_1}$ we have

$$\frac{W_1}{W_2} \times \frac{W_2}{T_1} = \frac{T_1}{T_2} \times \frac{W_2}{T_1} \text{ or } \frac{W_1}{T_1} = \frac{W_2}{T_2} \qquad \text{(i)}$$

Also since $$\frac{W_2}{W_3} = \frac{T_2}{T_3}.$$

Multiplying both ratios by $\frac{W_3}{T_2}$ we have

$$\frac{W_2}{W_3} \times \frac{W_3}{T_2} = \frac{T_2}{T_3} \times \frac{W_3}{T_2} \text{ or } \frac{W_2}{T_2} = \frac{W_3}{T_3} \qquad \text{(ii)}$$

Proceeding in this manner we get

$$\frac{W_1}{T_1} = \frac{W_2}{T_2} = \frac{W_3}{T_3} = \ldots \frac{W_7}{T_7},$$

and thus we find that if W varies directly as T, any two corresponding values of W and T are in a constant ratio, which in this case is 25 : 1.

[The symbol $\propto$ is used for the words "varies as"; thus $W \propto T$ is read "W varies as T."]

Then if $W \propto T$, the ratio $W : T$ is constant, and if we put for this constant value, we have

$$\frac{W}{T} = K \text{ or } W = KT$$ (K is known as the "variation constant").

To find the constant K *in any case, we only need to know one pair of* CORRESPONDING VALUES of W and T.

The value of K in our illustration is $\frac{2.5}{1}$ or 25.

Then $$W = 25\,T \qquad \text{(A)}$$

Hence given any value of T we may calculate the corresponding value of W.

Dividing both sides of (A) by 25,

$$T = \frac{W}{25} \qquad \text{(B)},$$

and from (B) given any value of W we may calculate the corresponding value of T.

(*b*) We have learnt that the length of the circumference of any circle is obtained by multiplying the length of its diameter by $3\frac{1}{7}$ (roughly) or by 3·1416 (for greater accuracy).

Then if C_1, C_2, C_3 be the number of units in the circumferences of various circles, and D_1, D_2, D_3 be the corresponding numbers of units in their diameters, we have

(i) $C_1 = \frac{22}{7} D_1$, (ii) $C_2 = \frac{22}{7} D_2$, (iii) $C_3 = \frac{22}{7} D_3$.

Dividing (i) by (ii), $$\frac{C_1}{C_2} = \frac{\frac{22}{7} D_1}{\frac{22}{7} D_2} = \frac{D_1}{D_2}.$$

Dividing (ii) by (iii), $$\frac{C_2}{C_3} = \frac{\frac{22}{7} D_2}{\frac{22}{7} D_3} = \frac{D_2}{D_3}.$$

Then by definition :—

"The circumference of a circle varies as its diameter" and

$$\frac{C}{D} = \text{a constant}$$
$$= \tfrac{22}{7} \text{ or } 3{\cdot}1416$$
$$= \pi.$$

(*c*) Again, the area of a circle is obtained by multiplying the area of the square on its radius by π.

Then if A_1, A_2, A_3, etc., be the number of units of area of various circles R_1, R_2, R_3, the corresponding units of length in their radii, then

$$A_1 = \pi R_1^2 \text{ (i)} ; \quad A_2 = \pi R_2^2 \text{ (ii)} ; \quad A_3 = \pi R_3^2 \text{ (iii)}.$$

Dividing (i) by (ii), $$\frac{A_1}{A_2} = \frac{\pi R_1^2}{\pi R_2^2} = \frac{R_1^2}{R_2^2}.$$

Dividing (ii) by (iii), $$\frac{A_2}{A_3} = \frac{\pi R_2^2}{\pi R_3^2} = \frac{R_2^2}{R_3^2}.$$

And by definition—

"The area of a circle varies as the square on its radius."

Hence $$\frac{A}{R^2} = \text{a Constant}$$
$$= \pi.$$

138. *Inverse Variation.*

(*a*) Suppose we take equal lengths of worsted yarns of various counts and compare their weights. We will take the worsted hank (560 yards) as our fixed length.

Then in a 1's count the weight of 1 hank = 1 lb.
in a 10's ,, ,, ,, ,, 10 hanks = 1 lb.
or the weight of 1 hank = $\frac{1}{10}$ lb.,
and in a 20's count the weight of 1 hank = $\frac{1}{20}$ lb., etc.

Putting our results in tabular form we have :—

Counts of yarn .	1's	10's	20's	30's	40's	45's	
Weight of 1 hank .	1	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{30}$	$\frac{1}{40}$	$\frac{1}{45}$	lb.

Here we see that as the counts increase the weight of the hank (or other fixed length) decreases in the same proportion. Thus if the counts be doubled from 10's to 20's, the weight is halved from $\frac{1}{10}$ lb. to $\frac{1}{20}$ lb., and if C_1, C_2, C_3, etc., represent the counts of the yarn, whilst W_1, W_2, W_3, etc., represent the corresponding weights in lbs.,

(i) $\frac{W_1}{W_2} = \frac{C_2}{C_1}$ $\left(i.e., \frac{1}{\frac{1}{10}} = \frac{10}{1}\right)$.

(ii) $\frac{W_2}{W_3} = \frac{C_3}{C_2}$ $\left(\frac{\frac{1}{10}}{\frac{1}{20}} = \frac{20}{10}\right)$.

(iii) $\frac{W_3}{W_4} = \frac{C_4}{C_3}$, etc.

From (i), If $\frac{W_1}{W_2} = \frac{C_2}{C_1}$,

Multiply both sides by W_2C_1 and

$$W_1C_1 = W_2C_2.$$

From (ii) and (iii) we obtain in a similar manner

$$W_2C_2 = W_3C_3 \text{ and } W_3C_3 = W_4C_4.$$

Thus (any weight) × (the corresponding count) = a constant = 1 (in this case).

We describe this connection between the varying quantities by saying that *the weight varies* INVERSELY *as the Counts.*

(*b*) Again, if a certain fixed sum of money (£50) is to be expended in buying cloth at varying prices per yard, then the number of yards which can be bought will vary.

Let us make a table :—

Cost per yard . . .	5/–	7/6	10/–	15/–
Number of yards which can be bought for £50 .	200	$133\frac{1}{3}$	100	$66\frac{2}{3}$

From this we see that if the price be increased the length purchased decreases in the same ratio, for if the price be doubled the length is halved, etc. Also

(Any length) × (the corresponding cost per yard) = a constant = £50 (in this case).

Hence we say : "For a fixed sum of money *the length of cloth purchased* VARIES INVERSELY *as the cost per yard.*"

And in general : "One quantity (y) is said to *vary inversely* as another quantity (x) when the product of any value of y and its corresponding value of x is constant."

Expressed in the form of an equation.

y varies inversely as x when $xy = K$ (some constant).

Also y varies inversely as x^2 when $yx^2 =$ a constant.

139. *Joint Variation.*

The cases considered up to this point have been those in which the variation of a quantity depended on the variation of *one* other quantity only.

Now the variation of a quantity may depend upon that of two or more other quantities.

For example :—

(*a*) *The weight of a rectangular piece of cloth of the same material depends upon its length and breadth.*

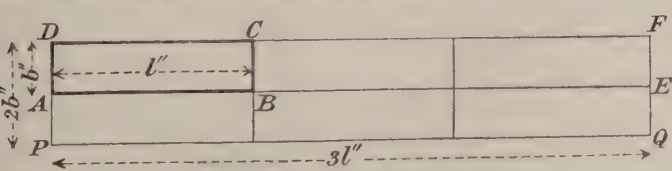


FIG. 113A.

Suppose we start with a strip $ABCD$, l inches long by b inches wide, and that its weight is w grains.

Then we shall have a certain number of warp threads each of length l inches across the width AD, and a certain number of weft threads each b inches long in the length (AB) of the strip.

If now we make the strip three times as long without altering the width ($AEFD$ in figure) it is easy to see that the number of weft threads is three times as great, and that the warp threads are three times as long though their number remains the same.

Thus the total length of yarn (warp and weft) in strip $AEFD$ is three times that in the original strip $ABCD$.

Hence the weight of strip $AEFD$ is three times the weight of the strip $ABCD$.

Again, if we keep the length of the strip $3l$ inches (AE) and double the breadth ($DP = 2b$ inches), we shall double the number of warp threads and double the length of each weft thread, and the total length of yarn in strip $DPQF$ is *six* times the total length in the original strip $ABCD$.

Hence the weight of the strip $DPQF$ is six times the weight of the strip $ABCD$.

If the weight of the strip $DPQF$ be W grains, then

$$\frac{W}{w} = \frac{6}{1}.$$

Also
$$\begin{aligned} &\text{Area of } ABCD &&= l \times b \text{ square inches,} \\ &\text{Area of } DPQF &&= 3l \times 2b = 6\ lb \text{ square inches.} \end{aligned}$$

and
$$\frac{\text{Product of length and breadth of } DPQF}{\text{Product of length and breadth of } ABCD} = \frac{6\ lb}{lb} = \frac{6}{1}.$$

Hence the weight varies as the product of the length and the breadth when they both vary.

(*b*) As a further illustration: The area of a triangle varies as the product of the base and height when both vary.

For if A_1, b_1, h_1, ; A_2, b_2, h_2 represent the respective units in the area, base, and height of any two triangles,

$$\text{(i) } A_1 = \tfrac{1}{2}b_1h_1\text{; (ii) } A_2 = \tfrac{1}{2}b_2h_2.$$

Dividing (i) by (ii),

$$\frac{A_1}{A_2} = \frac{\frac{1}{2}b_1h_1}{\frac{1}{2}b_2h_2} = \frac{b_1h_1}{b_2h_2}.$$

In general we say: "*One quantity is said to vary jointly as a number of others when it varies directly as their product.*"

Thus x varies jointly as y and z when

$$\frac{x}{yz} = K \text{ (some constant).}$$

Again x is said to vary directly as y and inversely as z when

$$\frac{xz}{y} = K \text{ (some constant).}$$

10. *Worked examples.*

EXAMPLE 1.

The extension of a spring varies as the stretching force. spring is stretched 2·5 inches by a weight of 1·3 lbs. How uch is the spring stretched by a weight of 4·3 lbs. ? What eight will stretch it 3·6 inches ?

Suppose the weight $= W$ lbs. and the stretch or extension = inches.

Since $E \propto W$, then

$$\frac{E}{W} = K \text{ (some constant).}$$

Now when $E = 2{\cdot}5$ inches, $W = 1{\cdot}3$ lbs.

$$\therefore \quad K = \frac{2{\cdot}5}{1{\cdot}3}.$$

(*N.B.*—Don't work out at this stage.)

(i) If $W = 4{\cdot}3$ lbs.,

$$\frac{E}{4{\cdot}3} = \frac{2{\cdot}5}{1{\cdot}3}$$

$$\text{or} \quad E = \frac{2{\cdot}5 \times 4{\cdot}3}{1{\cdot}3} = 8{\cdot}27.$$

Then a weight of 4·3 lbs. produces a stretch of 8·27 inches.

(ii) If $E = 3{\cdot}6$ inches.

$$\frac{3{\cdot}6}{W} = \frac{2{\cdot}5}{1{\cdot}3}$$

(Inverting both ratios) $$\frac{W}{3{\cdot}6} = \frac{1{\cdot}3}{2{\cdot}5}$$

$$\text{or} \quad W = \frac{1{\cdot}3 \times 3{\cdot}6}{2{\cdot}5} = 1{\cdot}872.$$

Then weight required to stretch the spring 3·6 inches is ·872 lbs.

EXAMPLE 2.

The volume of a quantity of gas varies inversely as the pressure when the temperature is constant. If the volume is 36·4 cubic inches when the pressure is 14·7 lbs. per square inch,

find the volume when the pressure is 12·9 lbs. per square inch.

Suppose the volume = V cubic inches and the pressure = P lbs. per square inch.

Then, since V varies inversely as P, we have

$$V \times P = K \text{ (some constant).}$$

Now when $V = 36{\cdot}4$, $P = 14{\cdot}7$.

$\therefore \quad 36{\cdot}4 \times 14{\cdot}7 = K.$

(Don't work this out.)

Then if $P = 12{\cdot}9$,

$$V \times 12{\cdot}9 = 36{\cdot}4 \times 14{\cdot}7$$

$$\text{or} \qquad V = \frac{36{\cdot}4 \times 14{\cdot}7}{12{\cdot}9} = 41{\cdot}48.$$

Hence when the pressure is 12·9 lbs. per square inch the volume is 41·48 cubic inches.

Example 3.

The weight of a rectangular piece of cloth varies jointly as its length and breadth. A piece of Scotch tweed 2 inches by 3 inches weighs 33·6 grains. Find the weight per yard of the cloth which is 56 inches wide and contains 37 inches to the yard.

If the weight = W grains, length = L inches, breadth = B inches.

Since W varies jointly as L and B, then

$$\frac{W}{LB} = K \text{ (some constant).}$$

When $W = 33{\cdot}6$, $L = 3$, $B = 2$.

$$\therefore \qquad K = \frac{33{\cdot}6}{3 \times 2}.$$

If $L = 37$ inches and $B = 56$ inches, then

$$\frac{W}{37 \times 56} = \frac{33{\cdot}6}{3 \times 2}$$

$$\text{or} \qquad W = \frac{33{\cdot}6 \times 37 \times 56}{3 \times 2} \text{ grains.}$$

Now *the weight per yard of a cloth is usually given in ounces*. Since there are 7000 grains in 1 lb.,

$$W = \frac{33{\cdot}6 \times 37 \times 56}{3 \times 2 \times 7000} \text{ lbs.} = \frac{33{\cdot}6 \times 37 \times 56 \times 16}{3 \times 2 \times 7000} \text{ ozs.}$$

$$= 26{\cdot}52 = 26\tfrac{1}{2} \text{ ozs.}$$

Example 4.

A fancy woollen fabric weighs $2\frac{1}{2}$ grains per square inch hen finished. If there is a loss in weight in scouring and ishing of 15%, find (*a*) weight per yard of cloth ($56'' \times 37''$) ished, (*b*) weight per yard ($56'' \times 36''$) in the loom.

If the weight $= W$ grains, length $= L$ inches, breadth = inches.

Since W varies jointly as L and B, then

$$\frac{W}{L \times B} = K \text{ (some constant).}$$

(*a*) When $W = 2{\cdot}5$, $L = 1$ and $B = 1$.

$$\therefore \quad K = \frac{2{\cdot}5}{1 \times 1} = 2{\cdot}5.$$

If $L = 37$ and $B = 56$, then

$$\frac{W}{37 \times 56} = 2{\cdot}5,$$

$$W = 2{\cdot}5 \times 37 \times 56 \text{ grains} = \frac{2{\cdot}5 \times 37 \times 56 \times 16}{7000} \text{ ozs.}$$

$$= 11{\cdot}84 \text{ ozs. } \textit{or } 11\tfrac{4}{5} \text{ ozs. practically.}$$

Then weight per yard finished $= 11\frac{4}{5}$ ozs.

(*b*) If the weight of one square inch in the loom $= x$ grains, d the loss in scouring and finishing is 15% or 3/20, then

$$x - \tfrac{3}{20}x = 2{\cdot}5,$$

$$\text{or} \qquad \tfrac{17}{20}x = 2{\cdot}5.$$

Divide by $\frac{17}{20}$ and $x = \dfrac{2{\cdot}5 \times 20}{17} = \dfrac{50}{17}$.

Then weight in the loom $= \frac{50}{17}$ grains per square inch.

Proceeding as in (*a*) we have

(i) $K = \frac{50}{17}$

(ii) $W = \frac{50}{17} \times 36 \times 56$ grains

(since $L = 36$; $B = 56$)

$$= \frac{50 \times 36 \times 56 \times 16}{17 \times 7000} \text{ ozs.}$$

$$= 13{\cdot}55 \text{ ozs. } \textit{or } 13\tfrac{1}{2} \text{ ozs. practically.}$$

Then weight per yard in loom $= 13\frac{1}{2}$ ozs.

) (*a*). *Note* on *Finding the Weight per yard of a Cloth.*

If the student examines the calculations in Examples 3 and) where the finished yard is (56×37 inches) he will notice tain numbers occur in each.

There are $\frac{37 \times 56 \times 16}{7000}$

and these occur whatever the weight and size of the sample taken.

Suppose a 3-inch square of the cloth weighs x grains, and we require the weight in ounces of a yard of cloth ($56'' \times 37''$) with the same notation as before we have

$$\frac{W}{L \times B} = K \text{ (a constant)},$$

when $W = x$, $L = 3$, and $B = 3$.

$$\therefore \quad K = \frac{x}{9}.$$

When $L = 37$, $B = 56$,

$$\therefore \quad \frac{W}{37 \times 56} = \frac{x}{9},$$

$$\text{or} \quad W = \frac{x}{9} \times (37 \times 56) \text{ grains}$$

$$= \frac{x}{9} \times \left(\frac{37 \times 56 \times 16}{7000}\right) \text{ ozs.}$$

Now $\frac{x}{9}$ gives the weight in grains of 1 square inch of the sample, and thus :

$$\left\{\begin{matrix}\text{Weight per yard} \\ \text{in ounces}\end{matrix}\right\} = \left\{\begin{matrix}\text{Weight of 1} \\ \text{square inch} \\ \text{in grains}\end{matrix}\right\} \times \left\{\begin{matrix}37 \times 56 \times 16 \\ 7000\end{matrix}\right\}$$

This *Constant multiplier* $\frac{37 \times 56 \times 16}{7000} = 4{\cdot}736.$

Hence, if one square inch weighs 5·6 grains,

$$\left\{\begin{matrix}\text{Weight of one yard} \\ (56'' \times 37'')\end{matrix}\right\} = 5{\cdot}6 \times 4{\cdot}736 \text{ ozs.}$$

$$= 26{\cdot}52 \text{ ozs.} \quad \text{(compare Ex. 3)}$$

or

If one square inch weighs 2·5 grains,

$$\left\{\begin{matrix}\text{Weight per yard} \\ (56'' \times 37'')\end{matrix}\right\} = 2{\cdot}5 \times 4{\cdot}736.$$

$$= 11{\cdot}84 \text{ ozs. (compare Ex. 4 } (a)).$$

The student should now work the constant multiplier which may be used when the cloth is 56 inches wide and 36 inches to the yard. His result should be 4·608.

Similarly the *constant multiplier* which may be used when the cloth is 54 inches wide and 36 inches per yard is 4·443.

Example 5.

It is found that 8 threads of a certain length of 2/36's cotton eigh the same as 10 threads of the same length extracted om a sample of cotton cloth. What are the counts of the tter ?

Here the weights of the threads are the same whilst the counts .d the lengths alter.

Suppose we fix on a weight of 1 lb. Then

ngth of 1 lb. of 2 36's or 1/18's = 18 hanks (18 × 840 yards).
,, .. 1 lb. of 1/36's = 36 ,, (36 × 840 yards).
,, ,, 1 lb. of 1/9's = 9 ,, (9 × 840 yards).

If the counts (18's, 36's, 9's) are C_1, C_2, C_3, and if the ngths (18 hanks, 36 hanks, 9 hanks) are L_1, L_2, L_3, we see at

$$\frac{C_1}{C_2} = \frac{18}{36} = \frac{L_1}{L_2} \quad \ldots\ldots\ldots \quad (1)$$

$$\frac{C_2}{C_3} = \frac{36}{9} = \frac{L_2}{L_3} \quad \ldots\ldots\ldots \quad (2)$$

d this shows that *the counts vary as the lengths* if the weights e constant.

Hence $\frac{C}{L} = K$ (variation Constant).

hen $C = 18$, $L = 8$, then $K = \frac{18}{8} = \frac{9}{4}$.

hen $L = 10$, $\frac{C}{10} = \frac{9}{4}$ or $C = \frac{45}{2} = 22\frac{1}{2}$.

ien the counts are 2/45's cotton.

1. *Variation represented graphically.*

In § 137, we have said that one quantity y varies directly another quantity x when the ratio of any value of y to s corresponding value of x is a constant.

Expressed in the form of an equation we have

$$\frac{y}{x} = K \text{ (the variation Constant)},$$

or $y = Kx$.

If we draw two lines OX, OY at right angles on squared paper nd plot points whose co-ordinates are corresponding values r x and y, we find the points lie on a straight line, and in hapter VII, § 90, we learnt that $y = Kx$ is the equation of a ne through the origin (O) and the gradient of the line is K (the ariation constant in this case).

As an example :—

"The extension of a spring varies as the load hung on it." In an experiment with a steel spring the following measurements were made.

Load in lbs.	0	1	2	3	4	5	6
Increase in length of spring in inches	0	0·23	0·46	0·69	0·93	1·15	1·37

Plotting these results as in Fig. 114—Loads along OX and Extensions along OY, we see that the points lie very near a

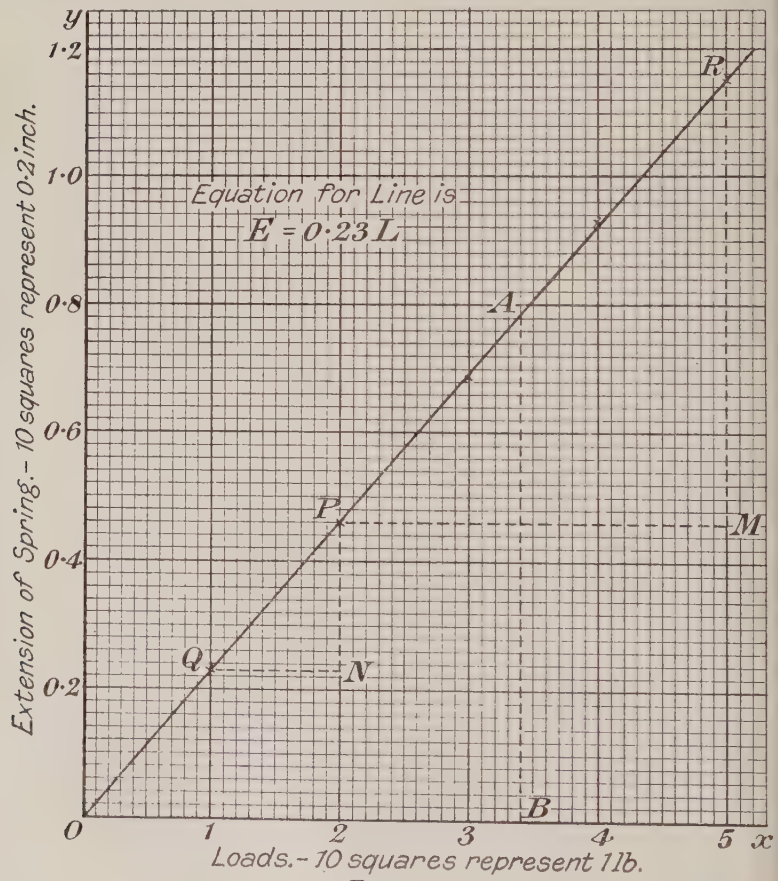


Fig. 114.

straight line passing through O, the deviations being due to inaccuracies in weighing and measuring.

As shown in Chapter VII, § 90,

The gradient of the line is :—

$$(a)\ \frac{PN}{QN} = \frac{0{\cdot}23}{1},$$

$$\text{or } (b)\ \frac{RM}{PM} = \frac{0{\cdot}69}{3} = \frac{0{\cdot}23}{1}.$$

Then if extension $= E$ inches, load $= L$ lbs.,

Equation for line is $E = 0{\cdot}23\,L$,

or, dividing both sides by L, $\frac{E}{L} = 0{\cdot}23$.

Taking any values of E, say 0·46, 1·15 from table and 0·78 (AB) from graph, with the corresponding values of L, say 2, 5 from table and 3·4 (OB) from graph, we have

$$\frac{E}{L} = \frac{0{\cdot}46}{2} = \frac{1{\cdot}15}{5} = \frac{0{\cdot}78}{3{\cdot}4} = 0{\cdot}23 \text{ (the variation constant).}$$

Thus if *one quantity* y *varies directly as another quantity* x and the corresponding values of the two quantities be plotted as the values of extension and load were plotted, then *the points so plotted will lie on a straight line.*

Conversely, *if the points so obtained lie on a straight line, then one quantity varies as the other.*

In some cases it is not easy to see, except by means of the graphical method, if two quantities vary as one another or not.

Example 1.

The following figures give corresponding values for the Diameter (D), Circumference (C), and Area (A) of various circles. They were obtained by measurements made of circular wooden slabs.

Values for D .	6	10	12·7	15	20	cms.
Values for C .	18·85	31·4	39·9	47·12	62·8	cms.
Values for A .	28·27	78·5	126·7	176·7	314·2	sq. cms.

Show that (i) C varies as D,
(ii) A ,, ,, D^2,
(iii) A ,, ,, $(C \times D)$,

and find the average variation constant in each case.

We first calculate values of D^2 and $(C \times D)$.

Values of D^2	36	100	161·29	225	400	sq. cms.
Values of $(C \times D)$	113·1	314	506·7	706·8	1256	sq. cms.

(*a*) *To show that* C *varies as* D.

Plot values for D^2 along the horizontal axis (OX) and corresponding values for C along the vertical axis (OY) as in Fig. 115.

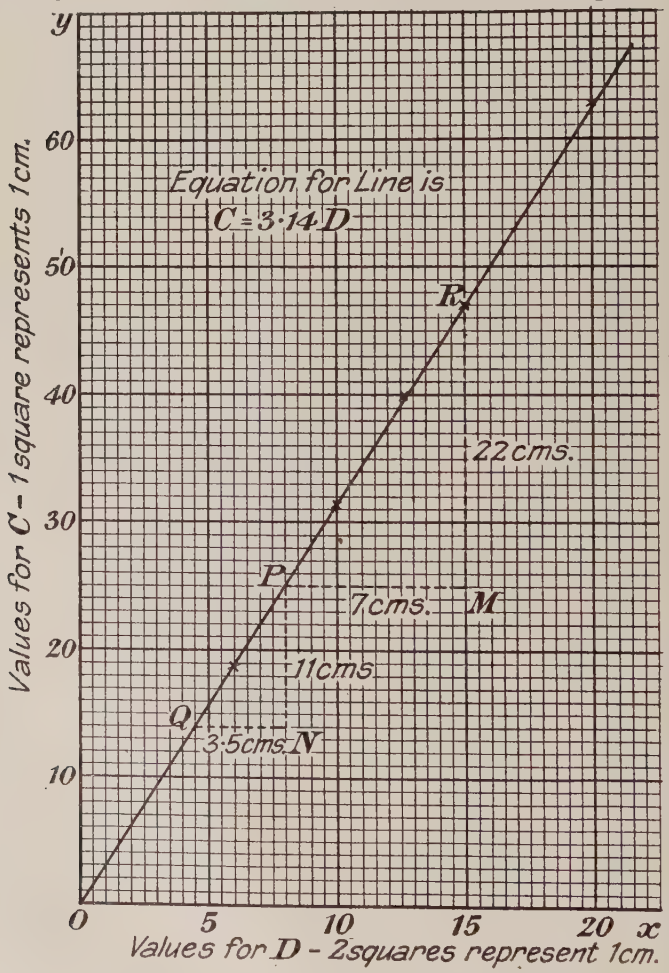


Fig. 115.

We see that the points lie approximately on a straight line through the origin (O).

(i) The gradient $= \frac{PN}{QN} = \frac{11}{3{\cdot}5} = \frac{3{\cdot}14}{1}$ to nearest $\frac{1}{100}$.

(ii) The gradient $= \frac{RM}{PM} = \frac{22}{7} = \frac{3{\cdot}14}{1}$ to nearest $\frac{1}{100}$.

Then equation of line is

$$C = 3{\cdot}14\ D \text{ (see § 90, Chapter VII).}$$

Dividing by D, $\qquad \frac{C}{D} = 3{\cdot}14,$

and C varies as D, whilst the variation constant is 3·14.

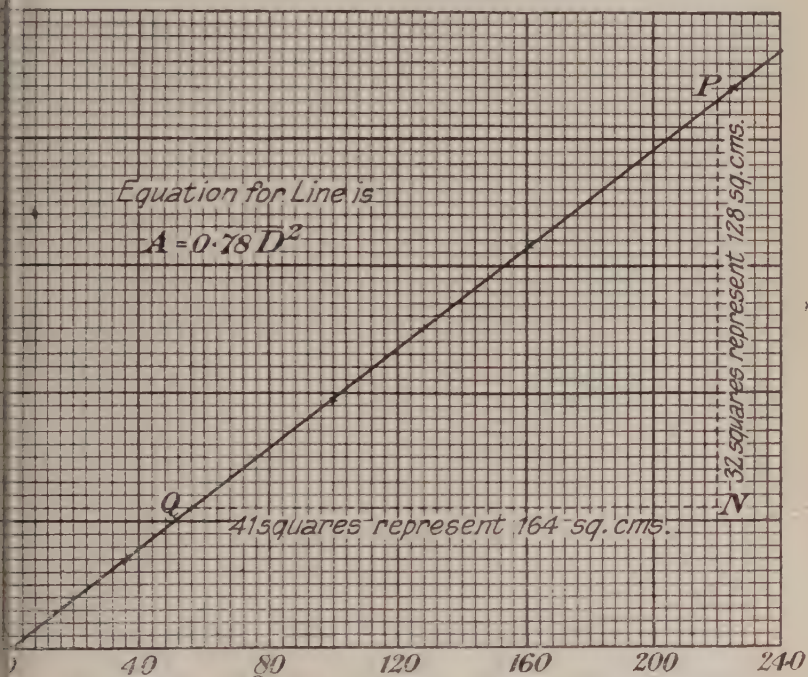


FIG. 116.

If we try values from the table we get

(i) $\frac{C}{D} = \frac{18{\cdot}85}{6} = 3{\cdot}14$ to nearest $\frac{1}{100}$.

(ii) $\frac{C}{D} = \frac{47{\cdot}12}{15} = \frac{9{\cdot}424}{3} = 3{\cdot}14$ to nearest $\frac{1}{100}$.

(*b*) *To show that* A *varies as* D^2.

Plot values for D^2 horizontally and corresponding values for A vertically.

This is shown in Fig. 116 and the points plotted lie on a straight line approximately.

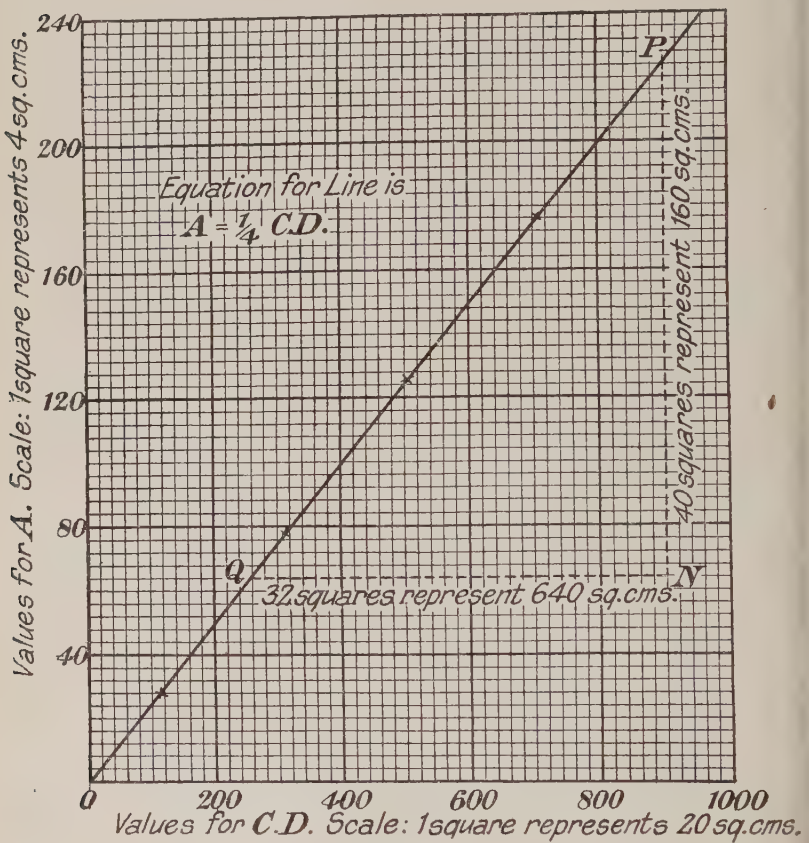


FIG. 117.

The gradient of the line $= \frac{PN}{QN} = \frac{128}{164} = 0{\cdot}78$ to nearest $\frac{1}{100}$

and the equation for the line is

$$A = 0{\cdot}78\ D^2,$$

$$\text{or } \frac{A}{D^2} = 0{\cdot}78 \text{ (the variation constant).}$$

Taking values from the tables we have

$$\text{(i)}\ \frac{A}{D^2} = \frac{78{\cdot}5}{100} = 0{\cdot}785.$$

$$\text{(ii)}\ \frac{A}{D^2} = \frac{126{\cdot}7}{161{\cdot}29} = 0{\cdot}786.$$

Then, since the ratio of any value of A to its corresponding value of D^2 is the same in every case approximately, we may say that A varies as D^2.

(*c*) *To show that* A *varies jointly as* C *and* D.

As before plot values of CD horizontally and corresponding values for A vertically (Fig. 117).

The points lie on a straight line through the origin.

The gradient of the line $= \frac{PN}{QN} = \frac{160}{640} = \frac{16}{64} = \frac{1}{4}$.

Then A varies as CD,

$$\text{or}\ \frac{A}{CD} = \frac{1}{4}\ \text{(the variation constant)}.$$

Example 2.

A body falls from rest and the distances fallen after various intervals of time are measured. These are given in the following table.

Time from start (t)	0	0·5	0·75	1	1·5	2	2·5	seconds
Distance travelled (d)	0	4	9	16	36	64	100	feet
Values of t^2	0	0·25	0·5625	1	2·25	4	6·25	

Find (*a*) if d varies as t, or (*b*) if d varies as t^2. We may draw the two graphs on the same paper with the same axes (Fig. 118).

Values of t and t^2 are plotted along OX and corresponding values of d along OY.

(*a*) On plotting values of t and d the points plotted lie on a curve OA, then d is *not proportional to* t.

(*b*) On plotting values of t^2 and d the points plotted lie on a straight line OB.

Then d varies as t^2.

The gradient of the line $= \frac{PN}{QN} = \frac{32}{2} = \frac{16}{1}$.

Then equation of line is $d = 16t^2$,

or $\frac{d}{t^2} = 16$ (the variation constant).

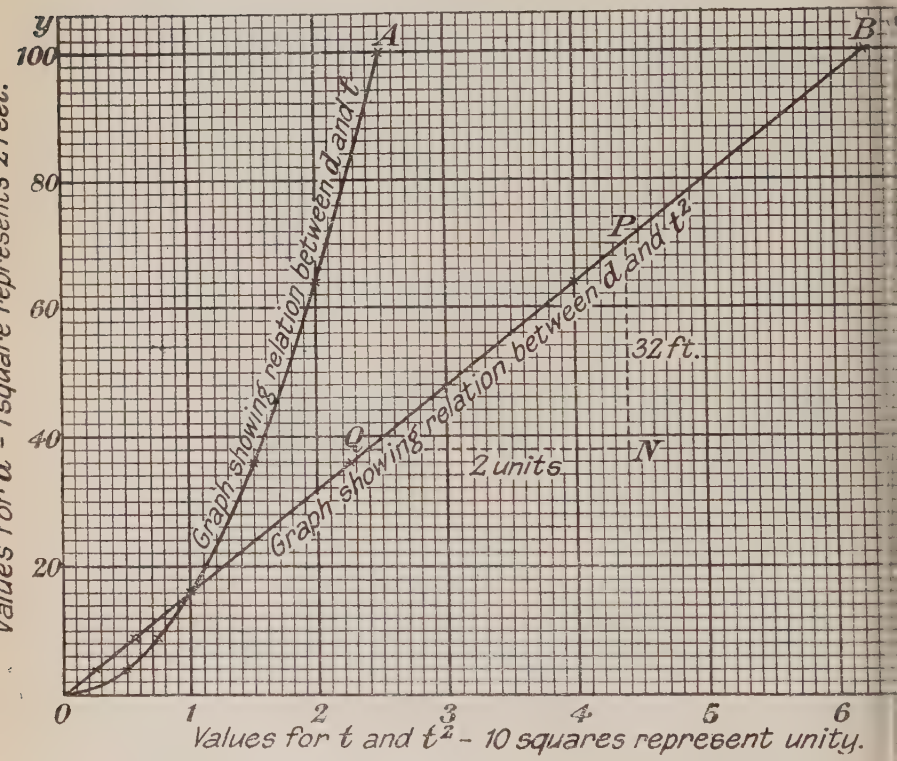


Fig. 118.

Example 3.

Inverse Variation.

A quantity y is said to vary inversely as another quantity x when $xy = K$ (the variation constant) . . (1)

Dividing both sides by x we may express the relationship as

$$y = \frac{K}{x}$$

$$\text{or } y = K\left(\frac{1}{x}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Dividing both sides of (2) by $\frac{1}{x}$ we have

$$\frac{y}{\frac{1}{x}} = K \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Written in this form (3) we notice that

y varies directly as $\frac{1}{x}$. (*Note.* $\frac{1}{x}$ is called the reciprocal of x.)

The following results were obtained in an experiment to show that the volume of a given mass of gas varies inversely as the pressure if the temperature be kept constant.

Total pressure in cms. of mercury (P) . . .	75·5	85·3	99·1	109·3
Volume of gas in c.c.'s (V).	11·3	10·0	8·6	7·8
Values for $\frac{1}{V}$	0·0885	0·1	0·116	0·128
Total pressure in cms. of mercury (P) . . .	131·2	142·2	147·0	155·1
Volume of gas in c.c.'s (V).	6·5	6·0	5·8	5·5
Values for $\frac{1}{V}$	0·154	0·167	0·172	0·182

(*a*) Plot values of P vertically and corresponding values of V horizontally and we see that the points plotted lie on a curve, Fig. 119. Examining this curve we notice,

(1) As V becomes larger P becomes smaller.
(2) As V ,, smaller P ,, larger.
(3) If we take any points A and B on the curve and draw lines AM, AN; BG, BK, parallel to the axes, then the areas of the rectangles so formed are equal.

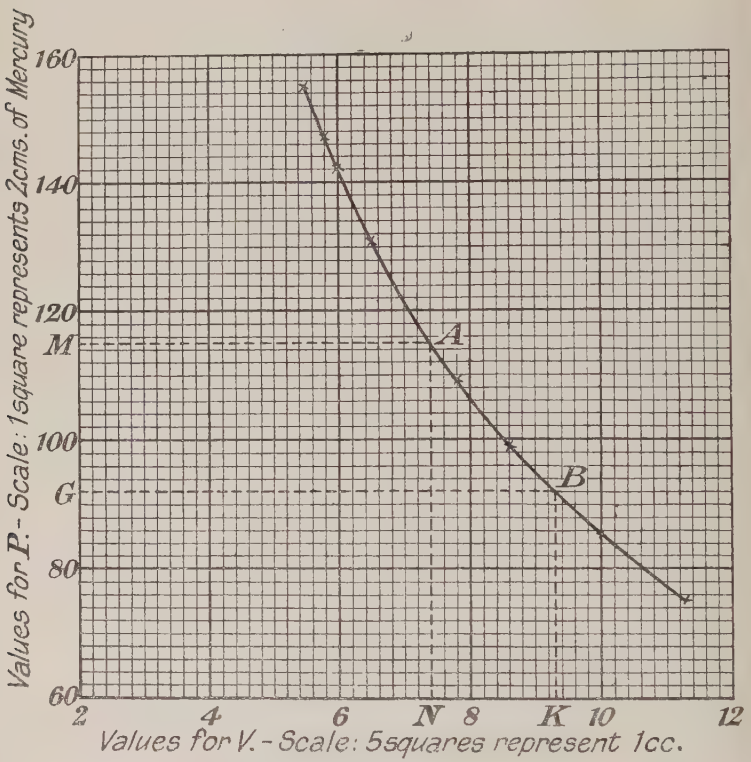


FIG. 119.

Thus,

Area of $AMON$ $= AN \times ON = 115 \times 7{\cdot}4 = 851.$
Area of $BGOK$ $= BK \times OK = 92 \times 9{\cdot}3 = 855{\cdot}6.$

Average area of rectangles $= 853{\cdot}3.$

Taking corresponding values of P and V from the table we have

(i) $P \times V = 109{\cdot}3 \times 7{\cdot}8 = 852{\cdot}54.$
ii) $PV = 85{\cdot}3 \times 10 = 853.$
(iii) $PV = 155{\cdot}1 \times 5{\cdot}5 = 853{\cdot}05.$

Then $PV = 853$ (the variation constant)
$=$ Area of rectangles shown in Fig. 119.

(*b*) Now plot values of P vertically and values of $\frac{1}{V}$ horizontally and the points plotted lie approximately on a straight line through the origin (Fig. 120).

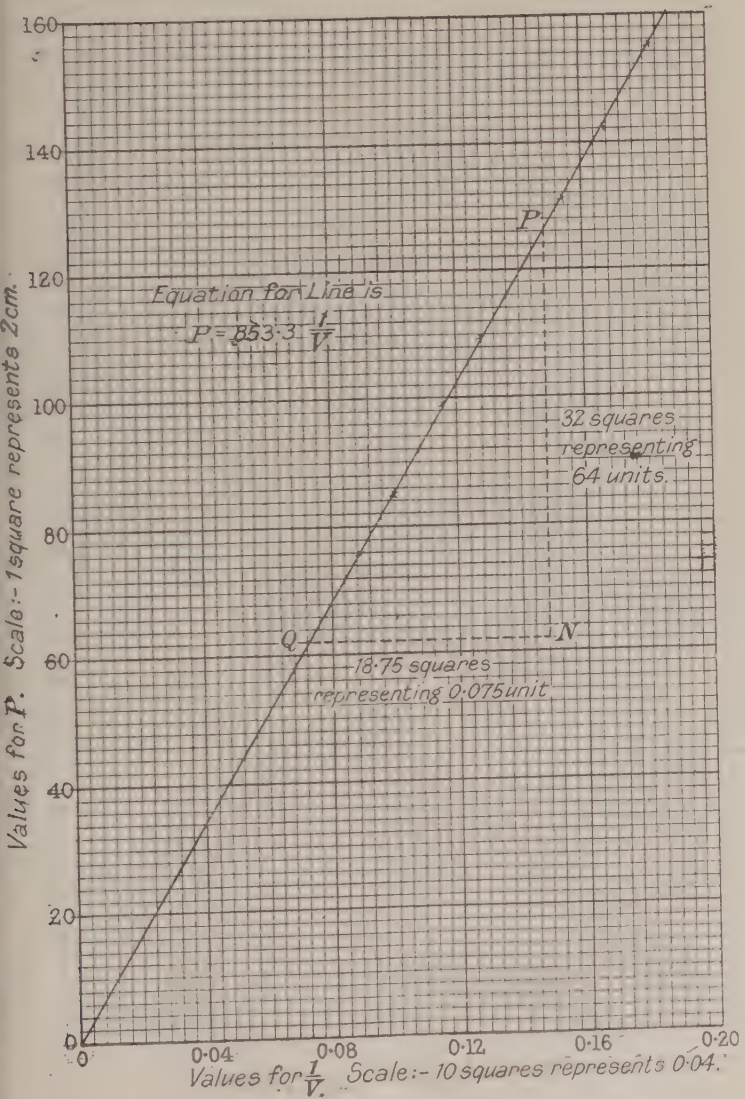


Fig. 120

Then P varies directly as $\frac{1}{V}$.

The gradient of the line $= \frac{PN}{QN} = \frac{64}{0{\cdot}075} = 853{\cdot}3.$

Then equation for the line is $P = 853{\cdot}3\ \frac{1}{V}$,

or $\frac{P}{\frac{1}{V}} = 853{\cdot}3$ (the variation constant) as above.

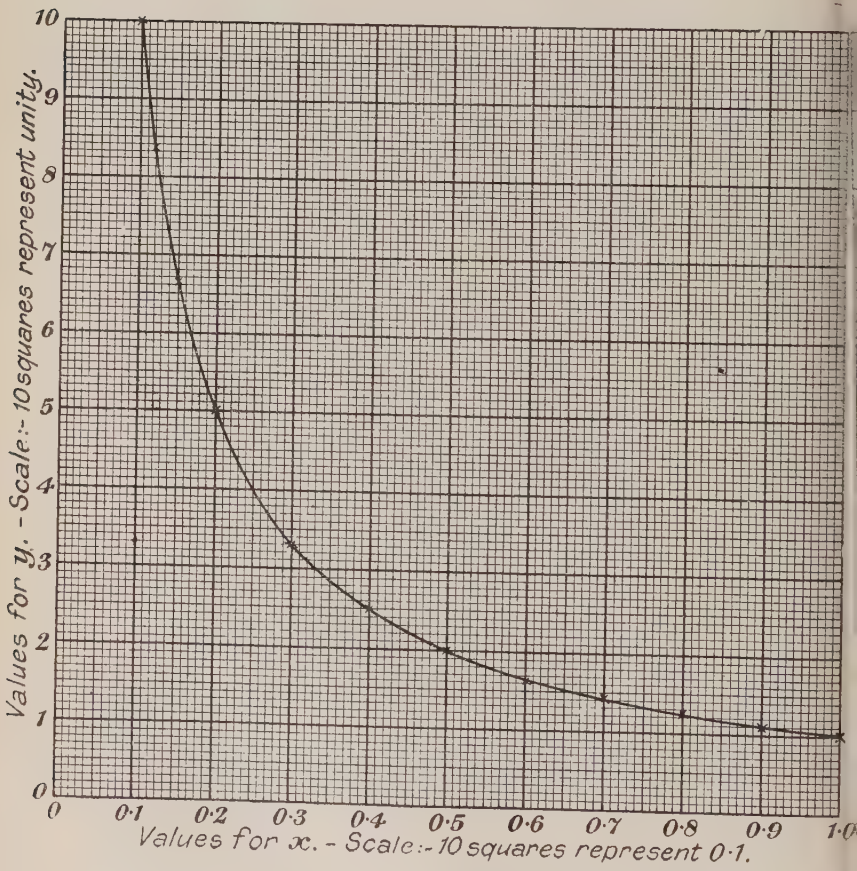


Fig. 121.

Example 4.

Suppose y varies inversely as x, then

$$xy = K \text{ (the variation constant)}$$

or $$y = K\left(\frac{1}{x}\right)$$

Suppose $K = 1$, then

$$xy = 1 \text{ or } y = \frac{1}{x} \text{ or } x = \frac{1}{y}.$$

Giving x various values and calculating the corresponding values for y we get the following :

Values for x .	0·1	0·2	0·3	0·4	0·5	0·6	0·8	0·9	1
Values for y .	10	5	$3\cdot\dot{3}$	2·5	2	$1\cdot6\dot{}$	1·25	$1\cdot\dot{1}$	1

The graph is shown in Fig. 121.

As in Example 3, the graph is a curve which, as x increases, approaches the axis OX, showing that y decreases.

As x decreases the curve approaches the axis OY, showing that y increases.

Since we have chosen 1 as the variation constant,

$$xy = 1,$$

or (any value of x) $\times$ (corresponding value of y) $= 1$.

Reciprocals.

Now when the product of two numbers is unity, each is said to be the Reciprocal of the other.

Thus, since $2 \times \frac{1}{2} = 1$, then $\frac{1}{2}$ is the reciprocal of 2, and 2 is the reciprocal of $\frac{1}{2}$.

Similarly, since $\frac{3}{5} \times \frac{5}{3} = 1$, then $\frac{3}{5}$ is the reciprocal of $\frac{5}{3}$, and $\frac{5}{3}$ is the reciprocal of $\frac{3}{5}$.

Hence, since $y \times \frac{1}{y} = 1$, then $\frac{1}{y}$ is the reciprocal of y and also $\frac{1}{x}$ is the reciprocal of x.

Then, since the co-ordinates of every point on the graph (Fig. 121) satisfy the equation

$$xy = 1$$

or $$x = \frac{1}{y},$$

we see that *any value of* x *is the reciprocal of the corresponding value of* y, and we may use the graph to obtain the reciprocals of numbers from 1 to 10.

Thus, suppose we want the value of $\frac{1}{5{\cdot}5}$.

Find $OA = 5{\cdot}5$ along the y axis.

Then AB gives the corresponding value of x, which is 0·18

and since $x = \frac{1}{y}$, $0{\cdot}18 = \frac{1}{5{\cdot}5}$ (the reciprocal of 5·5).

The student should carefully compare the curve for inverse variation with the curve (Fig. 118) obtained by plotting values of d and t in an example where d varies as t^2.

142. *Applications of Variation.*

The diameters of threads of the same material vary inversely as the square roots of the counts.

Let us consider the thread to be a long cylinder with a small diameter.

(i) Now the weights of cylinders are proportional to their volumes if they are made of the same material, for if the volume be doubled then the weight is doubled, etc. Hence

Weights of threads of same material vary as the volumes. (*a*)

(ii) Again suppose we take cylinders of same length as well as of same material ; then since

$$V \text{ (volume of cylinder)} = \frac{\pi D^2 H}{4} = \frac{\pi H}{4} \times D^2$$

$$= (\text{Constant}) \times D^2,$$

since H or length is constant

we have, if we keep the same length of yarn as well as the same material,

Volumes of 1 hank of thread of same material vary as the squares on the diameter. (*b*)

Combining results (*a*) and (*b*) we see that

Weights of equal lengths of same material vary as the squares on the diameters. (*c*)

Hence, if we consider 16's and 36's worsted yarn, and if we compare 1 hank of each thread and let diameter of 16's worsted $= D_1$ and diameter of 36's worsted $= D_2$, we have

Weight of 1 hank of 16's $= \frac{1}{16}$ lb.,

Weight of 1 hank of 36's $= \frac{1}{36}$ lb.,

1 from (*c*), $\frac{\frac{1}{16}}{\frac{1}{36}} = \frac{D_1^2}{D_2^2}$,

or $\frac{D_1^2}{D_2^2} = \frac{1}{16} \div \frac{1}{36} = \frac{36}{16}$.

'aking square root of both sides,

$$\frac{D_1}{D_2} = \frac{\sqrt{36}}{\sqrt{16}}. \qquad (d)$$

f C_1 = counts of the 16's and C_2 = counts of the 36's sted,

'rom (*d*),

$$\frac{D_1}{D_2} = \frac{\sqrt{C_2}}{\sqrt{C_1}}, \qquad (e)$$

ich shows that the *diameters vary inversely as the square roots the counts* (see § 138).

Example 1.

If the diameter of a 1/20's worsted is equal to $\frac{1}{95}$ inch, *or* if 95 reads of 1/20's worsted laid side by side occupy 1 inch, find e diameter of a 2/48's worsted.

If diameter of 2/48's or 1/24's = D_{24}'s and if diameter of 20's = D_{20}'s, we have from (*e*) above that

$$\frac{D_{24}\text{'s}}{D_{20}\text{'s}} = \frac{\sqrt{20}}{\sqrt{24}},$$

$$\text{or } \frac{D_{24}\text{'s}}{\frac{1}{95}} = \frac{\sqrt{20}}{\sqrt{24}}$$

$$\therefore \quad D_{24}\text{'s} = \frac{\sqrt{20}}{\sqrt{24}} \times \frac{1}{95} = \sqrt{\frac{20}{24}} \times \frac{1}{95}$$

$$= \sqrt{\frac{1}{1{\cdot}2}} \times \frac{1}{95} = \frac{1}{95\sqrt{1{\cdot}2}}$$

$$= \frac{1}{104{\cdot}1}, \text{ using log. tables,}$$

r 104 *threads of* 2/48's *can be laid side by side in a space of inch.*

Example 2.

The diameter of a 1/20's Y.S. woollen yarn is given as $\frac{1}{810}$ ch. What is the count of a woollen yarn whose diameter s $\frac{1}{840}$ inch ?

Suppose the required count $= x$'s Y.S., then

$$\frac{\sqrt{x}}{\sqrt{20}} = \frac{\text{Diameter of 20's}}{\text{Diameter of } x\text{'s}} = \frac{\frac{1}{60}}{\frac{1}{80}} = \frac{1}{60} \div \frac{1}{80}$$

$$= \frac{80}{60} \qquad \text{(i)}$$

or working as in Example 2 (page 283), suppose diameter $=$ inches ; counts $= C$.

Since the diameter varies inversely as square root of count

$$D \times \sqrt{C} = K \text{ (the variation constant).}$$

When $D = \frac{1}{60}$ inch $\quad C = 20$.

$$\therefore \qquad K = \frac{\sqrt{20}}{60}.$$

If $D = \frac{1}{80}$,

$$\frac{1}{80} \times \sqrt{C} = \frac{\sqrt{20}}{60}.$$

Multiply by 80 and

$$\sqrt{C} = \frac{\sqrt{20} \times 80}{60}.$$

Dividing by $\sqrt{20}$,

$$\frac{\sqrt{C}}{\sqrt{20}} = \frac{80}{60}. \qquad \text{(ii)}$$

Note.—Equation (ii) is the same as Equation (i).

Now square both sides of equation (i) and

$$\frac{x}{20} = \left(\frac{80}{60}\right)^2 = \left(\frac{4}{3}\right)^2.$$

Multiply by 20 and $\quad x = \frac{20 \times 16}{9} = 35\frac{5}{9}$.

$\therefore$ Required count $= 35{\cdot}5$'s Y.S. practically.

143. To show that "*for the same material the number of Dia meters per inch varies directly as the square root of the Counts.*

The diameter of a 1/20's Y.S. woollen is given as $\frac{1}{60}$ incl that is we may lay 60 of these threads *side by side* in a widt of one inch.

In practice we say there are 60 *diameters per inch.*

[*Note.*—The term "*ends per inch*" is used in the setting of warp See Chapter II on "Sett" Systems.]

Again, if the diameter of a 1/28's cotton is $\frac{1}{141}$ inch, then we all have 141 *diameters per inch*, and if the diameter of thread be D inches, the number of *diameters per inch* ill be $\frac{1}{D}$.

Now 60 is the reciprocal of $\frac{1}{60}$
141 " " " $\frac{1}{141}$
$\frac{1}{D}$ " " " D
} see note on reciprocals, par. 141, Ex. 4.

Hence we may say :—

ie Reciprocal of the Diameter } = { the number of threads which can be laid *side by side* in a width of one inch }
= the number of *diameters per inch* (i)

Since the diameters of yarn of the same material vary versely as the square roots of the counts, we have from (*e*), 142,

$$\frac{D_1}{D_2} = \frac{\sqrt{C_2}}{\sqrt{C_1}} \qquad \text{(ii)}$$

here D_1 C_1 ; D_2 C_2 represent the diameters and counts of two yarns.

If D_1 and D_2 represent the diameters, then $\frac{1}{D_1}$ and $\frac{1}{D_2}$ repre- nt the *diameters per inch.*

Now
$$\frac{\frac{1}{D_1}}{\frac{1}{D_2}} = \frac{1}{D_1} \div \frac{1}{D_2}$$
$$= \frac{1}{D_1} \times \frac{D_2}{1} = \frac{D_2}{D_1} \qquad \text{(iii)}$$

nd if we invert both ratios in (ii) above,
$$\frac{D_2}{D_1} = \frac{\sqrt{C_1}}{\sqrt{C_2}}.$$

Hence from (iii), $\frac{1}{\frac{D_1}{\frac{1}{D_2}}} = \frac{\sqrt{C_1}}{\sqrt{C_2}}$,

r $\frac{\textit{Diameters per inch} \text{ of 1st thread}}{\textit{Diameters per inch} \text{ of 2nd thread}} = \frac{\sqrt{\text{Counts of 1st thread}}}{\sqrt{\text{Counts of 2nd thread}}}$

e., the number of *diameters per inch* varies directly as the quare root of the counts (§ 137).

144. *Speeding and Gearing Problems.*

(1) *The speed of a pulley* or drum is usually expressed as "the number of revolutions made per minute."

In some problems we require to know the *Surface Speed* of a drum or cylinder (*e.g.*, the doffer in a carding machine).

Consider a point P (Fig. 121A) on the surface of a revolving cylinder represented in section by the circle PQR. If the cylinder were rolling on the ground, then in one revolution the point P would travel a distance equal to the circumference of the cylinder. This distance may be measured in feet or yards. If we know the number of revolutions made by the cylinder

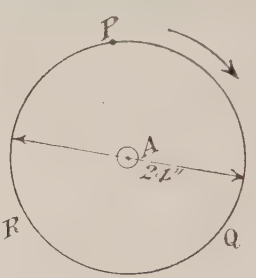


Fig. 121A.

per minute, then *the surface speed of the cylinder may be expressed in feet per minute or yards per minute* (usually "feet per minute").

Example 1.

A pulley, diameter 24, makes 126 revolutions per minute. Find the surface speed in feet per minute.

Here Diameter of pulley $= 24'' = 2$ feet.

Then Circumference $= \pi D = \frac{22}{7} \times 2$ feet,

and in one revolution the point P (Fig. 121A) travels $\frac{44}{7}$ feet.

Since pulley makes 126 revolutions per minute,

$$\text{Distance travelled by } P \text{ in one minute} = \frac{44 \times 126}{7} \text{ feet}$$

$$= 792 \text{ feet.}$$

$\therefore$ Surface speed $= 792$ feet per minute.

Example 2.

"A doffer in a carding machine has a diameter of 40 inches and is making 4 revolutions per minute. It is delivering

iver or carded top so that 1 yard of sliver weighs 20 drams. find the weight of sliver delivered per minute."

With every revolution the doffer delivers a length of sliver qual to the length of its circumference.

Since Diameter $= 40$ ins.

$$\text{Circumference} = \frac{22 \times 40}{7} \text{ ins.} = \frac{22 \times 40}{7 \times 36} \text{ yards.}$$

hen $$\text{Length of sliver delivered in one revolution} = \frac{22 \times 40}{7 \times 36} \text{ yards,}$$

nd since the doffer makes 4 revolutions per minute,

$$\text{Length of sliver delivered per minute} = \frac{22 \times 40 \times 4}{7 \times 36} \text{ yards.}$$

Now 1 yard of sliver weighs 20 drams.

$$\therefore \text{ Weight of sliver delivered per minute} = \frac{22 \times 40 \times 4 \times 20}{7 \times 36}$$
$$= 279{\cdot}4 \text{ drams.}$$

45. For *Two Pulleys connected by a Belt.* "*Speeds vary inversely as the Diameters of the Pulleys.*"

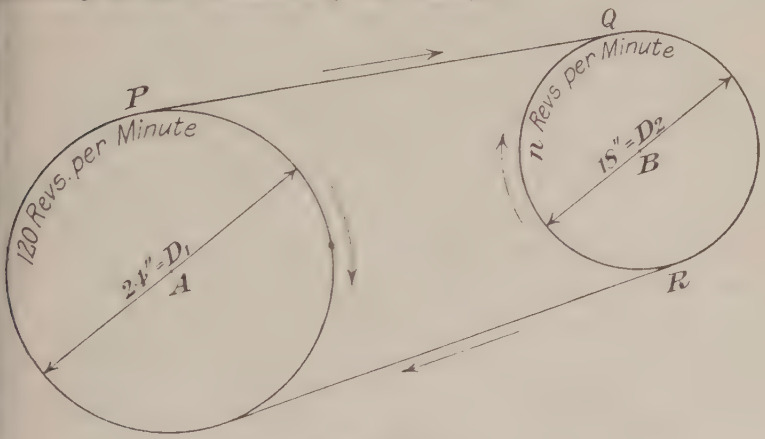


Fig. 122.

In the diagram (Fig. 122) let A and B represent two pulleys connected by a straight belt.

Suppose diameters of A and B are 24″ and 18″ respectively, and suppose A is making 120 revolutions per minute. We wish to calculate the speed of the pulley B.

Let its speed be n revolutions per minute.

Now it is easy to see that (if *there is no slipping of the belt*) whatever length of belt leaves the rim of the pulley A (at the point P, say) in one minute, the same length must leave the rim of the pulley B (at the point R) in the same time. This amounts to saying that

Surface speed of A = *Surface speed of* B (if the belt does not slip)

Since Diameter of A $= 24''$ and diameter of $B = 18''$,

$$\text{Circumference of } A = \frac{22 \times 24}{7} \text{ ins. and}$$

$$\text{Circumference of } B = \frac{22 \times 18}{7} \text{ ins.}$$

If A makes 120 revolutions per minute, then

$$\left.\begin{matrix}\text{Length of Belt leaving pulley } A \\ \text{at point } P \text{ per minute}\end{matrix}\right\} = \frac{22 \times 24 \times 120}{7} \text{ inches}$$

If B makes n revolutions per minute, then

$$\left.\begin{matrix}\text{Length of Belt leaving Pulley } B \\ \text{at point } R \text{ per minute}\end{matrix}\right\} = \frac{22 \times 18 \times n}{7} \text{ inches,}$$

and these lengths are equal.

$$\therefore \quad \frac{22 \times 18 \times n}{7} = \frac{22 \times 24 \times 120}{7}.$$

Divide both sides by $\frac{22}{7}$ and

$$18 \times n = 24 \times 120.$$

Divide both sides by 18 and

$$n = \frac{24 \times 120}{18} = 160. \qquad \text{(A)}$$

Then B makes 160 revolutions per minute.

(*b*) *Generalising.*

Suppose pulley A has a diameter D_1 inches and makes N_1 revolutions per minute, and B has a diameter D_2 inches and makes N_2 revolutions per minute (Fig. 123).

Reasoning as before,

Circumference of pulley $A = \pi D_1$ inches.

" " " $B = \pi D_2$ inches.

Surface speed of pulley $A = \pi D_1 N_1$ inches per minute.

" " " " $B = \pi D_2 N_2$ " " "

Since the surface speeds are equal (if there is no slipping on the belt),

$$\pi D_1 N_1 = \pi D_2 N_2.$$

Dividing both sides by π we have $D_1 N_1 = D_2 N_2$.

Divide both sides by $D_1 \times N_2$ and

$$\frac{D_1 N_1}{D_1 N_2} = \frac{D_2 N_2}{D_1 N_2}$$

or
$$\frac{N_1}{N_2} = \frac{D_2}{D_1} \qquad \text{(B)}$$

or
$$\frac{\text{Speed of pulley } A}{\text{Speed of pulley } B} = \frac{\text{Diameter of pulley } B}{\text{Diameter of pulley } A}. \qquad \text{(C)}$$

Referring to § 138, this result shows us that the speeds of the pulleys vary *inversely* as their diameters.

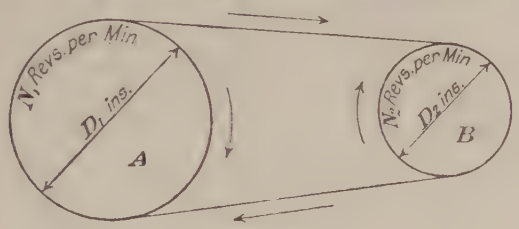


FIG. 123.

(*c*) In the numerical example (*a*)

$$N_1 = 120\,; \quad N_2 = n.$$
$$D_1 = 24''\,; \quad D_2 = 18''.$$

From (B)
$$\frac{120}{n} = \frac{18}{24}.$$

Inverting both ratios,

$$\frac{n}{120} = \frac{24}{18}.$$

Multiply both sides by 120 and

$$n = \frac{24 \times 120}{18} = 160,$$

which is the same as (A).

(*d*) Referring again to the figure (Fig. 123),

If the pulley B be on a line shaft and connected by a belt with a pulley A on a machine (*e.g.* a loom), then the pulley B is termed *the Driver* and the pulley A is termed *the Driven* or *the Follower*, and result (C) can be written

$$\frac{\text{Speed of Driven}}{\text{Speed of Driver}} = \frac{\text{Diameter of Driver}}{\text{Diameter of Driven}}. \qquad \text{(D)}$$

(*e*) There is always a certain amount of difference between the calculated speed of the driven pulley and its actual speed

owing to the *slipping* of the belt on both the driver and the driven. When the belt is made to work with its full width on the pulley the amount of loss should only be small, but when it is run with only half its width on the pulley the loss will be considerable. To lessen slipping the belt should be smooth and pliable.

If then we are told to allow 5% for slipping of the belt, we calculate the speed of the driven pulley as if there were no slip and then deduct 5% of this number of revolutions per minute. Thus, suppose the calculated speed of the driven pulley $= x$ revolutions per minute and the slip is 5% or $\frac{1}{20}$,

$$\text{Actual speed of Driven} = x - \tfrac{1}{20}x$$
$$= \frac{19}{20}x \text{ revolutions per minute.}$$

Again, if the slip is 2% and calculated speed is x revolutions per minute,

$$\text{Actual speed of Driven} = x - \frac{2}{100}x = \frac{98}{100}x$$
$$= 0{\cdot}98x \text{ revs./min.(using decimals)}$$
$$= \frac{49}{50}x \quad ,, \quad ,, \quad \text{(using fractions).}$$

EXAMPLE 1.

A line shaft in a weaving shed makes 120 revolutions per minute and carries a pulley 12 inches in diameter. The

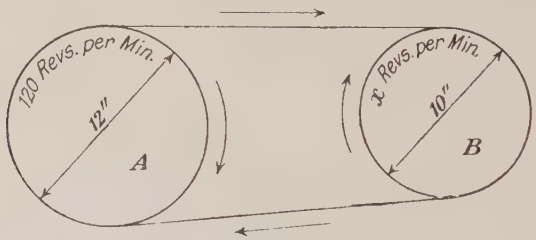


FIG. 124.

loom driven by it carries a pulley 10 inches in diameter. Find the speed of the loom (*a*) if there is no slip, (*b*) if there is 5% slip.

Suppose *A* (Fig. 124) represents the pulley on the line shaft (the Driver), and *B* represents the pulley on the loom (the Driven).

Then, (*a*) if there is no slipping of the belt,

$$\frac{\text{Speed of } B}{\text{Speed of } A} = \frac{\text{Diameter of } A}{\text{Diameter of } B}$$

$$\text{or } \frac{x}{120} = \frac{12}{10}.$$

Multiply each ratio by 120 and

$$x = \frac{12 \times 120}{10} = 144.$$

Then speed of loom pulley (B) = 144 revolutions per minute, and the loom makes 144 picks per minute.

(*b*) Allowing 5% for slipping,

$$\begin{aligned}\text{Actual speed of loom pulley } (B) &= 144 - (\tfrac{1}{20} \times 144) \\ &= \frac{19}{20} \times 144 = 136{\cdot}8 \\ &= 137 \text{ revs./min. approx.,}\end{aligned}$$

and the loom actually makes 137 picks per minute.

EXAMPLE 2.

A loom is required to run at 120 picks per minute. The loom pulley is 15″ diameter. The line shaft makes 150 revolutions

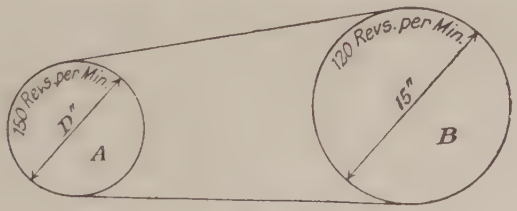


FIG. 125.

per minute. What must be the size of the drum on the line shaft (*a*) if there is no slip, (*b*) allowing 4% for slip ?

In the diagram (Fig. 125) let A and B represent the drum on the line shaft and the loom pulley respectively.

Suppose the diameter of the drum (A) = D inches.

(*a*) *If there is no slip,*

Then $$\frac{\text{Diameter of } A}{\text{Diameter of } B} = \frac{\text{Speed of } B}{\text{Speed of } A}$$

$$\text{or } \frac{D}{15} = \frac{120}{150}.$$

Multiply both sides by 15 and

$$D = \frac{120 \times 15}{150} = 12.$$

∴ Diameter of drum on line shaft must be 12 inches.

(*b*) *Allowing* 4% *for slip,*

The actual speed of loom pulley (*B*) must be 120 revolutions per minute.

Then its calculated speed must be greater.

Let its calculated speed be x revolutions per minute, and

$$\begin{aligned} x - 4\% \text{ of } x &= 120 \\ \text{or } x - \tfrac{1}{25}x &= 120 \\ \text{or } \tfrac{24}{25}x &= 120. \end{aligned}$$

Divide both sides by $\frac{24}{25}$ and

$$x = \frac{120 \times 25}{24} = 125.$$

We must therefore calculate the size of a drum which will give a speed of 125 revolutions per minute to the loom pulley if there were no slipping of the belt.

Then as in (*a*),

$$\frac{D}{15} = \frac{125}{150},$$

$$\text{or } D = \frac{125 \times 15}{150} = 12{\cdot}5.$$

$\therefore$ Diameter of drum on line shaft must be $12\frac{1}{2}$ inches.

Note.—The student should make it a rule to examine his answer to see if it is of the right order.

Thus in Example (1): The loom pulley (the Driven) is smaller than the pulley on the line shaft (the Driver); then we expect the speed of the loom to be greater than the given speed of the line shaft.

And in Example (2): The speed of the line shaft is greater than the speed of the loom; then the diameter of the drum on the line shaft will be less than the diameter of the loom pulley.

146. *For two Gear Wheels* (Spur or Toothed Wheels), *the "Speeds" vary inversely as the "Numbers of Teeth" on the wheels.*

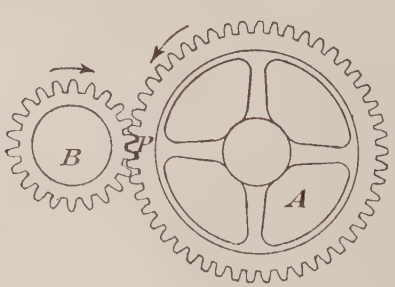


Fig. 126.

Fig. 126 shows two spur wheels in gear. Let A be the driver and B the follower.

Suppose A makes N_1 revolutions per minute, and B makes N_2 revolutions per minute.

If A has t_1 teeth and B has t_2 teeth, then $N_1 \times t_1$ teeth of the driver A pass the point P per minute, and $N_2 \times t_2$ teeth of the follower B pass the point P per minute.

Now every time a tooth belonging to A passes P it gears with a tooth belonging to B, so that whatever number of teeth belonging to A pass P per minute, the same number belonging to B must pass in the same time.

Then $$N_1 \times t_1 = N_2 \times t_2.$$

Dividing by $N_2 \times t_1$ we have

$$\frac{N_1 \times t_1}{N_2 \times t_1} = \frac{N_2 \times t_2}{N_2 \times t_1},$$

or $$\frac{N_1}{N_2} = \frac{t_2}{t_1},$$

or

$$\frac{\text{Speed of Driver } (A)}{\text{Speed of Driven (or Follower) } B} = \frac{\text{Number of teeth on Driven } (B)}{\text{Number of teeth on Driver } (A)}$$

or the "speeds" vary inversely as the "numbers of teeth" on the wheels.

147. *A Train of Wheels.*

(*a*) If several pairs of wheels are *geared* together as shown in Fig. 127, the combination is termed a *train*.

The wheel A has 60 teeth and drives B, which has 50 teeth. On the same shaft as B is the wheel C with 30 teeth, which drives D which has 45 teeth.

If A makes 150 revolutions per minute, find the speed of D. Since speeds of two wheels in gear vary inversely as the numbers of teeth in the wheel,

$$\frac{\text{Speed of } D}{\text{Speed of } C} = \frac{\text{Number of teeth in } C}{\text{Number of teeth in } D} = \frac{30}{45}. \quad \text{(i)}$$

Also $$\frac{\text{Speed of } B}{\text{Speed of } A} = \frac{\text{Number of teeth in } A}{\text{Number of teeth in } B} = \frac{60}{50}. \quad \text{(ii)}$$

Multiplying corresponding sides of equations (i) and (ii) we have

$$\frac{\text{Speed of } D}{\text{Speed of } C} \times \frac{\text{Speed of } B}{\text{Speed of } A} = \frac{30}{45} \times \frac{60}{50}.$$

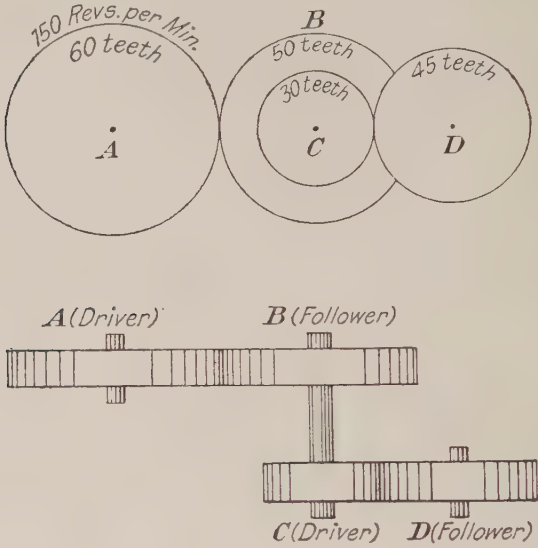


Fig. 127.

But speed of C = speed of B since they are both keyed to the same shaft.

Hence $$\frac{\text{Speed of } D}{\text{Speed of } A} = \frac{30 \times 60}{45 \times 50},$$

or $$\frac{\text{Speed of } D}{150} = \frac{30 \times 60}{45 \times 50}.$$

Multiply both sides of equation by 150 and

$$\text{Speed of } D = \frac{30 \times 60 \times 150}{45 \times 50} = 120 \text{ revolutions per minute.}$$

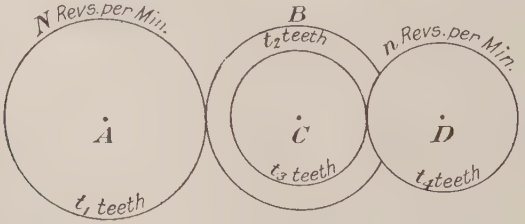


Fig. 128.

(*b*) *Generalising.*

Suppose the numbers of teeth in the various wheels of the train are as shown in the figure (Fig. 128).

Following the reasoning of (*a*),

$$\frac{\text{Speed of } D}{\text{Speed of } C} = \frac{\text{Number of teeth in } C}{\text{Number of teeth in } D}$$

$$= \frac{t_3}{t_4} \qquad \text{(i)}$$

and $$\frac{\text{Speed of } B}{\text{Speed of } A} = \frac{t_1}{t_2}. \qquad \text{(ii)}$$

Multiplying corresponding sides of equations (i) and (ii),

$$\frac{\text{Speed of } D}{\text{Speed of } C} \times \frac{\text{Speed of } B}{\text{Speed of } A} = \frac{t_3}{t_4} \times \frac{t_1}{t_2}.$$

But speed of C = speed of B.

Hence $$\frac{\text{Speed of } D}{\text{Speed of } A} = \frac{t_3 \times t_1}{t_4 \times t_2}. \qquad \text{(A)}$$

Referring to Fig. 127, we notice that A (with t_1 teeth) and C (with t_3 teeth) are drivers whilst B (with t_2 teeth) and D (with t_4 teeth) are followers.

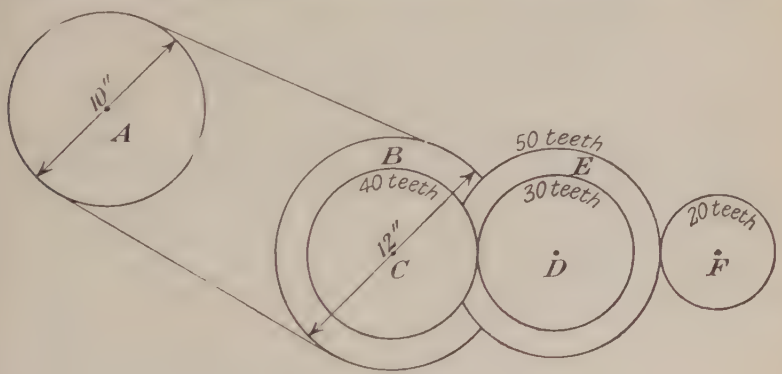


FIG. 129.

Thus in equation (A), $(t_3 \times t_1)$ is the product of the numbers of teeth in the drivers, and $(t_4 \times t_2)$ is the product of the numbers of teeth in the followers.

Hence we may write :—

$$\frac{\text{Speed of } D}{\text{Speed of } A} = \frac{\text{Product of the numbers of teeth on Drivers}}{\text{Product of the numbers of teeth on Followers}}$$

(*c*) Let us consider a longer train (Fig. 129).

The drum A, diameter 10 inches, drives by means of a belt the pulley B, diameter 12 inches. On the same shaft as B is

the toothed wheel C (40 teeth) which drives the wheel D (30 teeth). The wheel E (50 teeth), on the same shaft as D, drives the follower F (20 teeth).

If A makes 90 revolutions per minute, find the speed of F.

$$\text{Now } \frac{\text{Speed of } F}{\text{Speed of } E} = \frac{\text{Number of teeth in } E}{\text{Number of teeth in } F} = \frac{50}{20}. \quad \text{(i)}$$

$$\text{Also } \frac{\text{Speed of } D}{\text{Speed of } C} = \frac{40}{30} \quad \text{(ii)}$$

$$\text{and } \frac{\text{Speed of } B}{\text{Speed of } A} = \frac{\text{Diameter of } A}{\text{Diameter of } B} = \frac{10}{12}. \quad \text{(iii)}$$

(See §145 (*b*).)

Multiplying equations (i), (ii) and (iii),

$$\frac{\text{Speed of } F}{\text{Speed of } E} \times \frac{\text{Speed of } D}{\text{Speed of } C} \times \frac{\text{Speed of } B}{\text{Speed of } A} = \frac{50}{20} \times \frac{40}{30} \times \frac{10}{12}$$

Now B and C run at the same speed, and so do D and E. Then cancelling we have

$$\frac{\text{Speed of } F}{\text{Speed of } A} = \frac{50 \times 40 \times 10}{20 \times 30 \times 12} \quad \text{(B)}$$

$$\text{or } \frac{\text{Speed of } F}{90} = \frac{50 \times 40 \times 10}{20 \times 30 \times 12}$$

$$\text{or Speed of } F = \frac{50 \times 40 \times 10 \times 90}{20 \times 30 \times 12} = 250.$$

Then the follower F makes 250 revolutions per minute.

Referring to the diagram we see that A (diameter 10″), C (40 teeth), and E (50 teeth) are drivers, whilst B (diameter 12″), D (30 teeth), and F (20 teeth) are followers or driven wheels.

Then result (B) may be written :—

$$\frac{\text{Speed of last follower } (F)}{\text{Speed of first driver } (A)} = \frac{\text{Product of teeth and diameters of drivers}}{\text{Product of teeth and diameters of followers.}}$$

148. *Change Wheel and Gauge Point.*

(*a*) With the same train of wheels and dimensions as in § 147*c* (Fig. 130),

$$\frac{\text{Speed of } F}{\text{Speed of } A} = \frac{50 \times 40 \times 10}{20 \times 30 \times 12},$$

and if speed of A is constant at 90 revolutions per minute.

$$\text{Speed of } F = \frac{50 \times 40 \times 10 \times 90}{20 \times 30 \times 12}. \quad \text{(C)}$$

If the wheel D (30 teeth) be the *change wheel* (*i.e.*, it may conveniently be detached and its place taken by another wheel with a different number of teeth), since every other number in equation (C) remains the same, then the *speed of* F *alters if the wheel* **D** *be changed.*

Suppose the number of teeth in D (change wheel) $= x$.

Then the speed of $F = \dfrac{50 \times 40 \times 10 \times 90}{20 \times x \times 12}$

$$= \frac{50 \times 40 \times 10 \times 90}{20 \times 12} \times \frac{1}{x}.$$

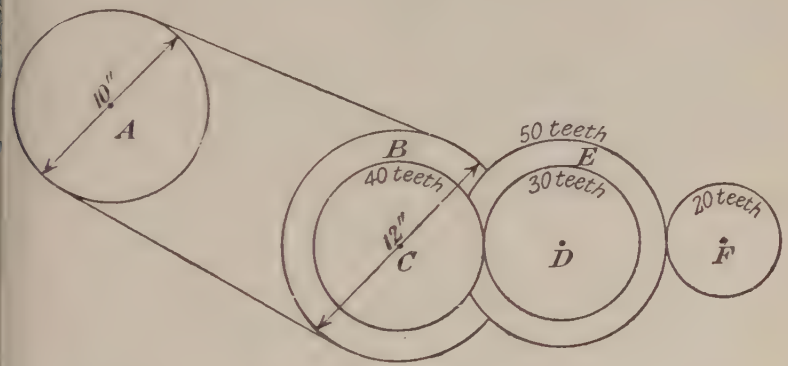


FIG. 130.

If this train of wheels belongs to the driving motion of a machine, *the constant* $\left(\dfrac{50 \times 40 \times 10 \times 90}{20 \times 12} = 7500\right)$, may be calculated once for all and is termed THE DIVIDEND or GAUGE POINT, and

$$\text{Speed of } F = \frac{\text{Dividend or Gauge point}}{x} \qquad \text{(D)}$$

Thus, if the change wheel has 30 teeth, $x = 30$, and

$$\text{Speed of } F = \frac{7500}{30} = 250 \text{ result as in (C).}$$

Again, if $x = 60$,

$$\text{Speed of } F = \frac{7500}{60} = 125 \text{ revolutions per minute,}$$

and if $x = 25$,

$$\text{Speed of } F = \frac{7500}{25} = 300 \text{ revolutions per minute.}$$

We may use Equation (D) to find *the number of teeth the change wheel must have so that* F *may run at a given speed.*

$$\text{Thus, Speed of } F = \frac{\text{Dividend or Gauge point}}{x \text{ (teeth in change wheel)}}$$

or $$\text{Speed of } F \times x = \text{Dividend.}$$

Then $$x = \frac{\text{Dividend}}{\text{Speed of } F}. \qquad \text{(E)}$$

Thus if speed of F must be 300 revolutions per minute,

$$x = \frac{7500}{300} = 25,$$

and change wheel must have 25 teeth.

Note.—The student should pay particular attention to this matter of change wheels and gauge points. We shall have to refer to it in loom driving, tappet driving, draft problems, etc.

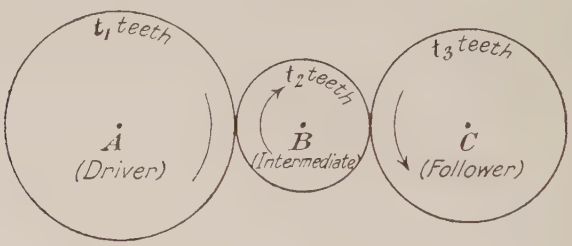


Fig. 131.

(*b*) Consider the train shown in Fig. 131.

Let the number of teeth in the wheels be those indicated in the figure.

Then $$\frac{\text{Speed of } C}{\text{Speed of } B} = \frac{t_2}{t_3} \qquad \text{(i)}$$

and $$\frac{\text{Speed of } B}{\text{Speed of } A} = \frac{t_1}{t_2}. \qquad \text{(ii)}$$

Multiply equations (i) and (ii).

$$\frac{\text{Speed of } C}{\text{Speed of } B} \times \frac{\text{Speed of } B}{\text{Speed of } A} = \frac{t_2}{t_3} \times \frac{t_1}{t_2}.$$

Cancelling, $$\frac{\text{Speed of } C}{\text{Speed of } A} = \frac{t_1}{t_3}.$$

This result is the same as if the driver (A) and the follower (C) were directly in gear. In a train of this kind the number of teeth on the intermediate wheel does not affect the ratio of the

speeds of C and A. Such a wheel is termed an *idle wheel*. The effect of this idle wheel is to make the follower rotate in the same direction as the driver.

Compare with Fig. 126, which shows that if two wheels are directly in gear they rotate in opposite directions.

Sometimes we may use more than one intermediate wheel if the driver and the follower are on shafts some distance apart in the machine, and if these intermediate wheels have the same number of teeth, then the velocity ratio of follower and driver is unchanged.

Thus in Fig. 132 the wheel A (t_1 teeth) drives, with the aid of the two intermediate wheels B and C (each 20 teeth), the follower D (t_2 teeth).

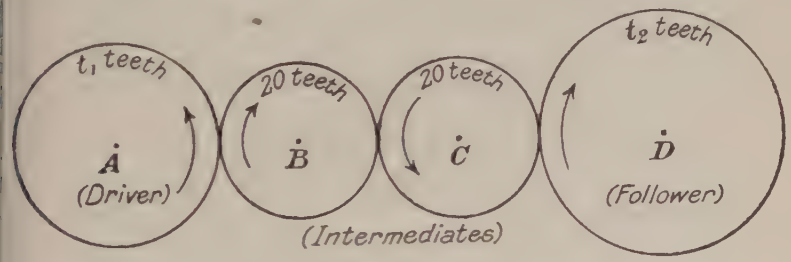


FIG. 132.

Then

$$\frac{\text{Speed of } D}{\text{Speed of } C} = \frac{20}{t_2} \text{ (i)}; \qquad \frac{\text{Speed of } C}{\text{Speed of } B} = \frac{20}{20} \text{ (ii)};$$

$$\frac{\text{Speed of } B}{\text{Speed of } A} = \frac{t_1}{20} \text{ (iii)}.$$

Multiplying corresponding sides of (i), (ii) and (iii),

$$\frac{\text{Speed of } D}{\text{Speed of } C} \times \frac{\text{Speed of } C}{\text{Speed of } B} \times \frac{\text{Speed of } B}{\text{Speed of } A} = \frac{20}{t_2} \times \frac{20}{20} \times \frac{t_1}{20}.$$

Cancelling, $$\frac{\text{Speed of } D}{\text{Speed of } A} = \frac{t_1}{t_2}.$$

This result is the same as if D and A were directly in gear and they also rotate in opposite directions.

(*c*) *Worked examples.*

EXAMPLE 1.

A shaft having a speed of 125 revolutions per minute drives another at 100 revolutions per minute on which is a bevel wheel with 40 teeth. How many teeth has the bevel wheel on the driving shaft ?

Suppose the bevel wheel on the driving shaft A (Fig. 133) has x teeth.

Then, since the speeds of the bevel wheels (or of shafts connected with them) vary inversely as the numbers of teeth on the wheels, we have :—

$$\frac{x}{40} = \frac{\text{Speed of driven shaft}}{\text{Speed of driving shaft}} = \frac{100}{125}.$$

Multiply both sides by 40 and

$$x = \frac{100 \times 40}{125} = 32.$$

Then bevel wheel on driving shaft has 32 teeth.

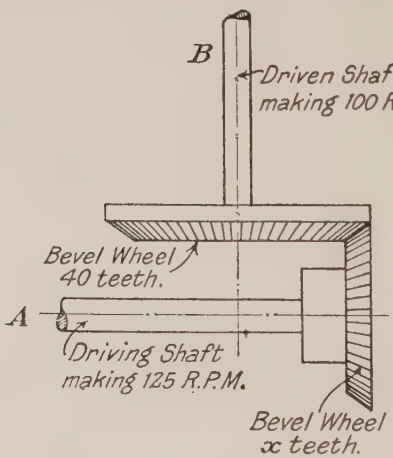


Fig. 133.

Example 2.

A shaft running at 96 revolutions per minute is required to drive a machine at 90 revolutions per minute ; what pulleys would be required to give this result, allowing 3% for loss by slipping, the belt speed not to exceed 650 feet per minute ?

In the diagram (Fig. 134),

Let A represent the pulley on the driving shaft making 96 revolutions per minute.

Let B represent the pulley on the machine shaft making 90 revolutions per minute.

Now the belt speed is the surface speed of the pulleys, which is the same for both pulleys.

If Diameter of pulley A $= x$ feet,

The Circumference of pulley A $= \frac{22x}{7}$ feet

and Surface speed $= \frac{96 \times 22x}{7}$ feet per minute.

The belt speed must not exceed 650 feet per minute, and supposing it is just 650 ft./min.,

$$\frac{96 \times 22 \times x}{7} = 650$$

$$\text{or } x = \frac{650 \times 7}{96 \times 22}$$

$$= 2{\cdot}15.$$

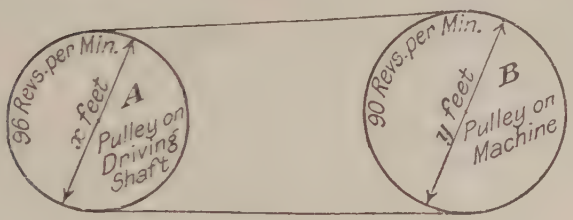


FIG. 134.

Thus, if the diameter of the pulley on the driving shaft be less than 2·15 feet, the belt speed will be less than 650 feet per minute.

Again, the speed of pulley B must be 90 revs./min. after allowing 3% for slip.

If calculated speed $= n$ revolutions per minute, actual speed $= n - 3\%\ n$ revolutions per minute.

Then $$n - \frac{3}{100}n = 90,$$

or $$\frac{97}{100}n = 90,$$

or $$n = \frac{90 \times 100}{97} = 92{\cdot}78.$$

Thus we must calculate the diameter of B for a calculated speed of 92·78 revolutions per minute.

Finally, if diameter of $B = y$ feet, and diameter of $A = x$ feet,

$$\frac{y}{x} = \frac{96}{92{\cdot}78} = \frac{2}{1{\cdot}93}.$$

Now 1·93 feet = 1 foot 11 inches nearly, and if $x = 1{\cdot}93$, $y = 2$.

Hence pulley A may be 1 foot 11 inches and pulley B may be 2 feet to satisfy the given conditions.

Example 3.

The cylinder of a card makes 167 revolutions per minute and the cylinder pinion has 56 teeth gearing into a 96 intermediate into a double carrier wheel 130, with 24 worker change on, into 134 on worker. Find (*a*) the speed of the worker,

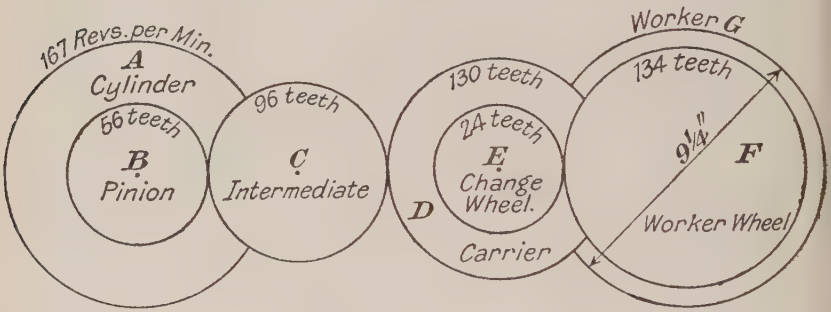


Fig. 135.

(*b*) the surface speed of the worker in inches per minute if diameter of worker is $9\frac{1}{4}$ inches from centre of pins.

Find also what are the revolutions of the worker per minute if the worker change has 40 teeth and the cylinder pinion is altered to 40 teeth.

Let diagram (Fig. 135) represent relative position of the train of wheels. (For calculation purposes we may ignore sizes and actual positions on the machine.)

The cylinder pinion B is the first driver and like the cylinder makes 167 revolutions per minute.

Intermediate C is driven by B and drives D so that it is both a driver and a follower. Or it may be treated as an *idle wheel* (Fig. 131).

The change wheel E is a driver and worker wheel F is a follower, and the speed of F is the speed of the worker G. Then

$$\frac{\text{Speed of } F}{\text{Speed of } A} = \frac{\text{Product of teeth of Drivers } (B \text{ and } E)}{\text{Product of teeth of Followers } (D \text{ and } F)}$$

(omitting the idle wheel (C))

or $$\frac{\text{Speed of worker}}{167} = \frac{56 \times 24}{130 \times 134}$$

or $$\text{Speed of worker} = \frac{56 \times 24 \times 167}{130 \times 134} \text{ revs. per min.} \quad (A)$$

$$= 12{\cdot}88 = 13 \text{ revs. per min. practically.}$$

If diameter of worker $= 9\frac{1}{4}$ inches, circumference of worker $= \frac{22 \times 37}{7 \times 4}$ inches, and it makes 13 revs./min.

$$\therefore \text{ Surface speed of worker} = \frac{13 \times 22 \times 37}{7 \times 4} = 378 \text{ ins. per min.}$$

If worker change E has 40 teeth and pinion B 40 teeth, from equation (A), we see

$$\text{Speed of worker} = \frac{40 \times 40 \times 167}{130 \times 134} = 15 \text{ revs. per min. (approx.).}$$

149. *The Drive and Speed of various parts of the Woollen or Worsted Carding Engine.**

I. *Description of Diagram* (Fig. 136).

A is the *Cylinder* or *Swift*—diameter 50 inches—driven by belt from the line shaft through pulley C—diameter 40 inches.

D is a *Doffer*—diameter 40 ins.—driven by pulley E (diameter 7 ins.) on Swift shaft through O (16 ins. diameter), spur wheel P (20 teeth) gearing into wheel on Doffer (232 teeth).

W_1, W_2, W_3 are *Workers*—diameters 12 inches. These are chain driven from the chain wheel G (10 teeth) *on the Doffer Shaft*. Pulleys on the workers have 13 teeth.

S_1, S_2, S_3 are *Strippers*—diameters 5 inches. These are belt driven from the pulley B (diameter 36 ins.) *on the Swift shaft*. Diameters of belt pulleys on Strippers 13 inches.

F is termed a *Fancy*. Diameter 12 inches. It is driven by the same belt as the Strippers. Diameter of belt pulley on Fancy is 7 inches.

S is an *Angle Stripper*—diameter 6 inch s—driven from pulley (diameter 10 inches) on Stripper shaft S_3. Diameter of belt pulley on S is 12 inches.

Suppose the swift A is making 100 revolutions per minute.

(*a*) *To find the Number of Revolutions of the Doffer.*

Referring to figure (Fig. 136),

Drivers are (1) pulley E (7″ diam.); (2) spur wheel P (20 teeth).

* See Chapters III and IV in "Woollen and Worsted" by Professor Beaumont.

Followers are (1) pulley O (16″ diam.); (2) toothed wheel on doffer shaft (232 teeth), and pulley E makes 100 revs./min. Find the speed of the swift.

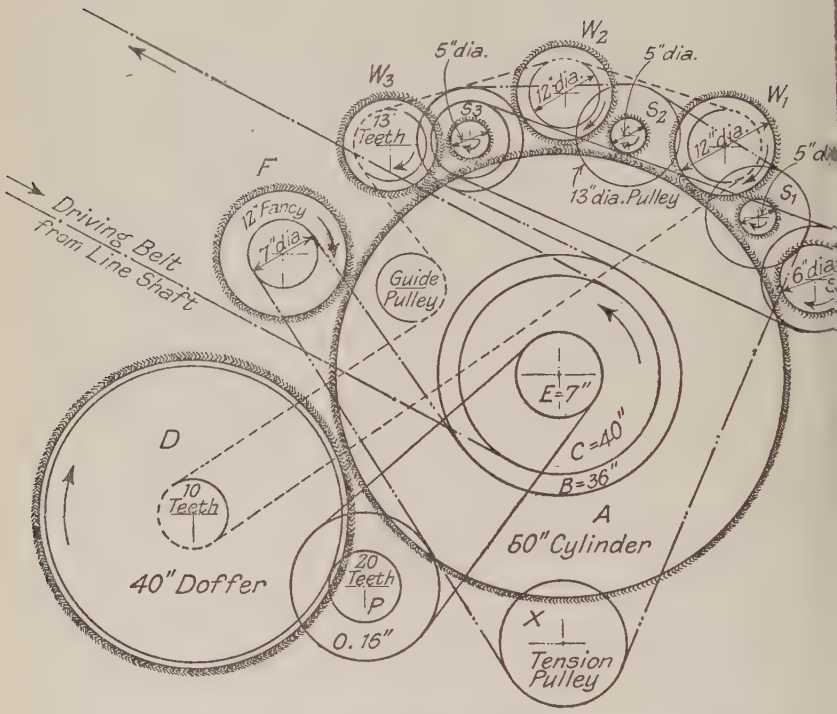


Fig. 136.—Portion of Worsted Carding Engine, showing Drives for Swift, Doffer, Workers, Strippers and Fancy.

Then

$$\frac{\text{Speed of Doffer}}{\text{Speed of } E} = \frac{\text{Product of teeth or diameters of Drivers}}{\text{Product of teeth or diameters of Followers}}$$

or
$$\frac{\text{Speed of Doffer}}{100} = \frac{7 \times 20}{16 \times 232'}$$

or
$$\text{Speed of Doffer} = \frac{7 \times 20 \times 100}{16 \times 232}$$
$$= 3{\cdot}771 \text{ (by logarithms)}$$
$$= 3{\cdot}8 \text{ revolutions per minute.}$$

(*b*) Since diameter of doffer = 40 inches (bare) or 41 inches allowing for clothing, then Circumference = 41 × 3·1416.
It is making 3·771 revs./min.

$$\therefore \text{ Surface speed of Doffer} = 41 \times 3{\cdot}1416 \times 3{\cdot}771$$
$$= 485{\cdot}9 \text{ (by logs.)}$$
$$= 486 \text{ inches per minute.}$$

(*c*) *To find Speed of Workers* (W_1, W_2, W_3).

These are chain driven from *G* (10 teeth) on doffer shaft. Chain wheel of worker shaft has 13 teeth.

$$\text{Then} \quad \frac{\text{Speed of worker}}{\text{Speed of } G} = \frac{\text{Number of teeth on } G}{\text{Number of teeth on Worker wheel}'}$$

$$\text{or} \quad \frac{\text{Speed of Worker}}{3{\cdot}771} = \frac{10}{13}'$$

$$\text{or} \quad \text{Speed of worker} = \frac{10 \times 3{\cdot}771}{13} = 2{\cdot}90 \text{ revs./min.}$$

Aliter.—The speed of the workers may be found without assuming the speed of the doffer.

Commencing with the driving pulley *E* on the Swift shaft,
The *Drivers* are (1) pulley *E* (7″ diam.), (2) spur wheel *P* (20 teeth), (3) chain wheel *G* (10 teeth).
The *Followers* are (1) pulley *O* (16″ diam.), (2) doffer wheel (232 teeth), (3) chain wheel on worker (13 teeth).

$$\text{Then} \quad \frac{\text{Speed of Worker}}{\text{Speed of } E \text{ or Swift}} = \frac{7 \times 20 \times 10}{16 \times 232 \times 13}'$$

$$\text{or} \quad \text{Speed of Worker} = \frac{7 \times 20 \times 10 \times 100}{16 \times 232 \times 13}$$
$$= 2{\cdot}90 \text{ revs. per minute.}$$

(*d*) *To find the Speed of the Fancy* (F).

This is driven from pulley *B* (36 inches diameter) on the Swift shaft by a belt passing over pulley (7 inches diameter) on Fancy shaft.

$$\text{Then} \quad \frac{\text{Speed of Fancy}}{\text{Speed of Swift}} = \frac{\text{Diameter of pulley } B}{\text{Diameter of pulley on Fancy}'}$$

$$\text{or} \quad \frac{\text{Speed of Fancy}}{100} = \frac{36}{7}.$$

$$\text{Multiply by 100 and Speed of Fancy} = \frac{36 \times 100}{7}$$
$$= 514{\cdot}3 \text{ revs./min.}$$

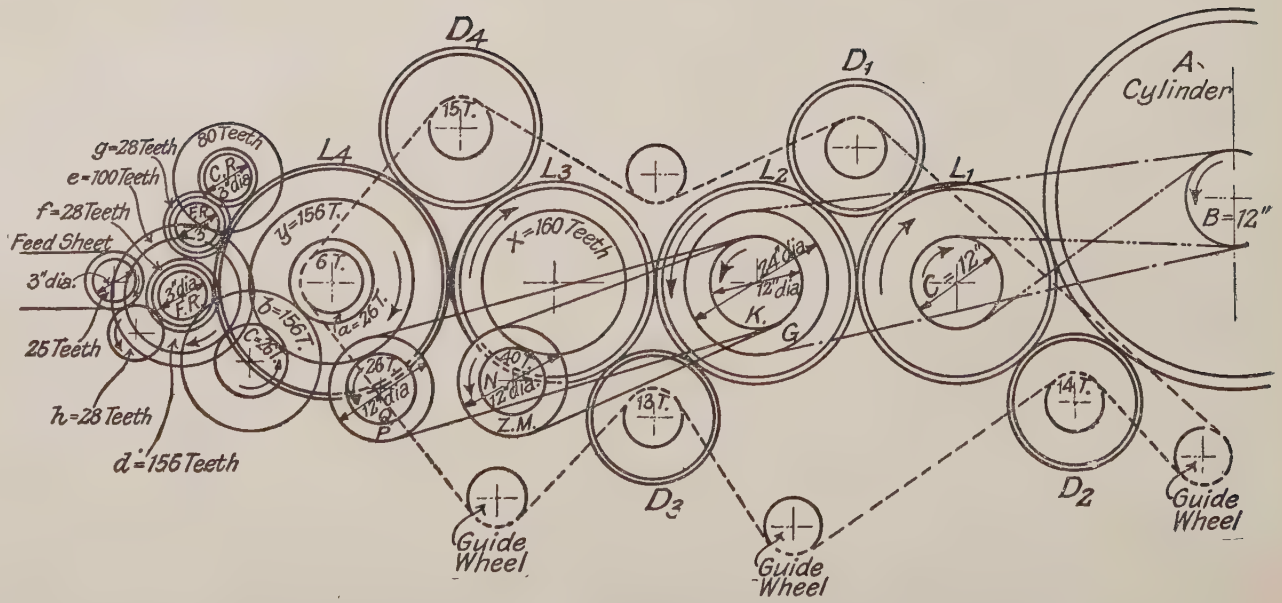


FIG. 137.—Portion of Worsted Carding Engine showing Drives for Lickers-in, Dividers, Feed Rollers and Feed Sheet.

(*e*) Since diameter of Fancy = 12 inches (bare) or 13 inches allowing for clothing, circumference of Fancy = 13 × 3·1416, and it makes 514·3 revs./min.

$$\therefore \text{ Surface speed of Fancy} = \frac{13 \times 3{\cdot}1416 \times 514{\cdot}3}{12} \text{ feet per min.}$$

$$= 1751 \text{ feet per minute.}$$

(*f*) If the student studies carefully the foregoing examples he will have little difficulty in verifying the following results.

(i) Speed of Strippers (S_1, S_2, S_3) $= \frac{36 \times 100}{13} = 277$ revs./m. (approx.)

(ii) Surface speed of Strippers $= 6 \times 3{\cdot}1416 \times 277$

$= 5223$ inches per minute.

(iii) Speed of Angle Stripper $S = \frac{36 \times 10 \times 100}{13 \times 12} = 230{\cdot}8$ revs./min.

II. *Description of Diagram* (Fig. 137).

A is the Cylinder or *Swift*. Diameter 50″.

L_1, L_2, L_3, L_4 are the *Lickers-in*. Their diameters are 26, 26, 26 and 30 inches respectively.

L_1 is belt driven from pulley *B* (12 inches diameter) on the Swift. Diameter of pulley *C* on shaft of L_1 is 12 inches.

L_2 is also driven from pulley *B* on the Swift. Driving pulley (*G*) on L_2 has a diameter of 24 inches.

L_3 is driven from pulley *K* (diameter = 12 inches) on L_2 through pulley *M* (12 inches diameter) and spur wheel *N* (40 teeth). Toothed wheel *X* on shaft of L_3 has 160 teeth.

L_4 is also driven from pulley *K* on shaft of L_2 through pulley *P* (12 inches diameter) and spur wheel *Q* (26 teeth). Toothed wheel *Y* on shaft of L_4 which gears with *Q* has 156 teeth.

D_1, D_2, D_3, D_4 are known as *Dividers*. Their diameters are 16, 16, 16 and 20 inches respectively. They are all chain driven from the chain wheel on licker-in L_4, which has 6 teeth. The chain wheels on the shafts of D_1, D_2, D_3 and D_4 have 12, 14, 13 and 15 teeth respectively.

FR is a *Feed Roller*. There are two of these known as the top and bottom feed rollers. The diameter of each is 3 inches. The *Bottom Feed Roller* is spur wheel driven from wheel *a* (26 teeth) through wheels *b* (156 teeth) and *c* (26 teeth) which gears with *d* (156 teeth) on the feed roller shaft.

The *Top Feed Roller* runs at the same speed as the bottom one, being driven from wheel *f* (28 teeth) on bottom roller shaft gearing with wheel *g* (28 teeth) on top roller shaft.

CR is a *Clearing Roller* (diameter 3 inches). This is driven from spur wheel *e* (100 teeth) on bottom feed roller shaft gearing with wheel (80 teeth) on *CR* shaft.

The *Feed Sheet* is carried over the *Feed Sheet Roller* (diameter 3 inches). This is driven from *f* (28 teeth) on bottom feed roller shaft, through the intermediate wheel *h* (28 teeth) gearing with a wheel (25 teeth) on the Feed Sheet Roller shaft.

Assuming the Swift A is making 100 revolutions per minute.

(i) *To find the speed of Licker-in* (L_2).

Referring to Fig. 137 and its description, we see

$$\frac{\text{Speed of } L_2}{\text{Speed of } A} = \frac{\text{Diameter of pulley } B \text{ on } A}{\text{Diameter of pulley } G \text{ on } L_2'}$$

or $$\frac{\text{Speed of } L_2}{100} = \frac{12}{24'}$$

or $$\text{Speed of } L_2 = \frac{12 \times 100}{24} = 50 \text{ revs. per minute.}$$

(ii) *To find speed of Licker-in* (L_4).

This is driven from L_2.

Referring to diagram (Fig. 137),

The Drivers are K (12″ diameter) and Q (26 teeth).

The Followers are P (12″ diameter) and Y (156 teeth).

Then

$$\frac{\text{Speed of } L_4}{\text{Speed of } L_2} = \frac{\text{Product of teeth or diameters of Drivers}}{\text{Product of teeth or diameters of Followers}}$$

or $$\frac{\text{Speed of } L_4}{50} = \frac{12 \times 26}{12 \times 156'}$$

or $$\text{Speed of } L_4 = \frac{12 \times 26 \times 50}{12 \times 156} = 8\tfrac{1}{3} \text{ revolutions per minute.}$$

(iii) *To find speed of the Bottom Feed Roller* (FR).

This is driven from Licker-in (L_4), making $8\frac{1}{3}$ revs./min. (Example ii).

Referring to diagram (Fig. 137)

The Drivers are a (26 teeth) and c (26 teeth).

The Followers are b (156 teeth) and d (156 teeth).

Then $$\frac{\text{Speed of } FR}{\text{Speed of } L_4} = \frac{26 \times 26}{156 \times 156'}$$

or $$\text{Speed of } FR = \frac{26 \times 26 \times 25}{156 \times 156 \times 3} = 0{\cdot}231 \text{ rev. per min.}$$

(iv) *To find speed of the Feed Sheet Roller.*

This is driven from the Bottom Feed Roller, making $\frac{25}{108}$ revolutions per minute (Example iii).

Referring to F g. 137, we see

The intermediate h has no effect on the relative speeds of the bottom Feed Roller and the Feed Sheet Roller.

Hence

$$\frac{\text{Speed of Feed Sheet Roller}}{\text{Speed of Feed Roller}} = \frac{\text{Number of teeth on } FR \text{ wheel}}{\text{Number of teeth on Sheet Roller wheel}},$$

or

$$\frac{\text{Speed of Feed Sheet Roller}}{\frac{25}{108}} = \frac{28}{25}.$$

Multiply both sides by $\frac{25}{108}$ and

$$\text{Speed of Feed Sheet Roller} = \frac{28 \times 25}{25 \times 108} = 0{\cdot}26 \text{ rev. per min.}$$

(v) *To find the surface speed of the Feed Sheet.*

This will be the same as the surface speed of the Feed Sheet Roller.

Diameter of Roller = 3 inches.
Circumf. of Roller = 3 × 3·1416 inches.
Speed of Roller = 0·2593 revs. per min. (Example iii).

∴ Surface speed of Feed Sheet = 0·2593 × 3 × 3·1416
= 2·444 (using logs.)
= 2·4 inches per minute.

(vi) We may find the speed of the Feed Sheet Roller without the intermediate steps of Examples (i), (ii) and (iii) thus :—

The *Drivers* are B (12″ diameter), pulley K (12″ diameter), wheel Q (26 teeth), wheel a (26 teeth), wheel c (26 teeth), wheel on FR (28 teeth).

The *Followers* are pulley G (24″ diameter), pulley P (12″ diameter), wheel Y (156 teeth), wheel b (156 teeth), wheel d (156 teeth), wheel on Feed Sheet Roller (25 teeth).

Then

$$\frac{\text{Speed of Feed Sheet Roller}}{\text{Speed of Swift } A} = \frac{\text{Product of the Drivers}}{\text{Product of the Followers}},$$

or

$$\frac{\text{Speed of Feed Sheet Roller}}{100} = \frac{12 \times 12 \times 26 \times 26 \times 26 \times 28}{24 \times 12 \times 156 \times 156 \times 156 \times 25}$$

or

$$\text{Speed of Feed Sheet Roller} = \frac{12 \times 12 \times 26 \times 26 \times 26 \times 28 \times 100}{24 \times 12 \times 156 \times 156 \times 156 \times 25}$$

$$= 0{\cdot}2593 = 0{\cdot}26 \text{ rev. per min. (Example iv).}$$

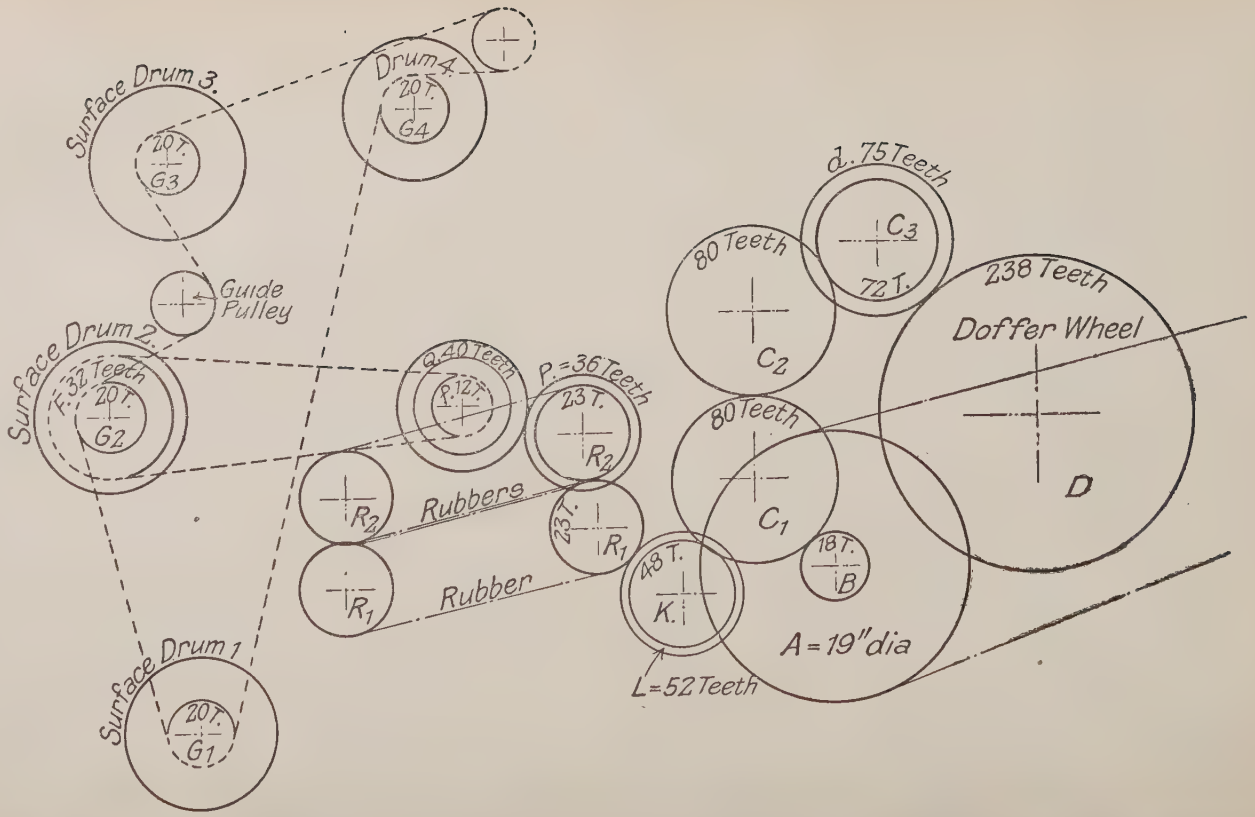


FIG. 138.—To illustrate the Driving of the Rubbers and Surface Drums in a Double Rubber Condenser

III. *Description of Diagram* (Fig. 138) *illustrating the Driving of Rubbers and Surface Drums in a Double Rubber Condenser.*

A is the driving pulley of the *Condenser*. Its diameter is 19″, and it is belt driven from a pulley 10″ diameter on the shaft of the last swift of the Carder.

R_1, R_2 are the *Rubber Rollers*. These are spur wheel driven from B (18 teeth) on shaft of A through the spur wheels C_1 (80 teeth), K (48 teeth), and L (52 teeth) which gears with R_1 (23 teeth) on the lower rubber roller shaft. R_2 on the upper rubber roller has also 23 teeth and gears with R_1, so that the rubbers travel at the same speed.

D_1, D_2, D_3, D_4 are the *Surface Drums*.

D_2 is chain driven from Q (40 teeth) which gears into the pinion wheel P (36 teeth) on the upper rubber roller shaft. The chain wheels f and F on Q and D_2 have 12 teeth and 32 teeth respectively.

D_1, D_3, and D_4 are chain driven from D_2. The chain wheels G_1, G_2, G_3 and G_4 have 20 teeth, so that they all run at the same speed.

(i) *To find the speed of the Driving Pulley* A.

Suppose the speed of the last swift of the carder is 80 revolutions per minute.

Then $$\frac{\text{Speed of } A}{\text{Speed of Swift}} = \frac{\text{Diameter of pulley on Swift}}{\text{Diameter of } A},$$

or $$\frac{\text{Speed of } A}{80} = \frac{10}{19},$$

or $$\text{Speed of } A = \frac{10 \times 80}{19} = 42{\cdot}11 \text{ revs. per minute.}$$

(ii) *To find the speed of the Rubber Rollers.*

Referring to Fig. 138, we see

The Drivers are B (18 teeth), L (52 teeth).

The Followers are K (48 teeth), R_1 (23 teeth).

Notice that C_1 is an intermediate wheel and may be omitted in the calculation, or if included it must be reckoned both as a Driver and a Follower.

Then

$$\frac{\text{Speed of Rubber Rollers}}{\text{Speed of } A} = \frac{\text{Product of teeth on Drivers}}{\text{Product of teeth on Followers}},$$

or $$\frac{\text{Speed of Rubber Rollers}}{42{\cdot}11} = \frac{18 \times 52}{48 \times 23},$$

or $$\text{Speed of Rubber Rollers} = \frac{18 \times 52 \times 42{\cdot}11}{48 \times 23},$$

$$= 35{\cdot}72 \text{ revs. per minute.}$$

(iii) *To find the speed of the Surface Drums.*

These are driven from the rubber roller shaft.

The Drivers are P (36 teeth) and f (12 teeth).

The Followers are Q (40 teeth) and F (32 teeth).

Then $$\frac{\text{Speed of Drums}}{\text{Speed of Rubbers}} = \frac{\text{Product of teeth on Drivers}}{\text{Product of teeth on Followers}},$$

or $$\frac{\text{Speed of Drums}}{35{\cdot}72} = \frac{36 \times 12}{40 \times 32},$$

or $$\text{Speed of Drums} = \frac{36 \times 12 \times 35{\cdot}72}{40 \times 32} = 12{\cdot}06 \text{ revs. per min.}$$

(iv) *To find the speed of the last Doffer* D.

This doffer it will be noticed from Fig. 138 is not driven direct from the swift, but by spur-wheel gearing from A through the intermediates C_1 and C_2 gearing with C_3 (72 teeth) mounted with the change wheel d (75 teeth) which imparts motion to the doffer wheel (238 teeth).

The Drivers are B (18 teeth), d (75 teeth).

The Followers are C_3 (72 teeth), doffer wheel (238 teeth).

Then $$\frac{\text{Speed of Doffer}}{\text{Speed of } A} = \frac{18 \times 75}{72 \times 238},$$

or $$\text{Speed of Doffer} = \frac{18 \times 75 \times 42{\cdot}11}{72 \times 238} \qquad \text{(A)}$$

$$= 3{\cdot}317 \text{ revolutions per minute.}$$

Note.—The speed of the doffer may be increased or decreased by altering the change wheel d, since all the other factors in the product (A) above remain constant.

This constant $\frac{42{\cdot}11}{4 \times 238}$ is sometimes called the *Dividend.*

Hence we see

$$\text{Speed of Doffer} = \left\{\begin{matrix}\text{Number of teeth} \\ \text{in Change Wheel}\end{matrix}\right\} \times \text{the Dividend,}$$

or $$\text{Teeth in Change Wheel} = \frac{\text{Speed of Doffer}}{\text{Dividend}}.$$

This enables us to determine the size of change wheel to impart a required speed to the doffer.

150. *Calculations on the Driving Motions of the Power Loom.*

There are two types of driving motions chiefly employed on power looms—(1) *Simple or Direct Driving*; (2) *Compound or Indirect Driving.*

In the Direct Drive the driving pulley is mounted on the crank-shaft. An indirect drive is one in which the crank or main shaft is driven by means of a train of wheels, from the low or bottom shaft.

I. *A Direct driving motion* is illustrated in Fig. 139.

A and *A'* are the loose and fast pulleys respectively ; *M* is the crank or main shaft ; *B* the brake wheel ; *D* is a spur driver on the main shaft, gearing with *E*, a spur follower on the low shaft.

The loom pulley *A* is belt driven from a drum on the line shaft.

Example 1.

The drum on the line shaft is 15 inches in diameter. It is making 84 revolutions per minute. If the diameter of the loom pulley *A* is 14 inches, find the number of picks per minute (*a*) assuming the belt does not slip, (*b*) allowing 10 per cent. for slipping.

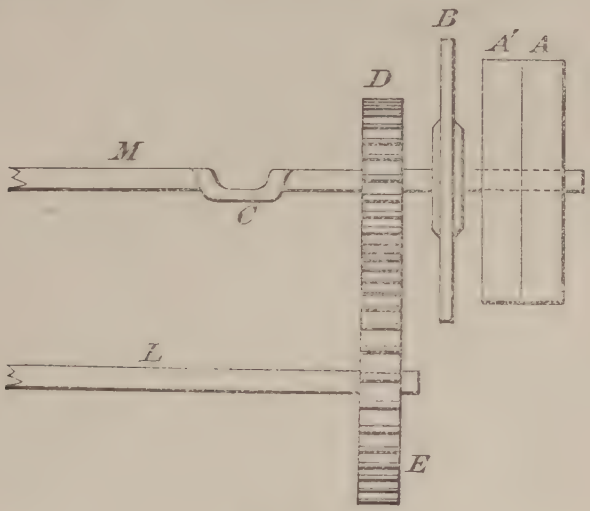


Fig. 139.

Remember one pick means one revolution of the crank-shaft, which in this type of drive is the same as one revolution of the loom pulley, since the loom pulley is keyed on to the crank-shaft.

$$\text{Now} \quad \frac{\text{Speed of Loom pulley}}{\text{Speed of Line shaft}} = \frac{\text{Diameter of Drum on Line shaft}}{\text{Diameter of Loom pulley}}$$

$$\text{or} \quad \frac{\text{Speed of Loom pulley}}{84} = \frac{15}{14},$$

$$\text{or} \quad \text{Speed of Loom pulley} = \frac{15 \times 84}{14} = 90 \text{ revs. per min.}$$

$\therefore$ (*a*) If there is no slipping of the belt,

Number of picks per minute $= 90$.

(*b*) Allowing 10% for slipping,

Number of picks per minute $= 90 - (10\%$ of $90)$

$= 90 - 9 = 81$.

Example 2.

Suppose the loom pulley is 14″ diameter and the speed of the line shaft is 70 revolutions per minute. If the loom makes 90 picks per minute allowing 10% for the slipping of the belt, what is the diameter of the drum on the line shaft?

We must first find what the speed of the loom pulley would be if there were no slipping of the belt.

Suppose this is x revolutions per minute.

Since $10\% = \frac{1}{10}$, then $x - \frac{1}{10}x = 90$ or $\frac{9}{10}x = 90$.

Divide by $\frac{9}{10}$ and $x = 100$.

If the diameter of the drum $= d$ inches, we have

$$\frac{\text{Speed of Loom pulley}}{\text{Speed of Line shaft}} = \frac{\text{Diameter of drum}}{\text{Diameter of Loom pulley}},$$

or

$$\frac{100}{70} = \frac{d}{14}.$$

Multiply both sides by 14 and $d = \frac{100 \times 14}{70} = 20$.

$\therefore$ Diameter of Drum required $= 20$ inches.

II. One type of *indirect driving* is shown in Figs. 140 and 141.

Example 1.

Suppose loom pulley A is making 255 revolutions per minute; bevel change wheel B has 18 teeth; bevel wheel C has 51 teeth; the spur driver D and spur follower E have 36 teeth each. Find the number of picks per minute.

(*a*) Since D and E have the same number of teeth, their speeds will be the same.

Since C and D are keyed to the same shaft, they will run at the same speed, so that the *speed of* C *will give us the speed of the loom.*

Also the speed of B = speed of loom pulley A.

$$\therefore \frac{\text{Speed of } C}{\text{Speed of } A} = \frac{\text{Speed of } C}{\text{Speed of } B} = \frac{\text{Number of teeth in } B}{\text{Number of teeth in } C}.$$

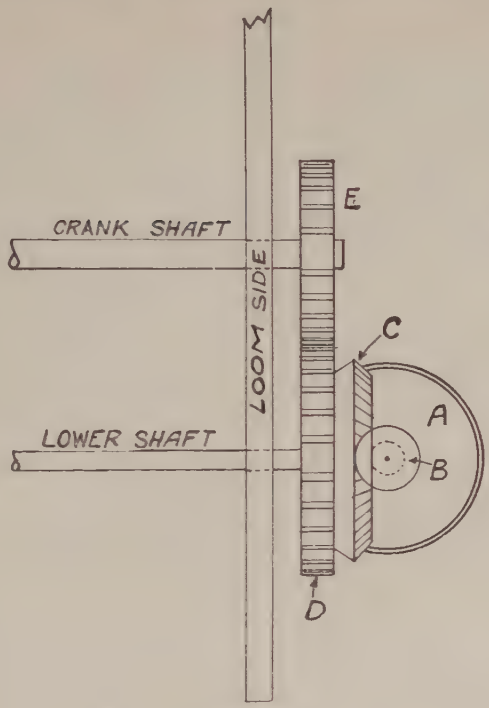


FIG. 140.

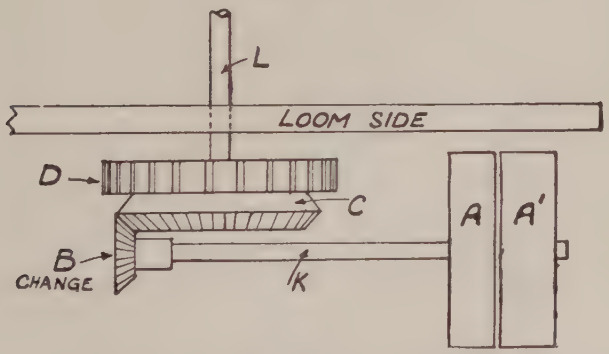


FIG. 141.

A' and A are the loose and fast loom pulleys respectively mounted on a countershaft K. At the other end of the countershaft the bevel *change wheel* B is in gear with the bevel wheel C which is keyed fast to the lower shaft L of the loom. Directly behind C on the lower shaft is the spur driving wheel D, and this is in gear with the spur follower E which is secured to the main or crank shaft of the loom.

Substituting the given values,

$$\frac{\text{Speed of } C}{255} = \frac{18}{51},$$

or $$\text{Speed of Loom} = \frac{18 \times 255}{51} = 18 \times 5 \qquad \text{(A)}$$

$$= 90 \text{ picks per minute.}$$

(*b*) If we refer to (A) above, we have :—

$$\text{Speed of Loom} = 18 \times 5$$

$$= \left\{\begin{matrix}\text{Number of teeth} \\ \text{in Change Wheel}\end{matrix}\right\} \times 5,$$

and if the speed of the loom pulley *A* is kept constant at 255 revolutions per minute, then by altering the change wheel *B* we alter the speed of the loom. This change wheel is usually supplied with the loom in three sizes having 18, 19, and 20 teeth.

Hence, if change wheel *B* has 20 teeth and loom pulley is making 255 revolutions per minute as before,

$$\text{Speed of Loom} = 20 \times 5 \text{ [see (A) above]}$$

$$= 100 \text{ picks per minute.}$$

Example 2.

Referring to Fig. 141, if bevel wheel *C* has 51 teeth, change bevel wheel *B* has 19 teeth, diameter of loom driving pulley *A* is 14 inches, diameter of drum on line shaft is 25 inches, speed of loom is 95 picks per minute, find (*a*) speed of loom pulley *A* ; (*b*) speed of the line shaft.

Since the loom is making 95 picks per minute, the bevel wheel *C* is making 95 revolutions per minute, for the spur driver *D* and spur follower *E* have the same number of teeth. Then

(*a*) $$\frac{\text{Speed of } A}{\text{Speed of } C} = \frac{\text{Speed of } B}{\text{Speed of } C} = \frac{\text{Number of teeth in } C}{\text{Number of teeth in } B},$$

or $$\frac{\text{Speed of } A}{95} = \frac{51}{19},$$

or $$\text{Speed of } A = \frac{51 \times 95}{19} = 255 \text{ revolutions per minute.}$$

Also

(*b*) $$\frac{\text{Speed of Line shaft}}{\text{Speed of } A} = \frac{\text{Diameter of } A}{\text{Diameter of Drum on Line shaft}},$$

or $$\frac{\text{Speed of Line shaft}}{255} = \frac{14}{25},$$

or $$\text{Speed of Line shaft} = \frac{14 \times 255}{25} = 142{\cdot}8 \text{ revs. per min.}$$

Example 3.

Referring again to Fig. 141, what must be the size of the change wheel B when the loom is making 105 picks per minute, given that the line shaft is making 150 revolutions per minute, diameter of drum on line shaft is 25 inches, diameter of loom pulley A is 14 inches, bevel wheel C has 51 teeth, D and E have 36 teeth ?

Suppose change wheel B has x teeth.

The Drivers are (1) drum on line shaft, (2) change wheel B, (3) spur wheel D on low shaft.

The Followers are (1) loom pulley, (2) bevel wheel C, (3) spur wheel E on main shaft.

Then

$$\frac{\text{Speed of Loom}}{\text{Speed of Line shaft}} = \frac{\text{Product of Diameters and teeth of Drivers}}{\text{Product of Diameters and teeth of Followers}},$$

or

$$\frac{105}{150} = \frac{25 \times x \times 36}{14 \times 51 \times 36}.$$

Multiply both sides by $(14 \times 51 \times 36)$ and

$$25 \times 36 \times x = \frac{105 \times 14 \times 51 \times 36}{150}.$$

Divide both sides by (25×36) and

$$x = \frac{105 \times 14 \times 51 \times 36}{150 \times 25 \times 36} = 20 \text{ (using the Slide Rule).}$$

Thus the change wheel must have 20 teeth.

Example 4.

How long will it take a loom to weave 50 pieces, each 48 ells, 60 picks per inch, the loom having a 12-inch pulley, 15-inch shaft drum ; speed of line shaft 80 revolutions per minute ? Allow 20% for stoppages.

Note.—An ell = 1¼ yards = 45 inches.

Then Length of a piece $= 45 \times 48$ inches.

Length of 50 pieces $= 45 \times 48 \times 50$ inches.

Since there are 60 picks per inch, then in weaving the 50 pieces,

Number of picks made by loom $= 45 \times 48 \times 50 \times 60$.

[Don't work out at this stage.]

$$\text{Now} \quad \frac{\text{Speed of Loom}}{\text{Speed of Line shaft}} = \frac{\text{Diam. of pulley on Line shaft}}{\text{Diam. of pulley on Loom}}$$

$$\text{or} \quad \frac{\text{Speed of Loom}}{80} = \frac{15}{12},$$

$$\text{or} \quad \text{Speed of Loom} = \frac{15 \times 80}{12} \text{ picks per minute.}$$

$$\text{Then} \quad \text{Time taken to weave 50 pieces} = \frac{45 \times 48 \times 50 \times 60 \times 12}{15 \times 80} \text{ mins}$$

$$= 1080 \text{ hours.}$$

This is the time required if there were no stoppage. Allowing 20% or $\frac{1}{5}$ for stoppages,

$$\text{Time taken} = 1080 + (\tfrac{1}{5} \times 1080) \text{ hours}$$

$$= 1080 + 216 = 1296 \text{ hours.}$$

Example 5.

A Carding Swift 55 inches in diameter is running at 90 revolutions per minute and driving a Fancy 12 inches in diameter from the body pulley 36 inches in diameter. What should be

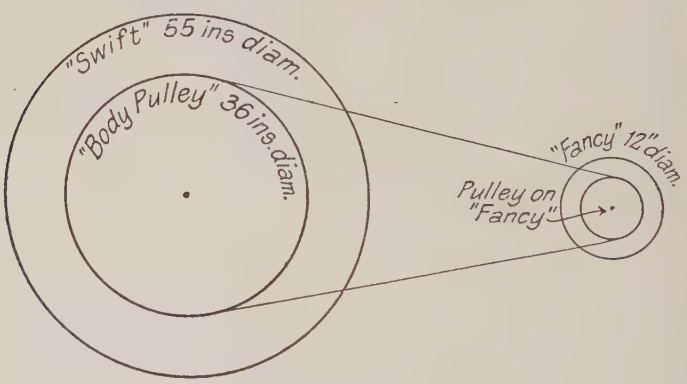


Fig. 142.

the diameter of the pulley on the Fancy to give 10% greater surface velocity of the Fancy than that of the Swift? Allow for card clothing $\frac{1}{2}$ inch deep from top of wire to bottom of foundation in the case of the Swift and for card clothing 1 inch deep in the case of the Fancy.

Allowing $\frac{1}{2}$ deep for clothing, diameter of *Swift* = 56 inches.

Since it makes 90 revolutions per minute,

$$\text{Surface speed of Swift} = \frac{22}{7} \times 56 \times 90$$
$$= 15840 \text{ inches per minute.}$$

If the Fancy has a 10% greater surface velocity,

$$\text{Surface speed of Fancy} = 15840 + (\tfrac{1}{10} \times 15840)$$
$$= 15840 + 1584$$
$$= 17424 \text{ inches per minute.}$$

Now the diameter of Fancy = 14 inches allowing 1 inch for clothing, and if speed of Fancy = x revolutions per minute,

Then Surface speed of Fancy $= \pi \times 14 \times x$ inches per min.

Hence $\pi \times 14 \times x = 17424$

or $$x = \frac{17424}{3{\cdot}1416 \times 14} = 396{\cdot}1.$$

Then Speed of Fancy $= 396{\cdot}1$ revs. per mins.
$= 396$ revs. per min. practically.

Now $$\frac{\text{Diameter of pulley on Fancy}}{\text{Diameter of pulley on Swift}} = \frac{\text{Speed of Swift}}{\text{Speed of Fancy}},$$

or $$\frac{\text{Diameter of pulley on Fancy}}{36} = \frac{90}{396},$$

or $$\text{Diameter of pulley on Fancy} = \frac{90}{11} = 8{\cdot}18 \text{ inches to nearest } \tfrac{1}{100}''.$$

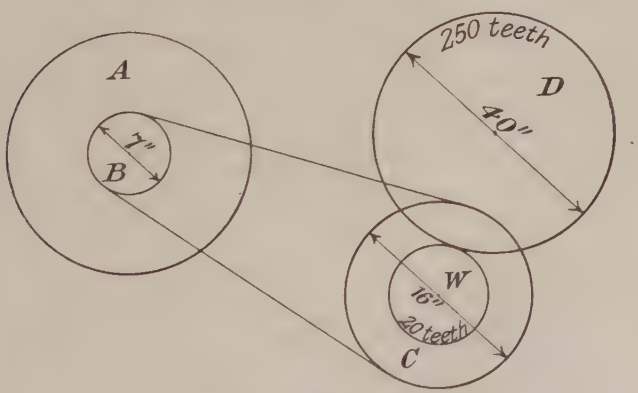


FIG. 143.

The following is left as an exercise for the student:—

A represents a Swift in a worsted carder making 100 revolutions per minute (Fig. 143). The pulley *B* on the Swift shaft is 7 inches in

diameter and drives by a belt the pulley C (16 inches diameter). This pulley shaft carries a spur change-wheel W (20 teeth) which drives the spur wheel (250 teeth) on the Doffer shaft (D); the Doffer is 40 inches in diameter. Find (i) the speed of the Doffer; (ii) its surface speed.

(Ans. :—$3\frac{1}{2}$, 440.)

151. *Calculations on the "Take-up" Mechanism of the Loom.*

The function of a "take-up" motion is to wind the cloth on to the piece-beam as it is woven.

The descriptions and diagrams given are sufficient for the practical student to make the necessary calculations. For fuller details and descriptions he should consult books like "Woollen and Worsted," by Prof. Roberts Beaumont, which contains chapters on the mechanism of Hand Looms and Power Looms.

The various types of "take-up" motions may be divided into two classes—(1) the negative or non-positive; (2) the change wheel or positive.

In the first class the amount of cloth drawn forward at each pick is determined by the force applied by the reed to the fell of the cloth at the beat-up. It is only as the cloth is pushed forward by the reed that it is possible for any "take-up" to take place.

In the second class the taking-up roller is driven in a positive manner by a suitable chain of wheels, actuated from some convenient part of the loom, *so that the same length of cloth is drawn forward on to the beam for each pick.*

Since the negative take-up motion does not lend itself to calculations, we shall deal only with motions of the positive type. The chief point of difference in motions of this class is the method of imparting motion to the train of wheels employed for driving the take-up roller.

I. *Positive Take-up Motion driven by means of a Catch and Ratchet Wheel.*

In Fig. 144, A is the take-up roller, sometimes called the friction roller. On the shaft of this roller is a spur wheel B which gears with the pinion wheel D which is compounded with the intermediate spur wheel E; the latter gears with the *change wheel* W, on the shaft of which is secured the ratchet wheel R. To operate the ratchet wheel there is a catch (or pawl) C which is pivoted on the swing lever L whose fulcrum is F. Working in the slot of the lever L is a stud G which is attached to the lay sword of the going-part of the loom. Thus when the sword moves towards the fell of the cloth, the stud G draws the lever L forward, thus pressing the catch forward and imparting motion to the ratchet wheel. Usually the stroke of the catch is such as to move the ratchet wheel one tooth forward at each pick of the shuttle. The holding-up catch K is merely to prevent the ratchet wheel from running back

when C is carried backwards by the receding motion of the sword The motion thus given to the ratchet wheel is transmitted by the train of wheels W, E and D to B and the friction roller A. The rough surface of this draws the piece forward and passes it on to the piece-beam, which is driven by frictional contact with the take-up or friction roller

In making calculations we must observe :

1. When the take-up roller A has made one revolution, it has drawn forward on to the piece-beam a piece of cloth equal in length to the circumference of the roller A. That length of warp is also drawn forward.

2. The speed of the ratchet wheel R depends upon the speed of the loom. Thus if the speed of the loom is 90 picks per

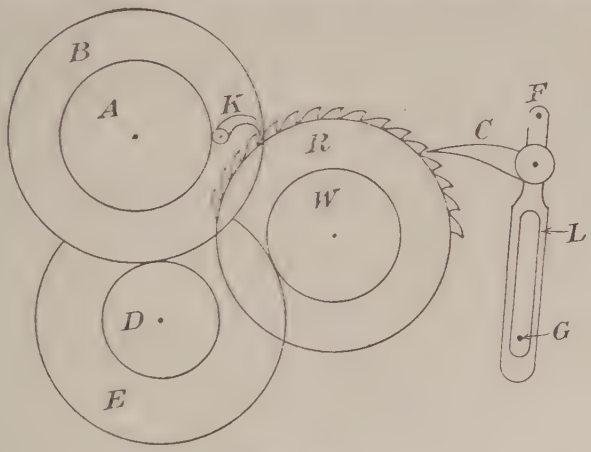


Fig. 144.

minute and the ratchet wheel has 60 teeth, then, since one tooth is driven forward at each pick,

Speed of ratchet wheel $= 1\frac{1}{2}$ revolutions per minute.

3. Speed of *change wheel* W = Speed of ratchet wheel.

4. If we alter the number of teeth in the change wheel W, the speed of the loom remaining the same, then we alter the speeds of E, D and B, and consequently that of the friction roller.

In our diagram W is a driver, and if we increase the number of teeth we increase the speeds of E, D and B, and consequently that of A. Hence a greater length of warp is drawn forward per minute. The loom makes the same number of picks per minute, so that in the cloth we shall have less picks per inch. Make the number of teeth in the change wheel less and we increase the number of picks per inch.

EXAMPLE.

To *find the number of picks per inch* given the following particulars for the motion sketched in Fig. 144.

(*a*) The catch C takes one tooth of ratchet wheel per pick; ratchet wheel (R) 60 teeth; change wheel (W) 55 teeth; intermediate wheel (E) 120 teeth; pinion wheel (D) 30 teeth; roller wheel (B) 110 teeth; circumference of take-up roller (A) 16 inches.

Since the catch C takes one tooth of R per pick and R has 60 teeth, then R makes one revolution every 60 picks.

If we know the length of cloth drawn forward by A whilst R makes one revolution, then in that length there must be 60 picks.

To find the speed of A,

The Drivers are W (55 teeth), D (30 teeth).

The Followers are E (120 teeth), B (110 teeth).

Then $$\frac{\text{Speed of } A}{\text{Speed of } R} = \frac{\text{Product of teeth in Drivers}}{\text{Product of teeth in Followers}},$$

or $$\frac{\text{Speed of } A}{1} = \frac{55 \times 30}{120 \times 110},$$

or $$\text{Speed of } A = \frac{55 \times 30}{120 \times 110} \text{ revolutions every 60 picks.}$$

Since circumference of $A = 16$ inches,

$$\text{Surface speed of } A = \frac{55 \times 30 \times 16}{120 \times 110} \text{ inches every 60 picks,}$$

and this length of cloth is drawn forward whilst the shuttle makes 60 picks.

Hence, in $\frac{55 \times 30 \times 16}{120 \times 110}$ inches of cloth there are 60 picks,

in 1 inch of cloth there are $\frac{60 \times 120 \times 110}{30 \times 16 \times 55}$,

or $$\text{Picks per inch} = \frac{60 \times 120 \times 110}{30 \times 16 \times 55} = 30. \qquad \text{I}$$

N.B.—The above calculation does not depend upon the speed of the loom, but *only on the number of teeth in the various wheels* and *the circumference of the take-up roller.*

(*b*) The wheel W (55 teeth in Example) on the ratchet-wheel shaft is a change wheel. All other particulars remain the same, and from I above.

$$\text{Picks per inch} = \frac{60 \times 120 \times 110}{30 \times 16} \times \frac{1}{\text{Teeth in Change Wheel}}$$

$$= \frac{1650}{\text{Teeth in Change Wheel}}, \qquad \text{II}$$

$$\text{or} \quad \text{Teeth in Change Wheel} = \frac{1650}{\text{picks per inch}}. \qquad \text{III}$$

The number 1650 is known as the *Dividend number* or the *Gauge point* for the particular motion we are considering. This is usually supplied to the Loom-tuner so that he may quickly find the number of teeth the change wheel must contain for a given number of picks per inch.

$$\text{Thus} \quad \text{Teeth in Change Wheel} = \frac{\text{Dividend number}}{\text{Picks per inch}}. \qquad \text{IV}$$

(*c*) If a piece has to be woven with 50 picks per inch, then in our example :—

$$\text{Number of teeth in Change Wheel} = \frac{1650}{50} = 33.$$

II. *Positive Take-up Motion driven by a Worm and Worm-wheel.*

This is illustrated in Fig. 145.

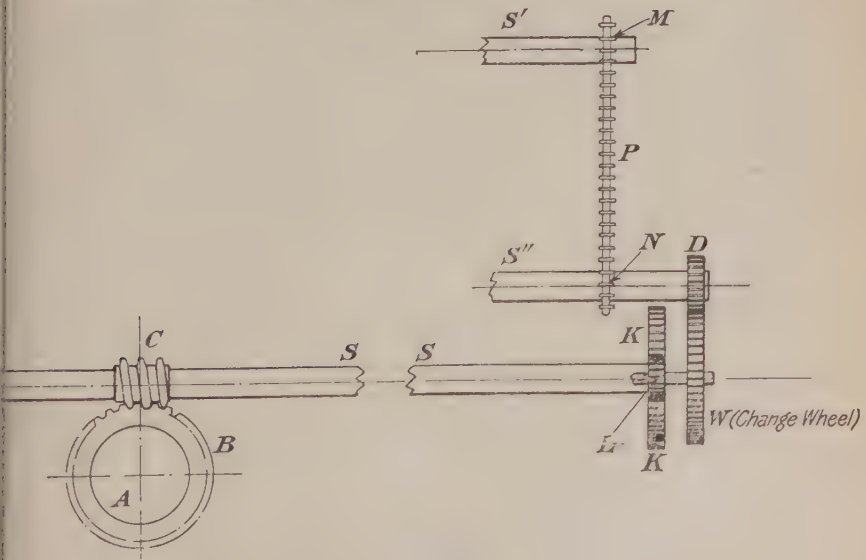


FIG. 145.

This motion is driven from the pattern chain cylinder. On the shaft S' is a star or sprocket wheel M driving by means of a chain P another star wheel N on shaft S''. Compounded with this is a spur wheel D which gears with W, the *change wheel* of the train. This through L drives the spur wheel K on the worm-shaft S. The worm C drives the worm-wheel B, which is mounted on the take-up roller A.

In making calculations we must remember that the star wheel M on the cylinder shaft makes one revolution each pick. Also one revolution of shaft S with a single worm C moves B forward one tooth only.

Example.

Assuming the following particulars (from the "Wool Year Book"):—Circumference of Take-up Roller A, 20 inches; Worm-wheel B, 85 teeth; single worm C; worm-shaft gear wheel (K), 84 teeth; Pinion L, 18 teeth; Spur wheel D, 20 teeth; Star wheels M and N, each 8 teeth.

(*a*) *Find the number of teeth in the Change Wheel* W *so as to weave a cloth with* 30 *picks per inch.*

Speed of Star wheel M = 30 revolutions every 30 picks
= 1 revolution every pick.

To find speed of take-up roller A,

The Drivers are M (8 teeth), D (20 teeth), L (18 teeth); Worm C (equivalent to 1 tooth).

The Followers are N (8 teeth), W (x teeth), K (84 teeth), B (85 teeth).

Then $$\frac{\text{Speed of } A}{\text{Speed of } M} = \frac{\text{Product of teeth in Drivers}}{\text{Product of teeth in Followers'}}$$

or $$\frac{\text{Speed of } A}{30} = \frac{8 \times 20 \times 18 \times 1}{8 \times 84 \times 85 \times x},$$

or $$\text{Speed of } A = \frac{8 \times 20 \times 18 \times 1 \times 30}{8 \times 84 \times 85 \times x} \text{ revs. every 30 picks.}$$

Since circumference of A = 20 inches,

$$\text{Surface speed of } A = \frac{8 \times 20 \times 18 \times 1 \times 30 \times 20}{8 \times 84 \times 85 \times x} \text{ inches every 30 picks}$$

$$= \frac{120 \times 30}{119 \times x} \text{ inches every 30 picks, (I)}$$

and this length of cloth or warp is drawn forward whilst the shuttle makes 30 picks.

Hence, in $\frac{120 \times 30}{119 \times x}$ inches of cloth there are 30 picks,

in 1 inch of cloth there are $\frac{30 \times 119 \times x}{120 \times 30}$ picks.

$$\therefore \text{ Number of picks per inch } = \frac{30 \times 119 \times x}{120 \times 30}$$

$$= x \quad \text{practically.} \qquad \text{(II)}$$

If the student will now work similar calculations step by step as indicated to try to find the number of teeth in the change wheel required to weave a cloth with, say, 20, 26, or 32 picks per inch. he will find that he arrives at the result (II) above, where

$$\left.\begin{matrix}\text{Number of Picks} \\ \text{per inch}\end{matrix}\right\} = x = \left\{\begin{matrix}\text{Number of Teeth in the} \\ \text{Change wheel.}\end{matrix}\right.$$

Thus with this particular gearing, if picks per inch = 30, 32, 40, 50, 70. then teeth in change wheel required = 30, 32, 40, 50, 70, respectively.

(*b*) Now along with the stock of change wheels a large star wheel containing 16 teeth is provided, and this may be substituted for the star wheel M on the cylinder shaft S' or be used in placed of the star wheel N on the shaft S''.

(1) Let us see the effect of substituting this wheel for M.

All the other particulars being the same as in (*a*), we shall get

$$\text{Surface speed of } A = \frac{2 \times 120 \times 30}{119 \times x} \text{ inches every 30 picks,}$$

[see result I above]

and finally :

$$\text{Number of picks per inch} = \frac{30 \times 119 \times x}{2 \times 120 \times 30}$$

$$= \tfrac{1}{2} \times x \quad \text{practically}$$

[see result II above]

$$= \tfrac{1}{2} \left\{\begin{matrix}\text{Number of teeth in} \\ \text{Change wheel}\end{matrix}\right\}$$

or $$\left.\begin{matrix}\text{Number of teeth in} \\ \text{Change wheel}\end{matrix}\right\} = \left\{\begin{matrix}\text{Number of picks} \\ \text{per inch}\end{matrix}\right\} \times 2. \qquad \text{(III)}$$

(2) Now substitute this star wheel (16 teeth) for N and let M have 8 teeth, and we get number of picks per inch $= 2x$

$$\text{Number of teeth in Change wheel} = \frac{1}{2}\left\{\begin{matrix}\text{Number of}\\ \text{picks per inch}\end{matrix}\right\} \quad \text{(IV)}$$

Thus with a small stock of change wheels and this large star wheel a very wide range of cloths can be woven. For example, assuming a stock of change wheels, the smallest having 30 teeth and the largest 60 teeth, each one differing from the next by 2 teeth, then :

(α) If M has 16 teeth, N (8 teeth), from (III) we have :

Teeth in change wheel = 30, 32, 34, 36 . . . 60.
Picks per inch = 15, 16, 17, 18 . . . 30.

(β) If M has 8 teeth, N has 8 teeth, from (II) we have :

Teeth in change wheel = 30, 32, 34, 36 . . . 60.
Picks per inch = 30, 32, 34, 36 . . . 60.

(γ) If M has 8 teeth, N has 16 teeth, from (IV) we have :

Teeth in change wheel = 30, 32, 34, 36 . . . 60.
Picks per inch = 60, 64, 68, 72 . . . 120.

Thus a range of cloths could be woven with the picks per inch varying from 120 down to 15.

152. *Speed of the Tappets.*

The tappets (T) (Fig. 146) are fixed to the tappet-shaft (A) which with the tappet-shaft wheel (W) runs loose on the low

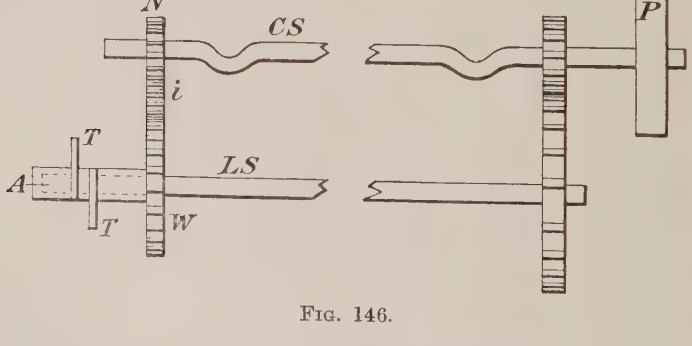


Fig. 146.

shaft (LS) of the looms. The tappet-shaft is driven from the crank-shaft (CS) by spur wheel gearing. The spur wheel (N) on the crank-shaft is a *change wheel* which drives through the intermediate wheel (i) the spur wheel (W) on the tappet-shaft.

Now the speed of the tappets is governed by the number of picks in one repeat of the pattern. Suppose there are 4 picks in one repeat and 4 tappets are keyed to the shaft (A), then the tappet-wheel W revolves once for every 4 revolutions of the crank-shaft.

In many looms the tappet wheel (W) has 120 teeth, this number being a multiple of all numbers from 1 to 10 with the exception of 7 and 9.

(*a*) *If there are* 4 *picks in the pattern, to find the number of teeth in the change wheel* N *given that* W *has* 120 *teeth.*

N makes 4 revolutions to every one revolution of W,

or
$$\frac{\text{Speed of } W}{\text{Speed of } N} = \frac{1}{4}.$$

Now the intermediate (i) does not affect the velocity ratio of W and N (see § 148 (*b*)), and if N has x teeth we have :—

$$\frac{\text{Speed of } W}{\text{Speed of } N} = \frac{\text{Number of teeth on } N}{\text{Number of teeth on } W'}$$

or
$$\frac{1}{4} = \frac{x}{120}. \qquad \text{(A)}$$

Multiply by 120 and

$$x = \frac{120}{4} = 30.$$

$\therefore$ Change wheel N must have 30 teeth.

(*b*) Again, if there are 6 picks in one repeat of the pattern,

$$\frac{\text{Speed of } W}{\text{Speed of } N} = \frac{1}{6'}$$

and
$$\frac{1}{6} = \frac{x}{120} \text{ from (A).}$$

Multiply by 120 and

$$x = \frac{120}{6} = 20.$$

$\therefore$ Change wheel N must have 20 teeth.

These results show us that

$$\text{Number of Teeth in change wheel } N = \frac{\text{Number of teeth in Tappet-shaft Wheel}}{\text{Number of picks in the pattern}}.$$

(c) Now 7 and 9 are not factors of 120, and though 8 and 10 are factors they would give change wheels (15 teeth and 12 teeth) which are too small for practical purposes.

Then if the pattern contains 7, 8, 9, or 10 picks in each repeat, we employ a train of wheels as shown diagrammatically in Fig. 147.

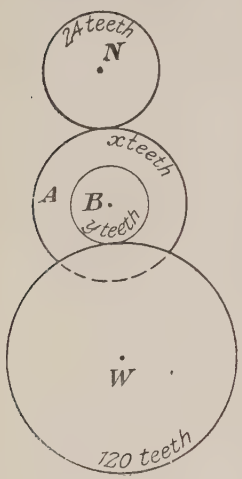


FIG. 147.

Here N is the change wheel on the crank-shaft, and let it be chosen with 24 teeth. A and B are intermediates with x and y teeth respectively. These are substituted for the intermediate (i) in Fig. 146. W is the tappet-shaft wheel with 120 teeth as before.

(i) Suppose the pattern has 7 picks for each repeat, and it is required to find suitable intermediate wheels A and B.

$$\text{Now} \quad \frac{\text{Speed of } W}{\text{Speed of } N} = \frac{1}{7}.$$

Also, the Drivers are N (24 teeth), B (y teeth). The Followers are A (x teeth), W (120 teeth).

$$\therefore \quad \frac{\text{Speed of } W}{\text{Speed of } N} = \frac{\text{Product of teeth on Drivers}}{\text{Product of teeth on Followers}},$$

or

$$\frac{1}{7} = \frac{24 \times y}{120 \times x}.$$

Inverting both ratios,

$$\frac{120 \times x}{24 \times y} = \frac{7}{1}.$$

Divide both sides by $\frac{120}{24}$ and

$$\frac{x}{y} = \frac{7 \times 24}{120} \qquad \text{(B)}$$

$$= \frac{168}{120} = \frac{42}{30} = \frac{7}{5}.$$

Thus the numbers of teeth in A and B are in ratio of 7 : 5, and theoretically any two wheels bearing that ratio to one another would give us the exact relative speed of the crank-shaft and the tappet-shaft.

Practically we choose A (42 teeth), B (30 teeth).

(ii) The student on working through the steps of (i) again with a different number of picks per pattern will get as in (B) above,

For an 8-pick pattern $\frac{x}{y} = \frac{8 \times 24}{120} = \frac{8}{5} = \frac{A \text{ (48 teeth)}}{B \text{ (30 teeth)}}$.

For a 9-pick pattern $\frac{x}{y} = \frac{9 \times 24}{120} = \frac{9}{5} = \frac{A \text{ (54 teeth)}}{B \text{ (30 teeth)}}$.

For a 10-pick pattern $\frac{x}{y} = \frac{10 \times 24}{120} = \frac{10}{5} = \frac{A \text{ (40 teeth)}}{B \text{ (20 teeth)}}$.

153. *Calculations involved in the Drafting and Spinning of Yarns.*

Drafting in yarn spinning comes under *three* heads :—

(1) Roller Draft. (2) Spindle Draft. (3) Draft between two sets of Fallers or wires or between Rollers and Fallers.

I. *Roller Draft.*

To illustrate this system let Fig. 148 represent 4 pairs of rollers, *A*, *B*, *C*, *D*. Carded top or sliver is fed to the first pair of rollers *A* and passed through the rollers *B* and *C* to the delivery rollers *D*.

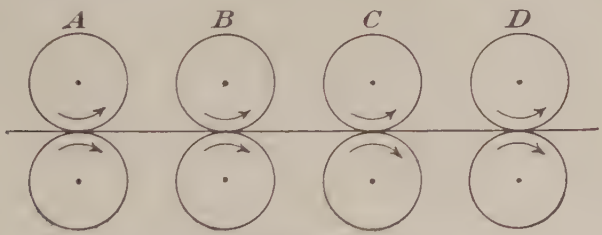


Fig. 148.

(*a*) Let us suppose that the rollers have equal diameters, say $3\frac{1}{2}$ inches.

Suppose *A* makes 3 revolutions per minute ; *B* revolves twice as fast as *A* and thus makes 6 revolutions per minute ; *C* revolves twice as fast as *B* and makes 12 revolutions per minute ; *D* revolves twice as fast as *C* and makes 24 revolutions per minute. Suppose 3 yards of carded top weighing 32 drams are passed through the set of rollers from *A* to *D*.

Since the diameter of each roller is $\frac{7}{2}$ inches, then circumference of each roller $= \frac{22}{7} \times \frac{7}{2} = 11$ inches, and in one revolution each pair of rollers delivers 11 inches of sliver to the pair in front.

Then

A delivers	11	×	3	or	33	inches of sliver per minute.		
B ,,	11	×	6	or	66	,,	,,	,,	,,	,,
C ,,	11	×	12	or	132	,,	,,	,,	,,	,,
D ,,	11	×	24	or	264	,,	,,	,,	,,	,,

Thus the rollers D deliver 8 times the length delivered by A in the same time, *or* a certain weight of sliver fed to A, whatever its length, will have 8 times that length when delivered by D.

Then, under these conditions, Draft applied = 8.

Again, 3 yards of sliver weighing 32 drams become 24 yards long, but still weighing 32 drams, *or* 3 yards delivered by D weigh 4 drams.

(*b*) To increase or decrease the weight of say 3 yards delivered we must alter the speeds of one or more of the rollers. Given the same particulars as in (*a*), suppose we wish 3 yards of sliver delivered by D to weigh 6 drams, and let us alter the speed of the rollers C to x revolutions per minute.

Then

A delivers	11	×	3	or	33	inches of sliver per minute.		
B ,,	11	×	6	or	66	,,	,,	,,	,,	,,
C ,,	11	×	x	or	$11x$	,,	,,	,,	,,	,,
D ,,	11	×	$2x$	or	$22x$	,,	,,	,,	,,	,,

Then D delivers $\frac{22x}{33}$ times the length delivered by A in the same time, or 3 yards weighing 32 drams become $\frac{3 \times 22x}{33}$ yards weighing 32 drams.

If $\frac{3 \times 22x}{33}$ yards delivered at D weigh 32 drams,

1 yard delivered at D weighs $\frac{32 \times 33}{3 \times 22x}$ drams,

and 3 yards delivered at D weigh $\frac{32 \times 33 \times 3}{3 \times 22x}$ drams.

But 3 yards delivered by D must weigh 6 drams.

$$\therefore \quad \frac{32 \times 33 \times 3}{3 \times 22 \times x} = 6,$$

$$\text{or} \quad \frac{48}{x} = 6. \quad \therefore \ x = 8.$$

Then C must make 8 revolutions per minute.

II. *Spindle Draft.*

The system occurs in the Woollen Mule. The carded rovings pass through one set of rollers (top and bottom), and after having been wrapped round the spindles are ready for the application of the draft. The *stretch* of the mule is 72 inches, *i.e.*, the greatest distance the spindle points move from the rollers.

The carriage in which the spindles revolve retreats from the rollers to the end of the stretch or draw, thus imparting draft to the rovings. The rollers are so governed that they will deliver the carded rovings for a certain fraction of the stretch and then stop delivery. Then the distance which the carriage and spindles travel after the rollers have ceased to revolve determines the amount of draft applied.

(*a*) Suppose a carded yarn is 8 skeins, *i.e.*, 8 yards weigh 1 dram, and it is to be reduced to 16 skeins, *i.e.*, 16 yards weigh 1 dram, then the yarn must be stretched to twice its length. In this case the rollers would deliver 36 inches or half the stretch before they stopped. The spindles travelling the remaining 36 inches would stretch the thread to 72 inches or twice its length.

(*b*) Again, if the same yarn 8 skeins has to be reduced to 24 skeins, *i.e.*, 24 yards must weigh 1 dram, then the yarn must be stretched to 3 times its length. In this case the rollers deliver for $\frac{1}{3}$ of the stretch, *i.e.*, 24 inches, before they stop, and the spindles travelling the remaining 48 inches increase the length to 72 inches or three times its length.

III. *Draft between Sets of Rollers and Fallers.*

In this system as in the roller draft, the extension of the sliver is attained by causing the surface speed of the rollers to differ from that of the fallers.

In Fig. 149 is shown the gearing of a preparing gill-box for worsted. Like all gill-boxes it possesses back rollers, fallers, and front rollers. In this way it is possible to apply three drafts which are known as (1) Draft between the back rollers and the fallers or *Back Draft*; (2) Draft between the fallers and front rollers or *Front Draft*; (3) Draft between the back and front rollers or *Whole Draft.*

A, *A'*	represent the	fast and loose pulleys . .	12 inches diameter.
B	represents ,,	driving speed wheel . . .	42 teeth.
C	,, ,,	driven ,, ,, . . .	70 teeth.
W	,, ,,	*change* or *draft wheel* . . .	*x* teeth.
E, *E'*	represent ,,	intermediate wheels . . .	equal numbers of teeth.

[*Note.*—The ratio of the speed of D to that of W is unaffected whatever the numbers of teeth in E and E' if these are equal. [§ 148 (*b*).]

D	represents the	front roller wheel . . .	60 teeth.
FR	,, ,,	front roller . . .	3·5 ins. diameter.
K	,, ,,	back shaft wheel . . .	18 teeth.
L	,, ,,	large double stud wheel	70 teeth.
H	,, ,,	small ,, ,, ,,	15 teeth.
g	,, ,,	intermediate wheel . . .	suitable number of teeth.
G	,, ,,	back roller wheel . . .	70 teeth.
BR	,, ,,	back roller . . .	3·5 ins. diameter.
F_1, F_2, F_3	represent ,,	fallers.	
S, S'	,, ,,	faller screws . . .	1 inch pitch.
N, N'	,, ,,	driving bevel wheels . . .	24 teeth.
O, O'	,, ,,	driven ,, ,, . . .	18 teeth.

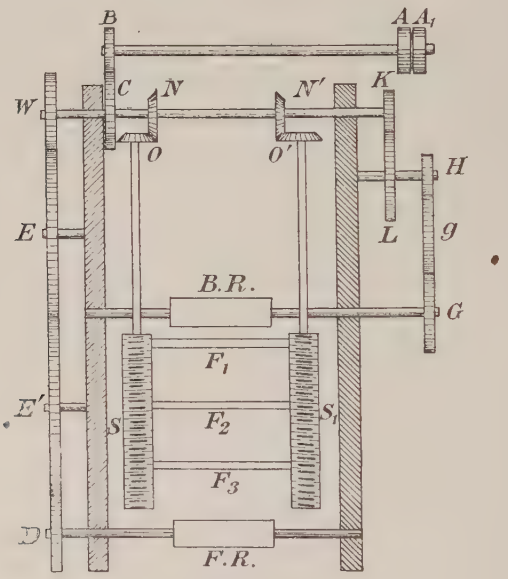


Fig. 149.

(*a*) *Back Draft.*

This is effected by making the surface speed of the fallers F_1, F_2, F_3 greater than the delivery or surface speed of the back roller.

Let us assume that the pulley A and hence B are making 100 revolutions per minute, but as we go along we shall notice that any alteration in the speed of A, B, or C has no effect on the draft.

(1) *To find the surface speed of the Fallers.*

Since the pitch of the screws S, S' is one inch, then each revolution of the screws carries the fallers F_1, F_2, F_3, and hence the sliver, forward 1 inch.

Now the drivers are B (42 teeth), bevel wheel N (24 teeth).
The followers are C (70 teeth), bevel wheel O (18 teeth).
Then

$$\frac{\text{Speed of } S}{\text{Speed of } B} = \frac{\text{Product of numbers of teeth on Drivers}}{\text{Product of numbers of teeth on Followers}},$$

or $$\frac{\text{Speed of } S}{100} = \frac{42 \times 24}{70 \times 18},$$

or $$\text{Speed of } S = \frac{42 \times 24 \times 100}{70 \times 18} \text{ revolutions per minute.}$$

$$\therefore \text{ Surface Speed of Fallers} = \frac{42 \times 24 \times 100 \times 1}{70 \times 18} \text{ ins. per min.}$$

[*Note.*—Don't work out at this stage.]

(2) *To find the Surface Speed of the Back Roller.*

The Drivers are B (42 teeth), K (18 teeth), stud wheel H (15 teeth).

The Followers are C (70 teeth), stud wheel L (70 teeth), back roller wheel G (70 teeth).

Then $$\frac{\text{Speed of } BR}{100} = \frac{42 \times 18 \times 15}{70 \times 70 \times 70} = \frac{\text{(Drivers)}}{\text{(Followers)}}$$

or $$\text{Speed of } BR = \frac{42 \times 18 \times 15 \times 100}{70 \times 70 \times 70} \text{ revs. per min.}$$

Since Diameter of back roller $= 3{\cdot}5$ inches,

$$\therefore \text{ Circumference of back roller} = \frac{22}{7} \times 3{\cdot}5 = 11 \text{ inches.}$$

Hence

$$\text{Delivery of back roller} = \frac{42 \times 18 \times 15 \times 100 \times 11}{70 \times 70 \times 70} \text{ inches per min.}$$

$$\therefore \text{ Back Draft} = \frac{\text{Surface Speed of Fallers}}{\text{Surface Speed of Back Roller}}$$

$$= \frac{42 \times 24 \times 100 \times 1}{70 \times 18} \div \frac{42 \times 18 \times 15 \times 100 \times 11}{70 \times 70 \times 70}$$

$$= \frac{42 \times 24 \times 100 \times 1}{70 \times 18} \times \frac{70 \times 70 \times 70}{42 \times 18 \times 15 \times 100 \times 11.}$$

[Notice here the quantities cancelling are 42 (teeth in B), 70 (teeth in C), 100 (revs. of pulley A), so that whatever the size of B and C and whatever the speed of A, the *draft is unaltered.*]

Thus Back Draft $$= \frac{24 \times 1 \times 70 \times 70}{18 \times 18 \times 15 \times 11} = 2{\cdot}2. \qquad \text{(A)}$$

(*b*) *Front Draft.*

This is caused by making the surface speed of the front roller greater than that of the fallers.

(1) *To find the Surface Speed of the Front Roller.*

The Drivers are B (42 teeth), change wheel W (x teeth).

The Followers are C (70 teeth), front roller wheel D (60 teeth).

Then $$\frac{\text{Speed of } FR}{100} = \frac{42 \times x}{70 \times 60} = \frac{\text{(Drivers)}}{\text{(Followers)}},$$

or $$\text{Speed of } FR = \frac{x \times 42 \times 100}{70 \times 60} \text{ revs. per minute.}$$

Since Diameter of front roller $= 3{\cdot}5$ inches.

$\therefore$ Circumference of front roller $= \frac{22}{7} \times 3{\cdot}5 = 11$ inches.

Hence

$$\left.\begin{matrix}\text{Delivery or Surface} \\ \text{Speed of } FR\end{matrix}\right\} = \frac{x \times 42 \times 100 \times 11}{70 \times 60} \text{ inches per min.}$$

Then $$\text{Front Draft} = \frac{\text{Surface Speed of Front Roller}}{\text{Surface Speed of Fallers}}$$

$$= \frac{x \times 42 \times 100 \times 11}{70 \times 60} \div \frac{42 \times 24 \times 100 \times 1}{70 \times 18}$$

$$= \frac{x \times 42 \times 100 \times 11}{70 \times 60} \times \frac{70 \times 18}{42 \times 24 \times 100 \times 1}.$$

[Notice again that the quantities cancelling are 42 (teeth in B), 70 (teeth in C), 100 (revs. of pulley A), so that the draft is unaltered by changing B and C or the speed of A.]

Thus $$\text{Front Draft} = \frac{x \times 11 \times 18}{60 \times 24 \times 1} = x \times \frac{11}{80}. \qquad \text{I}$$

Now x is the number of teeth in the *change or draft wheel* (W) and $\frac{11}{80}$ is a constant for this particular machine. The overlooker may thus find this Constant or *Gauge Point* for any machine under his charge and thus readily determine the change wheel required for a particular draft, or ascertain what draft any change wheel will give him.

Thus from I, Front Draft = (Change Wheel) $\times$ (Gauge Point)

or $$\text{Change Wheel} = \frac{\text{Front Draft}}{\text{Gauge Point}}.$$

In our example, if $x = 40$,

$$\text{Front Draft} = \frac{40 \times 11}{80} = 5{\cdot}5.$$

(c) *Whole Draft.*

(i) By this is meant the stretch to which the material is subjected from its passing through the back rollers to its leaving the front rollers. The student might easily assume that the whole draft is the sum of the back draft and the front draft. But this is not so. Let us see why.

Take our example :—

$$\text{Back Draft} = 2{\cdot}2.$$

This means that any length delivered by the back roller will be 2·2 times its length when delivered by the fallers to *FR*.

Also $$\text{Front Draft} = 5{\cdot}5.$$

This means that any length delivered by the fallers to the front rollers will be 5·5 times that length when delivered by the front roller. So that in passing from *BR* to *FR* the material is stretched to (2·2 × 5·5) times its length.

$$\therefore \text{ Whole Draft} = \text{Back Draft} \times \text{Front Draft}$$
$$= 2{\cdot}2 \times 5{\cdot}5$$
$$= 12{\cdot}1.$$

(ii) The whole draft, however, may be calculated at once by comparing the surface speed of the front rollers with the surface speed of the back rollers, thus :—

$$\text{Whole Draft} = \frac{\text{Surface speed of } FR}{\text{Surface speed of } BR}$$
$$= \frac{x \times 42 \times 100 \times 11}{70 \times 60} \div \frac{42 \times 18 \times 15 \times 100 \times 11}{70 \times 70 \times 70}$$
$$= \frac{x \times 42 \times 100 \times 11}{70 \times 60} \times \frac{70 \times 70 \times 70}{42 \times 18 \times 15 \times 100 \times 11}$$

[Notice our particulars with regard to *B* and *C* and the speed of *A* again cancel]

$$= \frac{x \times 11 \times 70 \times 70}{60 \times 18 \times 15 \times 11} = x \times 0{\cdot}3025.$$

Then *Gauge Point for Whole Draft* = 0·3025.

Since $x = 40$ in our example,

$$\text{Whole Draft} = 40 \times 0{\cdot}3025 = 12{\cdot}1 \text{ as before.}$$

To make an alteration in the whole draft, either the back draft or the front draft or even both may be altered. If there is to be an alteration in the back draft, then either the back shaft wheel *K* or the small stud wheel *H* is made the change wheel and the gauge point worked out.

Let us make H (15 teeth) the change wheel and substitute x for the 15 in the back draft result (A)

Then Back Draft $= \dfrac{24 \times 1 \times 70 \times 70}{18 \times 18 \times 11 \times x} = \dfrac{33}{x}$.

Then Gauge point for Back Draft = 33.

Note in this case. Change wheel (x teeth) $= \dfrac{\text{Gauge Point (33)}}{\text{Back Draft}}$.

154. *Worked Examples.*

EXAMPLE 1.

The diagram (Fig. 150) shows the draft gearing for an ordinary drawing box. Given the following particulars, find the gauge point for draft.

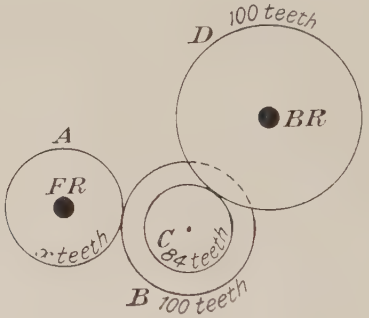


FIG. 150.

Front roller FR is 4 inches diameter.

Back roller BR is 2 inches diameter.

Draft or change wheel A on the front roller shaft has x teeth. This gears with stud wheel B (100 teeth), on whose shaft is the small stud wheel C (84 teeth) gearing with the back roller wheel D (100 teeth). The back roller is thus driven from the front roller.

Let us suppose that F roller is making K revolutions per minute. Then

Surface speed of front rollers $= \pi \times 4 \times K$ inches per minute

To find surface speed of Back Rollers.

The Drivers are A (x teeth), C (84 teeth).

The Followers are B (100 teeth), D (100 teeth).

Then $\dfrac{\text{Speed of } BR}{K} = \dfrac{x \times 84}{100 \times 100} = \dfrac{\text{(Drivers)}}{\text{(Followers)}}$;

or Speed of $BR = \dfrac{x \times 84 \times K}{100 \times 100}$ revs. per minute.

Since Diameter of $BR = 2$ inches, the Circumference of BR $\pi \times 2$ inches.

$$\therefore \text{ Surface speed of Back Rollers } = \frac{x \times 84 \times K \times \pi \times 2}{100 \times 100} \text{ inches per min.}$$

$$\therefore \quad \text{Draft} = \frac{\text{Surface speed of Front Rollers}}{\text{Surface speed of Back Rollers}}$$

$$= \frac{\pi \times 4 \times K}{\frac{x \times 84 \times K \times \pi \times 2}{100 \times 100}} = \frac{\pi \times 4 \times K \times 100 \times 100}{x \times 84 \times K \times \pi \times 2}.$$

Notice the quantities cancelling are K (revs. of front roller) and π. nce altering the speed of the front rollers does not alter the draft, l since π cancels we may use the diameters of BR and FR in place the circumferences.]

$$\text{Thus} \quad \text{Draft} = \frac{4 \times 100 \times 100}{x \times 84 \times 2} = \frac{238}{x}.$$

d Gauge point = 238.

Then suppose we desire a draft of 6.

$$6 = \frac{238}{x}.$$

Multiply by x and $6x = 238$,

$$x = \frac{238}{6} = 39{\cdot}6.$$

$\therefore$ Change wheel required must have 40 teeth.

Example 2.

The drawing roller of a spinning frame is 4 inches in diameter nd makes 36 revolutions per minute ; the cylinder driving the harl is 9 inches in diameter and makes 500 revolutions per inute, and the wharl is $1\frac{3}{4}$ inches diameter ; find the turns r inch in the yarn.

The amount of twist put into the yarn depends upon the fference between the speeds of the rollers and the spindles. o that if the roller delivers 3 inches of yarn whilst the spindles volve 45 times, the yarn will contain 15 turns per inch livered.

In our example Circumference of roller $= \pi \times 4$ inches. It akes 36 revolutions per minute.

$$\therefore \text{ Delivery or Surface Speed of Roller } = \pi \times 4 \times 36 \text{ inches per minute.}$$

on't work out at this stage.]

Again $\dfrac{\text{Speed of Spindle Wharl}}{\text{Speed of Driving Cylinder}} = \dfrac{\text{Diameter of Cylinder}}{\text{Diameter of Wharl}}$

or $\dfrac{\text{Speed of Spindles}}{500} = \dfrac{9}{1\frac{3}{4}}$

or $\text{Speed of Spindles} = \dfrac{9 \times 500 \times 4}{7}$ revs. per min

and these turns of the spindle are made whilst ($\pi \times 4 \times 3$ inches are delivered by the drawing roller.

$$\therefore \quad \text{Turns per inch} = \frac{9 \times 500 \times 4}{7} \div (\pi \times 4 \times 36)$$
$$= 5{\cdot}68.$$

Example 3.

Suppose it is required to spin 48's counts on a worsted spinni mule from a roving weighing 4 drams per 40 yards. What dra will be required ?

(*a*) Now 48's worsted counts means 48×560 yards weig 1 lb. or 256 drams.

Then Number of yards of yarn per dram $= \dfrac{48 \times 560}{256}$ (

$$= 105$$

and Number of yards of roving per dram $= \dfrac{40}{4}$ (I

$$= 10.$$

Then 10 yards of roving must be stretched to 105 yards,

$$\text{Draft} = \frac{105}{10} = 10{\cdot}5.$$

(*b*) If we do not multiply out the fractions (I) and (II) abo we may express the result thus :—

$$\text{Draft} = \frac{\text{Number of yards in 1 dram of yarn}}{\text{Number of yards in 1 dram of roving}}$$
$$= \frac{48 \times 560}{256} \div \frac{40}{4} = 48 \times 4 \times \left\{ \frac{560}{256 \times 40} \right\}$$
$$= \frac{48 \times 4}{18{\cdot}3} \quad \left[\text{since } \frac{560}{256 \times 40} = \frac{14}{256} = \frac{1}{18{\cdot}3} \right].$$

Now it is customary to express the weight of any roving so many drams per 40 yards.

Then if we require to spin x's counts worsted from a roving eighing y drams per 40 yards we get: x's worsted counts eans $(x \times 560)$ yards weigh 1 lb. or 256 drams.

Then Number of yards of yarn per dram $= \dfrac{x \times 560}{256}$

d Number of yards of roving per dram $= \dfrac{40}{y}$.

$\therefore$ Draft $= \dfrac{\text{Number of yards in 1 dram of yarn}}{\text{Number of yards in 1 dram of roving}}$

$$= \frac{x \times 560}{256} \div \frac{40}{y}$$

$$= x \times y \times \left\{ \frac{560}{256 \times 40} \right\} = \frac{x \times y}{18{\cdot}3}.$$

This constant 18·3 which occurs in all examples of this type termed the *Gauge point*, and you find in textile books this le: "To obtain from given weight of roving the draft to roduce required yarn: multiply counts (x) and weight of oving (y) and divide by gauge point (18·3)."

Example 4.

Find the weight of roving (per 40 yards) to give a 40's orsted counts with a draft of 8.

Suppose 40 yards of roving weigh x drams,

Then Number of yards of roving per dram $= \dfrac{40}{x}$.

Now 40's worsted counts means that (40×560) yards weigh lb. or 256 drams.

Then Number of yards of yarn per dram $= \dfrac{40 \times 560}{256}$

nd Draft $= \dfrac{\text{Number of yards in 1 dram of yarn}}{\text{Number of yards in 1 dram of roving}}$

$$= \frac{40 \times 560}{256} \div \frac{40}{x}$$

$$= 40 \times x \times \left\{ \frac{560}{256 \times 40} \right\} = \frac{40 \times x}{18{\cdot}3}.$$

[Note the Constant 18·3 occurs as in Example 3.]

But Draft $= 8$; then $\dfrac{40 \times x}{18{\cdot}3} = 8$.

Divide both sides by $\dfrac{40}{18{\cdot}3}$ and $x = \dfrac{8 \times 18{\cdot}3}{40} = 3{\cdot}66$.

$\therefore$ Weight of roving $= 3{\cdot}7$ drams per 40 yards.

The following is left as an exercise for the student :—

The diagram (Fig. 151) illustrates the driving of the front rollers a spindles in a spinning frame. Assume the following particulars: Main shaft, 100 revs. per min.; drum on main shaft, 40 inch diameter; driving pulley on frame and keyed to the cylinder sha 11 inches diameter; cylinder, 9 inches diameter; cylinder pull 9 inches diameter; twist-pulley, 15 inches diameter; twist-wh (change-wheel), 28 teeth; front roller wheel, 268 teeth; circumferen of front rollers, 9½ inches; diameter of wharl, 1 inch.

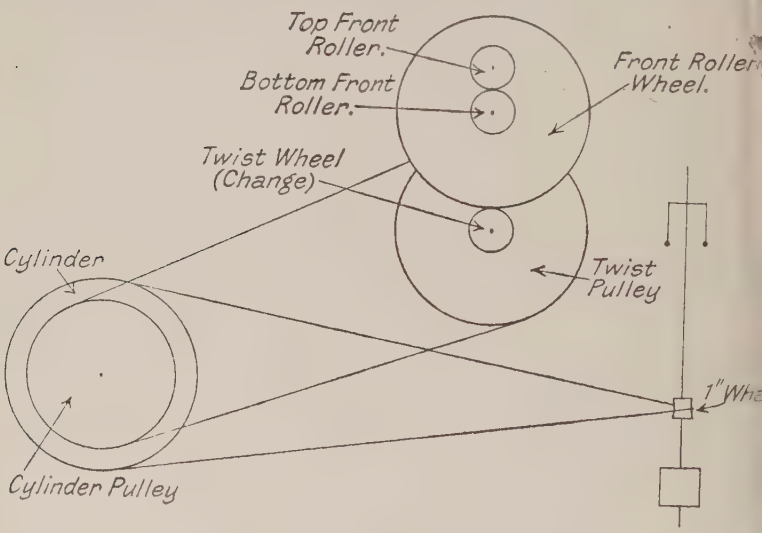


Fig. 151.

Calculate (*a*) delivery of rollers in inches per minute; (*b*) speed spindles in revs. per min.; (*c*) twist in turns per inch; (*d*) assume t the twist-wheel has x teeth and find the gauge-point for this particu machine

(Ans. :—(*a*) 238; (*b*) 3,600; (*c*) 15; (*d*) 420.)

CHAPTER XII.

Loci—Cams and Tappets.

155.—Loci. 156.—Cams. 157.—Harmonic Motion. 158.—Construction of Tappets. 159.—Movement of the Going Part of the Loom.

155. *Loci.*

In modern Geometry we speak of a moving point, as tracing or generating a line. Thus the point of a pencil moving across the paper leaves a mark which represents a geometrical line. The finer the pencil point the nearer the trace is to the true line.

The *Locus* of a point is the line traced out by it when it moves in accordance with some given law.

The line may be straight or curved according to the conditions under which the point moves.

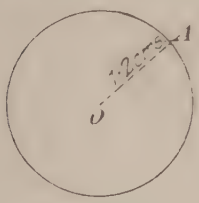


Fig. 152.

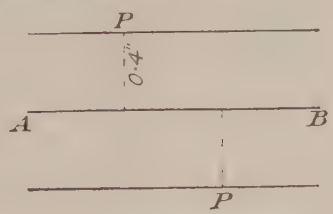


Fig. 153.

Example 1.

Suppose the point A (Fig. 152) moves so that its distance from a fixed point O is constant (1·2 cms. say). Then the locus of A is evidently the circumference of a circle whose centre is O and radius 1·2 cms.

Example 2.

Suppose the point P (Fig. 153) moves at a constant distance (0·4 inch say) from a fixed straight line AB. Then the locus of P is one or other of two straight lines parallel to AB one on each side, and at a distance of 0·4 inch from it.

Example 3.

To find the locus of a point P which moves so that its distances from two fixed points A and B are always equal to one another.

Take A and B (Fig. 154) any distance apart (say 2 inches). With centres A and B and radius 1·6 inches draw arcs above and below the line AB cutting at the points P_1. Then P_1 is 1·6 inches from A and 1·6 inches from B.

Now find points P_2, P_3, P_4, etc., whose distances from A and B are 1·8″, 2″, 2·4″, etc.

Notice all these points lie on *a straight line which bisects* AB *at right angles*. This then is the locus of P.

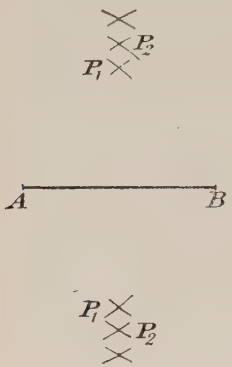


Fig. 154.

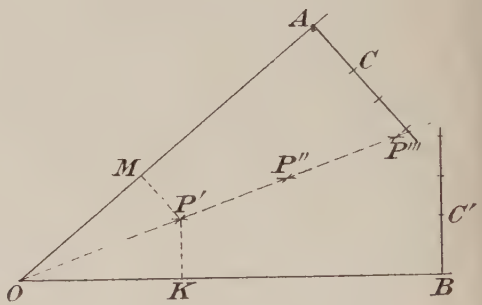


Fig. 155.

Example 4.

Show that the locus of a point P, which moves so that its perpendicular distances from the arms of an angle AOB are equal to one another, is the line which bisects the angle AOB.

Draw the angle AOB (Fig. 155) any size (40° say) with your protractor. From A and B draw perpendiculars to OA and OB respectively. Make $AC = BC' = 0{\cdot}3''$ say.

Through C draw line parallel to AO and through C' draw line parallel to BO, and these cut at P'; then P' is equidistant from OA and OB, *i.e.*, $P'M = P'K$.

In a similar manner find other positions as P'', P''', which in the figure are 0·5″ and 0·7″ from each arm respectively.

These points you should find lie on a straight line which passes through O and bisects the angle AOB. Measure the angles at O.

Example 5.

Make AB (Fig. 156) 5″ long. Construct right-angled triangles ABC_1, ABC_2, etc., with AB as hypotenuse and angles AC_1B, AC_2B, etc., right angles.

Thus make $B\hat{A}C_1 = 20°$; $A\hat{B}C_1 = 70°$; then $\hat{A}C_1B = 90°$.

Or make $B\hat{A}C_2 = 40°$; $A\hat{B}C_2 = 50°$; then $\hat{A}C_2B = 90°$.

Find 5 or 6 positions for C, satisfying the given condition that ACB must be a right angle.

Then if with centre O (mid-point of AB) and radius OC_1 you draw a semicircle it will pass through C_2, C_3, etc.

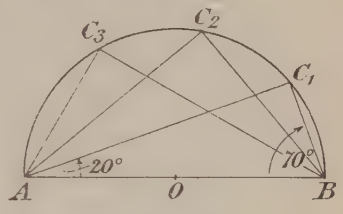


Fig. 156.

Hence "the locus of the vertices of right-angled triangles having the same hypotenuse is a semicircle with the given hypotenuse as diameter."

Example 6.

AB (Fig. 157) is 3 inches long. Find the locus of a point P which moves so that the sum of its distances from A and B is always 3·8 inches.

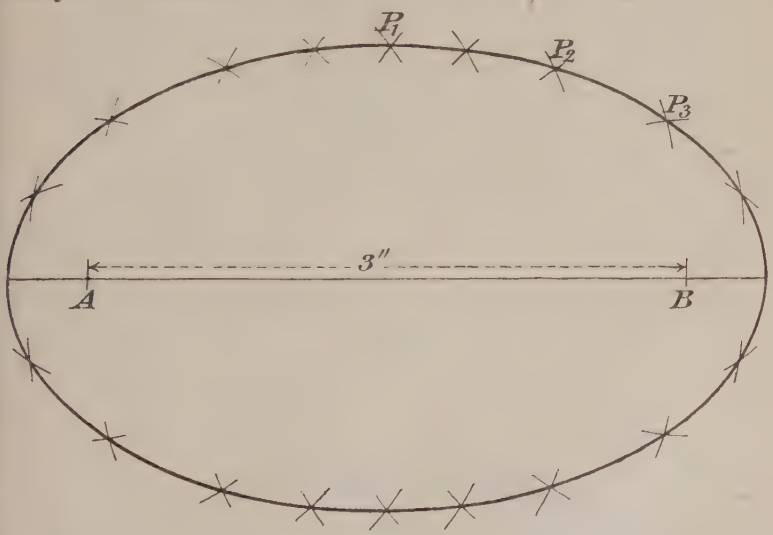


Fig. 157.

Choose values for PA and PB so that $PA + PB = 3{\cdot}8$ inches, thus:

Values for PA .	3·4	3·0	2·5	2·6	2·2	2·0	ins.
Values for PB .	0·4	0·8	1·3	1·2	1·6	1·8	ins.
Values for PA .	1·9	1·8	1·5	1·0	0·8	0·4	ins.
Values for PB .	1·9	2·0	2·3	2·8	3·0	3·4	ins.

As in the figure you will find that the locus of P is *an Ellipse*.

Example 7.

MX (Fig. 158) is a fixed line of infinite length. S is a point 1 inch distant from MX. Draw the locus of a point which moves in such a manner that its distance from MX is always equal to its distance from the fixed point S.

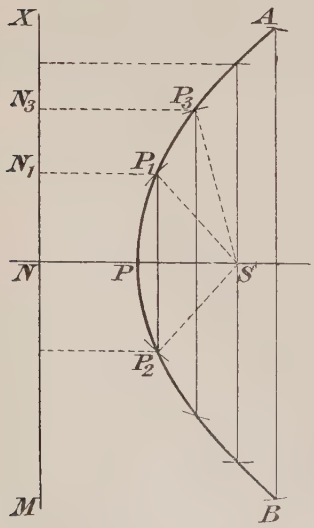


Fig. 158.

In the figure $PN = PS = 0{\cdot}5''$
$P_1N_1 = P_1S = 0{\cdot}6''$
$P_3N_3 = P_3S = 0{\cdot}8''$
etc.

The curve drawn through the points P, P_1, P_2, P_3, etc., is *a Parabola*.

156. *Cams and Tappet Shedding.*

A cam is a member of a machine so shaped that its motion imparts a definite motion to another member upon which it acts by direct contact or through a friction roller.

In seeking the motion of the lay pin of the going-part of the loom we are given the mechanism (crank and connecting rod). We have now the inverse problem, generally more difficult: given the required motion, to design the constraining mechanism.

Thus in one form of loom we have the treadles which are connected with the healds placed under the lower shaft. These require (1) to be raised and lowered a certain distance, to give a fixed depth of shed; (2) to be kept raised or lowered a certain length of time according to the pattern woven; and (3) the movement must be decidedly eccentric, moving slowly at first with a gradually increased speed to the centre of the motion and from that gradually decreasing.

How these conditions of motion are obtained will be made clear in the following work.

One form of cam is shown in Fig. 159. The cam-plate S rotates on a fixed centre C and so gives a reciprocating

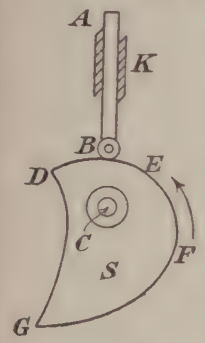


Fig. 159.

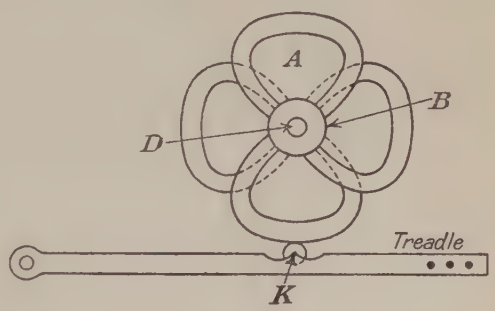


Fig. 160.

motion to the bar BA which slides in a fixed guide K. The roller B bears on the cam-plate and diminishes the friction.

Now let the cam rotate counter-clockwise with the roller at E and the slider in the lowest position.

Then during first half of the revolution, whilst the cam surface EFG makes contact with the roller, the slider will be raised to its highest position. During the third quarter the slider lowers to its initial position and remains there for the last quarter of the revolution.

Diagram (Fig. 160) shows one method of arranging tappets A on a sleeve B, which sleeve is placed over the end of the low shaft of the loom D.

Example 1.

To set out a cam so that a point reciprocated by it in a straight line will move as follows; whilst the cam is uniformly rotating 180°, uniformly upwards $1\frac{1}{2}$"; interval of rest for 30°;

uniformly down $\frac{1}{4}''$ for 60°, and the remaining $1\frac{1}{4}''$ for the remaining 90°. Diameter of roller $= \frac{1}{4}''$. Centre of cam is in a straight line with the slider and $\frac{1}{4}''$ from lowest point of the roller.

Method, see Fig. 161.

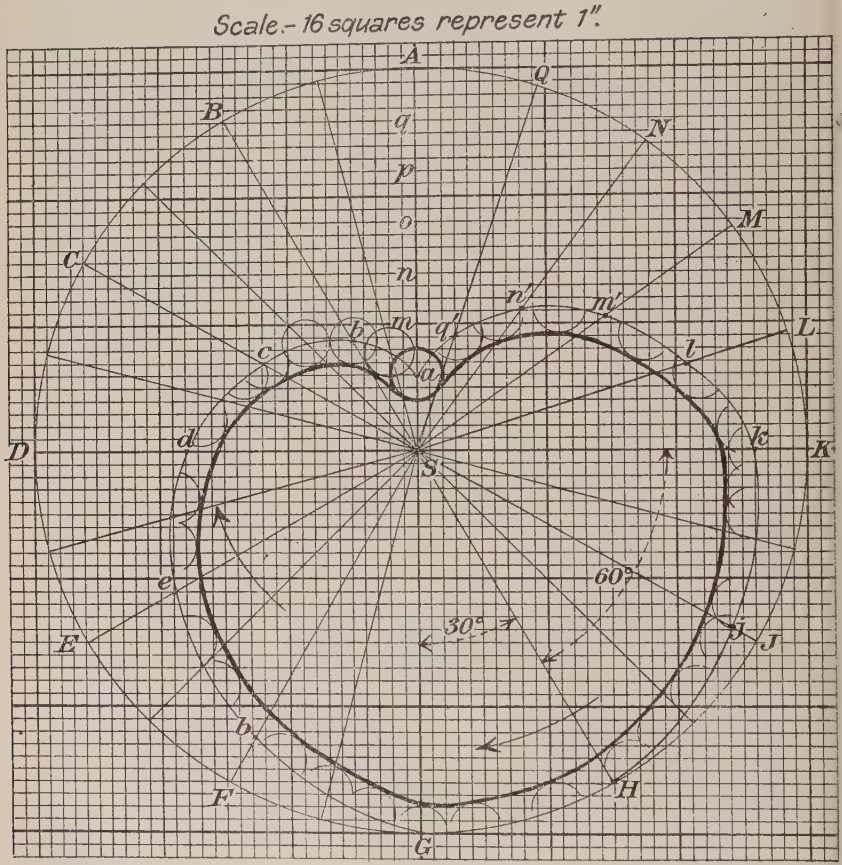


FIG. 161.

(*a*) *Upward Movement.*—Draw any straight line AG. Take any point S on AG as the centre of the cam. Mark (*a*) $\frac{3}{8}''$ from S. Then (*a*) is lowest position of the centre of the antifriction roller. Make $aA = 1\frac{1}{2}''$. Divide aA into any number of equal parts, say 6 or 12, at points m, n, o, p, q.

With centre S and radius SA describe a semicircle meeting AG at G. Divide the semicircumference into 6 equal arcs at B, C, D, E, F by making the angle ASB, etc., at centre = 30°.

Now with centre S and radii Sm, Sn, So, etc., describe arcs of circles cutting SB, SC, etc., respectively at b, c, d, e and f. These are the positions of the centre of the roller to give the upward movement of the straight piece.

Draw an even curve through a, b, c, d, e, f and G, and with these as centres and other points on the curve draw circles radius $\frac{1}{8}''$. The curve tangential to these circles represents the edge of the cam-plate.

(*b*) *Downward movement.*—For the dwell, make angle $GSH = 30°$. Then (*a*) moves on circumference of circle with radius SA or SG. Make

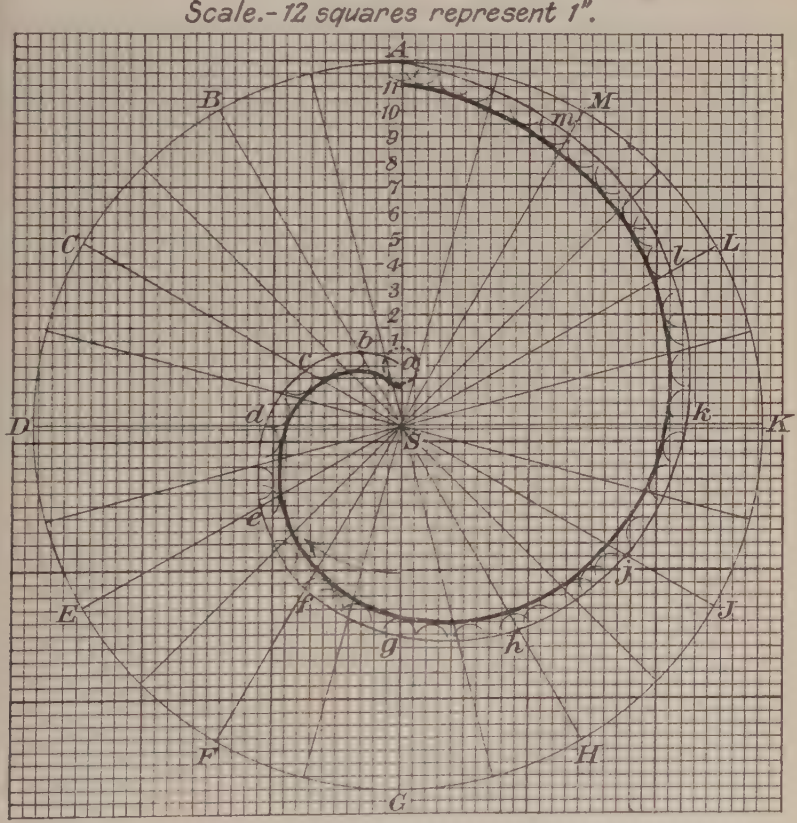


FIG. 162.

angle $HSK = 60°$. Divide $Aq = \frac{1}{4}''$ into 4 equal parts and also arc HK, and as before find positions for (*a*) during that period of rotation. For the remainder of downward movement, divide arc KA into 5 equal parts, and with centre S and radii Sq, Sp, So, etc., find positions for (*a*). Draw the curve through these points plotted and as before find the curve representing the cam-plate edge.

Example 2.

Design a cam which shall give a rise of 2 inches at a uniform speed during the entire revolution, with an instantaneous drop at the end. Diameter of roller $\frac{1}{4}$ inch; least distance of its centre from the axis of the cam to be $\frac{1}{4}$ inch.

Method, see Fig. 162.

Draw any straight line AG. Take a point S on AG as the centre of the cam. Mark (a) $\frac{3}{8}''$ from S. Then (a) is the lowest position of the centre of the roller. Make $aA = 2''$. Divide aA into 12 or 24 equal parts. With centre S and radius SA describe a circle. Divide the circumference into 12 equal arcs by making angles ASB, BSC, CSD, etc., equal to 30°. Now with centre S and radii $S1$, $S2$, $S3$, etc., cut the lines SB, SC, SD at b, c, d, etc. These points mark the positions taken up by (a), the centre of the roller, every succeeding $\frac{1}{12}$th of a revolution.

Draw a freehand curve through b, c, d, e, f, etc., and with these points as centres and also any other points on the curve, draw semicircles radius $\frac{1}{8}$ inch.

The curve tangential to these circles represents the edge of the cam-plate. Since there is an instantaneous drop at the end of the revolution, then the edge of the cam-plate is drawn straight along the line AG.

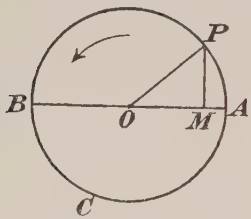


Fig. 163.

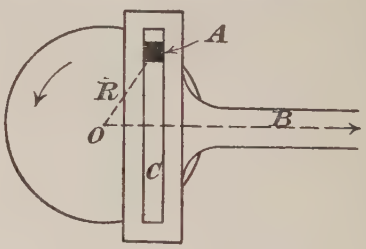


Fig. 164.

157. *Simple Harmonic Motion.*

Suppose a point P (Fig. 163) to move at a constant speed in a circular path, and let M be its projection on a fixed diameter AB.

Then as P moves on circumference APB, M moves from A to B, and as P moves on circumference BCA, M moves from B to A.

M is said to execute a simple harmonic motion or a simple vibration. Length of radius of circle, *i.e.*, half the travel, is called the *Amplitude.*

A practical embodiment occurs in the mechanism of a donkey-pump or of a small slotting or shaping machine. In diagram (Fig. 164) the crank (R) carries a block (A) which works in a slotted link (C) guided so as to reciprocate parallel to OB as the crank rotates.

Motion of M *as* P *rotates* (see Fig. 165). Draw a circle with radius 3 inches. Let P travel counter-clockwise from A at a uniform speed. Let unit of time be that taken for M or P to travel from A to B. Then in $\frac{1}{10}$ unit of time Angle $P_1OA = 18°$.

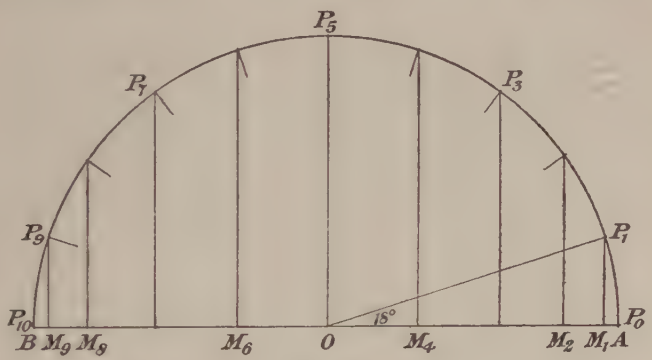


FIG. 165.

Find position of P after each $\frac{1}{10}$ unit of time and corresponding positions of M. Measure AM in each case.

Time in fractions of unit.	Displacement of $M = AM = s$ in inches.	ds inches.	dt	$\frac{ds}{dt}$ = Average velocity during time $\frac{1}{2}\,dt$ in inches per unit of time at middle of each interval.
0	0			
		0·16	0·1	1·6
0·1	0·16			
		0·43	0·1	4·3
0·2	0·59			
		0·67	0·1	6·7
0·3	1·26			
		0·83	0·1	8·3
0·4	2·09			
		0·91	0·1	9·1
0·5	3·0			
		0·91	0·1	9·1
0·6	3·91			
		0·83	0·1	8·3
0·7	4·74			
		0·67	0·1	6·7
0·8	5·41			
		0·43	0·1	4·3
0·9	5·84			
		0·16	0·1	1·6
1·0	6·0			

Now plot on squared paper :

(1) Times—horizontally,
(2) Displacements (AM_1, AM_2, etc.)—vertically,

and thus obtain a *Displacement-Time Graph* (see Fig. 166).

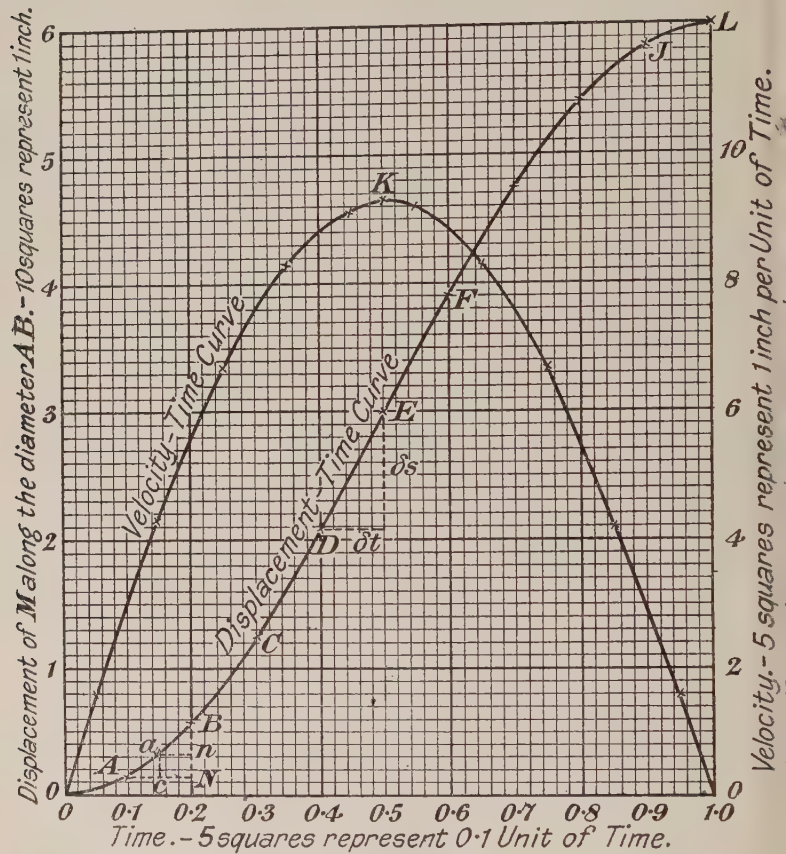


FIG. 166.

As P travels round the circumference of the circle from P_0 to P_1 in $\frac{1}{10}$ unit of time, A travels along the diameter to M_1, a distance of 0·16″. If the speed were uniform, then M's speed would be $\frac{0{\cdot}16}{0{\cdot}1}$, or 1·6 *inches per unit of time*. But the freehand curve through the points plotted shows that distance travelled in 0·05 unit of time is 0·05 inch, giving an average speed of $\frac{0{\cdot}05}{0{\cdot}05} = 1$ *inch per unit of time*.

In the next 0·05 unit of time distance travelled is (0·16 — 0·05) = 0·11 inch, and average speed during that interval $= \frac{0{\cdot}11}{0{\cdot}05} = 2{\cdot}2$ *inches per unit of time*, Thus the *speed is increasing* as M moves to M_1, and since its speed is constantly changing, we can only state the *average speed of M* during the first 0·1 unit of time is $\frac{0{\cdot}16}{0{\cdot}1}$ or 1·6 inches per unit of time, and we represent it by $\frac{ds}{dt}$ which means

$$\frac{\text{A small increment of distance travelled}}{\text{The small increment of time taken}}.$$

The *speed of M* is *less than* this *average* during the first half of the interval and *greater than* this *average* during the second half. We therefore assume that at the middle of the interval the velocity is 1·6 [see column 5 in Table, p. 367].

Now examine the path of the curve marked *AB*, Fig. 166. The distance travelled in the second interval of time (0·1) is shown by *BN* and is 0·43 inch, and average velocity during that interval $= \frac{0{\cdot}43}{0{\cdot}1} = 4{\cdot}3$ inches per unit of time. But the velocity of M during this interval has not been the same throughout the interval, for during the first half the distance travelled (ac) = 0·17″, and during the second half the distance travelled (Bn) = 0·26″.

Thus average speed in first half of this time interval $= \frac{0{\cdot}17}{0{\cdot}05} = 3{\cdot}4''$ per time unit, and average speed in second half of this time interval $= \frac{0{\cdot}26}{0{\cdot}05} = 5{\cdot}2''$ per time unit. And thus again we assume that at the middle of this interval the velocity is 4·3.

These *Average Velocities* for the 10 intervals of time in our example are given in column 5 of the Table (p. 367).

On the same squared paper we plot :

(1) Times—horizontally,

(2) Velocity at mid-intervals —vertically,

and thus obtain a *Velocity-Time Graph* (see Fig. 166).

Examining the two curves we notice :—

(1) From the displacement-time curve, that the displacements are greatest in the parts marked *DE*, *EF*, *i.e.*, during the fourth and fifth intervals, and the displacements are least during the first and last intervals, marked *OA*, *JL*.

(2) From the velocity-time curve, that the velocity of M gradually increases to a maximum of 9·3 inches per unit of time when M is at O, the mid-point of *AB*, and then gradually decreases, becoming zero again when M is at B.

This kind of motion then is very suitable for the opening and closing of the shed, so that shedding may commence slowly, increase in speed to the middle of the traverse of the healds, and then diminish to the end of the traverse.

158. *Construction of Tappets.*

There are two important factors in connection with thi construction of a tappet which require consideration, viz.. (1) the healds must rise and fall with an eccentric motion and we have seen that harmonic motion is satisfactory (2) the time to be allowed during one revolution of thi crank-shaft for the change and dwell of the healds.

The length of dwell will depend upon the width and speed o the loom and the character of the material, etc. The studen will have all these details explained in his lessons on th power loom or by reading some such work as "Woolle and Worsted," by Prof. Roberts Beaumont.

Thus we may have $\frac{1}{2}$ of a revolution of crank-shaft (or of a pick) allowed for "dwell" and $\frac{1}{2}$ of a pick for "change," *or* $\frac{2}{3}$ pick allowed for "change" and $\frac{1}{3}$ pick for "dwell."

Example 1.

As an example of the simplest form, let us draw a tappe to weave a plain cloth. In this there are only two picks i the complete pattern, so that the tappet must be so constructe as to raise and depress the treadle alternately for each succeed ing pick. Also one revolution of the tappet-shaft correspond to two revolutions of the crank-shaft.

Let us assume that the diameter of the inner circle of th tappet is 6 inches; throw of the tappet is 3 inches; diamet of treadle pulley 3 inches.

Suppose we are dealing with "open" shedding and tha the warp threads moved by the treadle are in their lowes position.

Suppose the time of "dwell" is $\frac{1}{2}$ pick and the time of chang $\frac{1}{2}$ pick.

Method, see Fig. 167.

Draw a straight line AC. Take a point S in AC as the centre of th tappet. With centre S and radius 3 inches describe the circle xy Make $Sa = 4\frac{1}{2}$ inches, then a is the highest position of the centre of th treadle pulley. Make $aA = 3$ inches. Then A is the lowest positio of the centre of the treadle pulley. With centre S and radius S describe a circle. Since the tappet revolves once in 2 picks of the loon then in $\frac{1}{2}$ pick it turns through an angle of 90° ASB. During $\frac{1}{2}$ pic the centre of the treadle bowl moves from a to A and the motion harmonic. Divide aA harmonically as we divided AB in Fig. 165 b making angle $AdP_1 = 30°$, $AdP_2 = 60°$, etc., and drawing perper diculars P_1f, P_2e, etc., to Aa. Then aA is divided at b, c, d, e, f (th student may make 10 or 12 divisions along aA if he likes). Divide th arc AB into six equal arcs at E, F, G, H, K by making angles ASE ASF, etc., equal to 15°, 30°, etc The tappet turns through the ang

ASE in $\frac{1}{6}$th of $\frac{1}{2}$ pick and a moves a distance ab. Then with centre S and radius Sb cut SE at e'.

In two-sixths of $\frac{1}{2}$ a pick a moves to c. Then with centre S and radius Sc cut SF at f'. Similarly with centre S and radii Sd, Se, Sf cut SG, SH, SK at g, h and k. Draw a freehand curve through the points e', f', g, h, k, and B. This represents the path of the centre of the treadle bowl during the upward motion of the heald.

During the "dwell" the centre of the treadle bowl remains at its lowest point, *i.e.*, $7\frac{1}{2}$ inches from S; hence for the next quarter of a

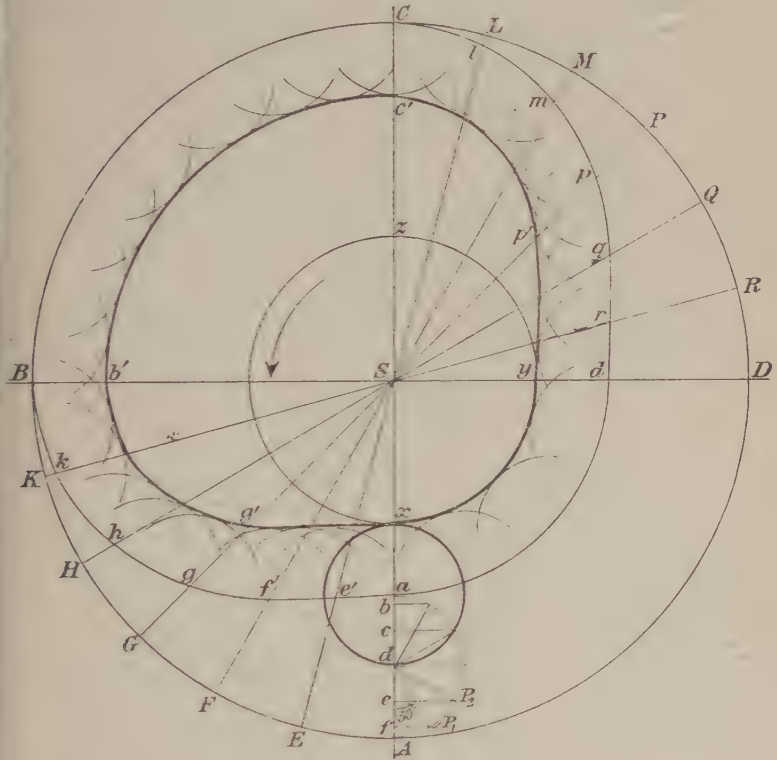


FIG. 167.

revolution of the cam its path is along the arc BC of the circle ABC. During the next quarter of a revolution the heald lowers, and dividing the arc CD into 6 equal parts at L, M, P, Q, and R, with centre S and radii Sf, Se, Sd, Sc, Sb, Sa, we cut SL, SM, SP, etc., at the points l, m, p, etc., and draw an even curve through C, l, m, p, q, r, d'. During the next $\frac{1}{2}$ pick the centre of the treadle pulley is at its highest position, *i.e.*, $4\frac{1}{2}$ inches from S, and its path is along the arc $d'a$ of a circle with centre S and radius Sa. Take any number of points on this curve thus drawn as centres and with radius $1\frac{1}{2}$ inches draw semicircles or

circles if you like, and *the curve tangential to these circles represents the edge of the tappet*

A similar tappet for the other treadle will be keyed to the shaft, but 180° (one pick) behind that on the first treadle. When the crank of the loom is 30° to 45° behind its highest position and approaching the fell of the cloth, the warp threads moved by the first treadle are in the lowest position and the point x of the cam-plate is in contact with the treadle pulley. One-eighth or one-twelfth of a revolution of crank before the "beat-up" ($\frac{1}{4}$ pick later) the point g' of tappet is in contact with the pulley and the shed is closed. During the next $\frac{1}{4}$ pick the crank travels to within 30° or 45° of its lowest position; the edge $g'b'$ presses against the pulley and the warp threads are carried to their highest position. They remain in that position whilst the shuttle crosses and the crank travels to within 30° or 45° of its top position and edge $b'c'$ of the tappet makes contact with the roller. Next $\frac{1}{4}$ pick the edge $c'p'$ presses against the pulley, the threads lower, and the shed is closed again just before the "beat-up." Next $\frac{1}{4}$ pick the crank travels again to within 30° or 45° of its lowest position; edge $p'y$ of tappet presses against the pulley and the warp threads reach their lowest position and remain there for the next $\frac{1}{2}$ pick, the edge yx of the tappet making contact with the treadle pulley.

So that the warp threads worked by the first treadle have been raised and lowered once in two revolutions of the crank or whilst the shuttle has picked twice. The tappet over the second treadle causes the other set of warp threads to work alternately.

Example 2.

Construct a tappet for Fig. 168. This twill occupies eight shafts and there are eight picks to one repeat of the pattern. If the changes of the first thread in the weave are analysed we find it is *up* on the first pick; *down* on the second pick; *up* on the third, fourth, and fifth picks; *down* on the last three picks.

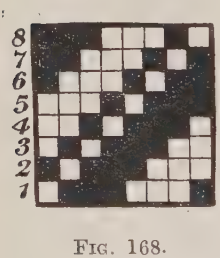


Fig. 168.

Let us assume that the diameter of the inner circle of the tappet is 6 inches; throw of the tappet is 3 inches; diameter of treadle pulley 2 inches. Allow $\frac{1}{2}$ pick for time of "dwell" and $\frac{1}{2}$ pick for time of "change."

Method, see Fig. 169.

Draw a line KA. Take any point S in KA to represent the centre of the tappet. With centre S and radius 3 inches draw the inner circle XYZ. Along SA measure Sa 4″. Then a marks the highest position of the centre of the treadle pulley. Make SA 7 inches. Then A marks the lowest position of the centre of the treadle pulley. With centre S and radius SA describe a circle AKH.

Since the tappet revolves once every 8 picks, then in $\frac{1}{2}$ pick, or as the

crank moves from its highest to its lowest position or *vice versa*, the tappet will revolve through an angle of 22½°.

Then make angles *ASB*, *BSC*, *CSD* equal to 22½°.

Divide *aA* harmonically at *b*, *c*, *d*, *e*, *f* as before. Divide the arcs *AB* and *CD* into 6 equal parts and with centre *S* and radii *Sa*, *Sb*, *Sc*, etc., mark the points 1, 2, 3, 4, etc. Draw freehand curves evenly joining the points *a*, 1, 2 . . . *B* and *C*, 5, 4 . . . *a'*. With any points on this curve *aBCa'* as centre and radius 1 inch describe semicircles, and the curve *XB'C'D'* tangential to the circles marks the *edge of the cam-plate*, which as it makes contact with the treadle pulley will (1) raise the warp threads worked by this treadle from their lowest position to

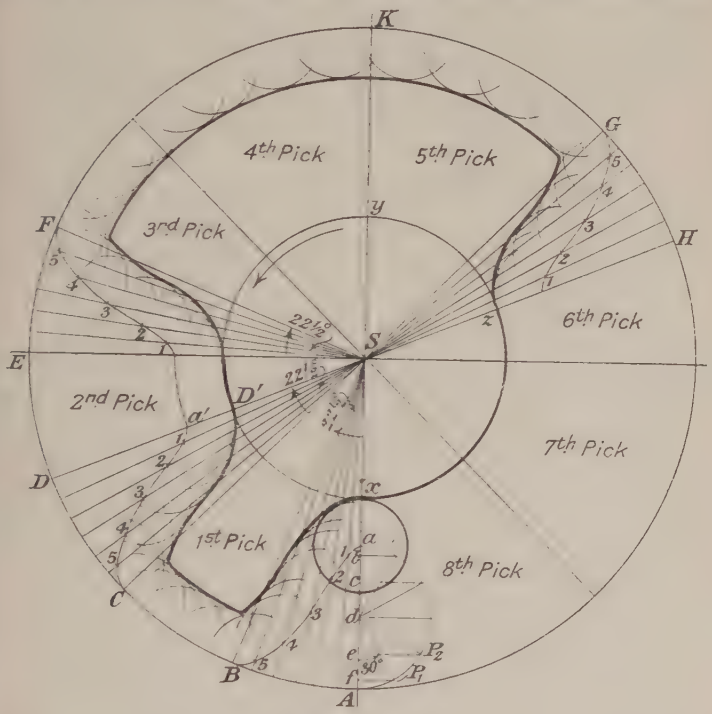


Fig. 169.

their highest position during ½ pick, (2) keep them in their highest position for ½ pick, and (3) lower them in the next ½ pick.

They remain depressed whilst the cam rotates through the angle *DSE* (22½°) in the next ½ pick. The angle *ESF* is 22½° and represents a period when the threads are raised. They remain raised whilst the tappet revolves through the angle *FSG* (112½°). Angle *GSH* (22½°) represents the period of depression, and they remain depressed for the last 3 picks (angle *HSA* = 112½°).

As every thread in this weave is like the first, only commencing on different picks, the same shape of tappet would be

used for all the eight treadles. To obtain the twilled effect the tappets would be keyed on to the tappet shaft, each one section in advance of the other.

Example 3.

Sketch the tappet for weaving a twill $\frac{3}{2}\frac{1}{2}$. This occupies eight shafts, and there are 8 picks to one repeat of the pattern.

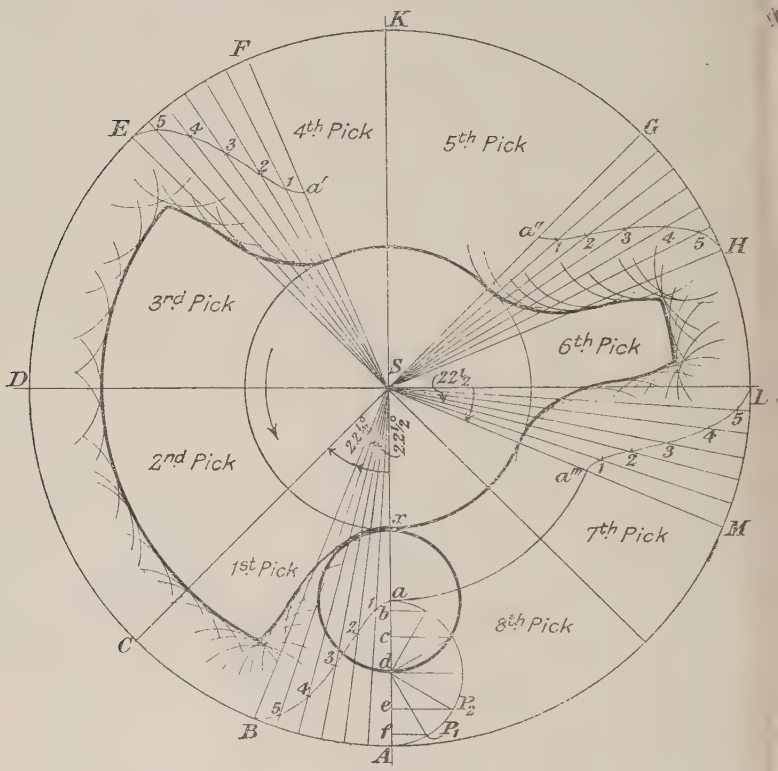


Fig. 170.

The first heald changes in the following manner: *up* for first three picks; *down* on the fourth and fifth picks; *up* on the sixth pick; *down* on the last two picks.

Let us assume as before that the diameter of the inner circle of the tappet is 6 inches; diameter of treadle pulley 3 inches; stroke of the tappet 3 inches.

The method of construction is similar to that in Example 2, and we leave the student to follow it from the diagram (Fig. 170) in which all the construction lines are shown.

The student must make these drawings for himself; a mere study of the diagrams never gives a perfect grasp of the method of construction.

159. *Movement of the Going Part of the Loom.*

The "going part" of the loom serves two purposes: (1) it beats up the weft when in its front position; (2) it provides the medium, when in its back position, on which the shuttle may pass from box to box.

Below is a hand sketch (Fig. 171) to show the method of actuating the going part.

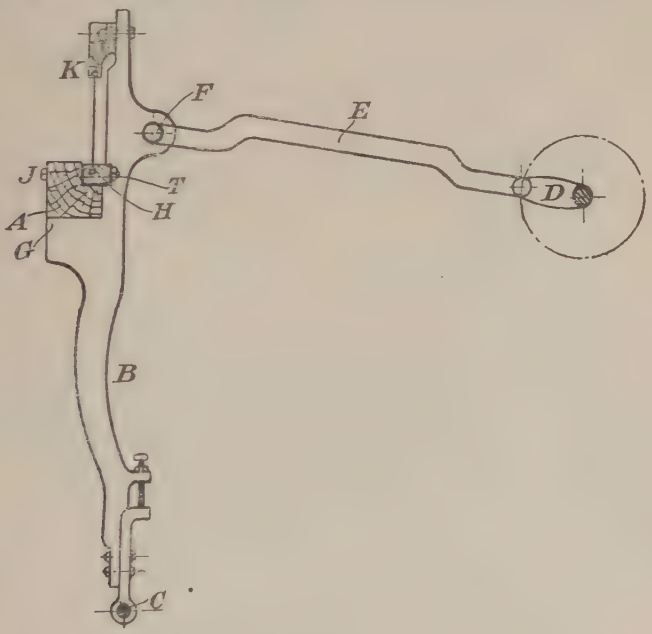


FIG. 171.

The lay (*A*) is a beam of wood. This is supported on the lay-sword (*B*) usually provided with a projecting arm to which the lay is firmly bolted. The lay-sword is centred on the rocking-shaft (*C*). Motion is conveyed to the going part from the crank (*D*) by means of the connecting arm (*E*), this being connected to the crank at one end and to the connecting pin (*F*) at the other end.

Now the motion communicated to the driving or crank shaft of the loom is a continuous circular motion. The lay (A) moves backwards and forwards along what is approximately a straight line (really a small arc of a circle with centre C). This rectilinear motion must not be regular.

The going part carries the shuttle as well as the reed, therefore its movement must be so arranged as to give as much time as possible of comparative rest at the period when the shuttle is passing across the loom, whilst at the other extremity of its movement, when the weft is beaten up to the fabric, it must be moving quickly so as to deliver a smart quick blow.

We will now examine the conversion of motion produced by means of a crank and connecting rod.

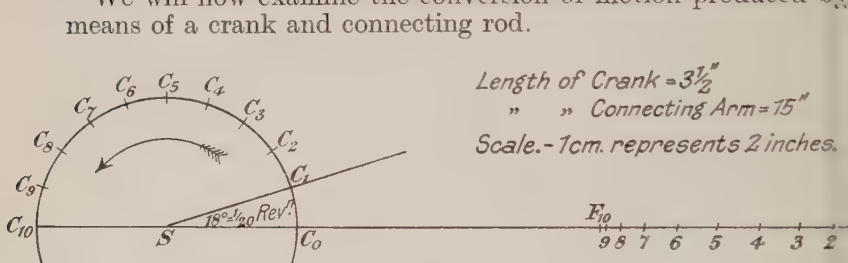


Fig. 172.

In fast-running looms the length of the crank is from 2½ to 3 inches, giving a stroke to the going part of from 5 to 6 inches. The length of the connecting arm is from 12 to 15 inches.

In broad looms, slow running, we find a large crank 4 inches, say, and a short connecting arm about 9 inches long.

Let us examine the motion of the connecting pin (F) in the two cases :—

Case I. Assume a *short crank* 3½ inches long and a *long connecting arm* 15 inches.

Let the motion of the crank be counter-clockwise, and suppose the time for half a revolution is the unit (1 second). Draw a line C_{10}, F (Fig. 172). Take a point S on this line as centre, and draw a circle radius 3½ inches cutting the line at C_0 and C_{10}. Make angles C_0SC_1, C_1SC_2, etc., 18°. Then C_0 represents position of the crank-pin *at the inner dead centre* or " beat-up " ; C_1, C_2, C_3, etc., represent the position of the crank-pin at intervals of $\frac{1}{10}$ second, whilst C_{10} represents the position of the crank-pin at *the outer dead centre* (shuttle crossing or " dwell ").

With C_0, C_1, C_2 . . . C_{10} as centres and radius 15 inches describe arcs cutting the line C_{10}, C_0 at points F_0, F_1, F_2 . . F_{10}. The

F_9, F_1, F_2 . . . F_{10} represent the positions of the lay-pin at intervals of $\frac{1}{10}$ second.

Now measure the distances F_0F_1; F_0F_2; F_0F_3 . . . F_0F_{10}, and these are the distances travelled in 0·1, 0·2, 0·3 . . . 1 second from the start.

Tabulate your results thus :—

Distance travelled by "lay-pin" (F) = s inches from the start.	Time taken = t secs.	Distance travelled between each successive interval of 0·1 sec. ds.	Small increase in time dt.	$\frac{ds}{dt}$ = Average velocity during time dt in inches per sec. *at middle of each small interval* of time.
inches.	secs.	inches.	secs.	inches per sec.
0	0			
		0·2	0·1	$\frac{0·2}{0·1} = 2.$
0·2	0·1			
		0·6	0·1	$\frac{0·6}{0·1} = 6$
0·8	0·2			
		0·88	0·1	$\frac{0·88}{0·1} = 8·8$
1·68	0·3			
		1·07	0·1	$\frac{1·07}{0·1} = 10·7$
2·75	0·4			
		1·11	0·1	$\frac{1·11}{0·1} = 11·1$
3·86	0·5			
		1·06	0·1	$\frac{1·06}{0·1} = 10·6$
4·92	0·6			
		0·88	0·1	$\frac{0·88}{0·1} = 8·8$
5·8	0·7			
		0·62	0·1	$\frac{0·62}{0·1} = 6·2$
6·42	0·8			
		0·43	0·1	$\frac{0·43}{0·1} = 4·3$
6·85	0·9			
		0·15	0·1	$\frac{0·15}{0·1} = 1·5$
7·0	1·0			

As the crank-pin completes its revolution moving on the other semicircumference from the outer dead centre back to the inner dead centre, the lay-pin will travel from F_{10} to F_9,

F_8, etc., to F_0 during each $\frac{1}{10}$ second, and the displacements will be 6·85″, 6·42″, etc., to 0.

(*a*) Examining the table, we find that in the first quarter revolution of the crank the lay-pin travels 3·86 inches in 0·5 second, giving an *average velocity* of $\frac{3{\cdot}86}{0{\cdot}5}$ or 7·72 inches per second. During the second quarter of a revolution of the crank the lay-pin travels 3·14 inches in 0·5 second, giving an *average velocity* of $\frac{3{\cdot}14}{0{\cdot}5}$ or 6·28 inches per second.

The average velocities of the lay-pin for the fourth and third quarters of a revolution of the crank will be those for the first and second quarters respectively. This gives a greater velocity at the moment of beating up and a less time of dwell than at the moment when the shuttle is crossing.

(*b*) Let us look at the results in another way.

Plot on squared paper :

Times—horizontally,

Distances (*s*)—vertically,

and obtain a Displacement-Time curve (*OAD* in Fig. 173). We notice :—

(1) The freehand curve through the points plotted will give us the distance travelled by the lay-pin at any interval of time from the start. Thus at *the point P* distance travelled = 0·2 inch in 0·1 second from the start. If this speed (2 inches per second) were uniform we should have a distance of 0·1 inch travelled in 0·05 second from start.

But the curve (at the point *Q*) shows a distance of 0·08 inch in 0·05 second or an average speed of $\frac{0{\cdot}08}{0{\cdot}05}$ or 1·6 inches per second, and this is less than the average speed for the first $\frac{1}{10}$ second.

In the second 0·05 sec. (see curve from *Q* to *P*) distance travelled (Pa) = 0·12 inch and average speed during this interval = $\frac{0{\cdot}12}{0{\cdot}5} = 2{\cdot}4$ inches per second, and this is greater than the average speed for the first $\frac{1}{10}$ second.

Thus the speed of the lay-pin is increasing as it travels from F_0 to F_1, and we can only speak of the *average speed* of the lay-pin during the first $\frac{1}{10}$ second.

Also, since its speed throughout the first half of this interval is *less than* the average for the whole interval, and *greater than the average* during the second half, we assume that at the middle of the interval the speed is 2 inches per second. [See column 5 in Table, p. 377, $\frac{ds}{dt} = 2$.]

(2) Examine the portion *AR* of the curve.

Distance travelled by lay-pin in 0·1 second = RN = 1·07 inch, giving an average speed of $\frac{1{\cdot}07}{0{\cdot}1} = 10{\cdot}7$ inches per second.

During *first half of this interval* (0·1 second) distance travelled $= Sn = 0{\cdot}47$ inch, and average speed $= \frac{0{\cdot}47}{0{\cdot}05} = 9{\cdot}4$ inches per second.

During the *second half of this interval* distance travelled $= Rm = 0{\cdot}6$ inch, and average speed $= \frac{0{\cdot}6}{0{\cdot}05} = 12$ inches per second.

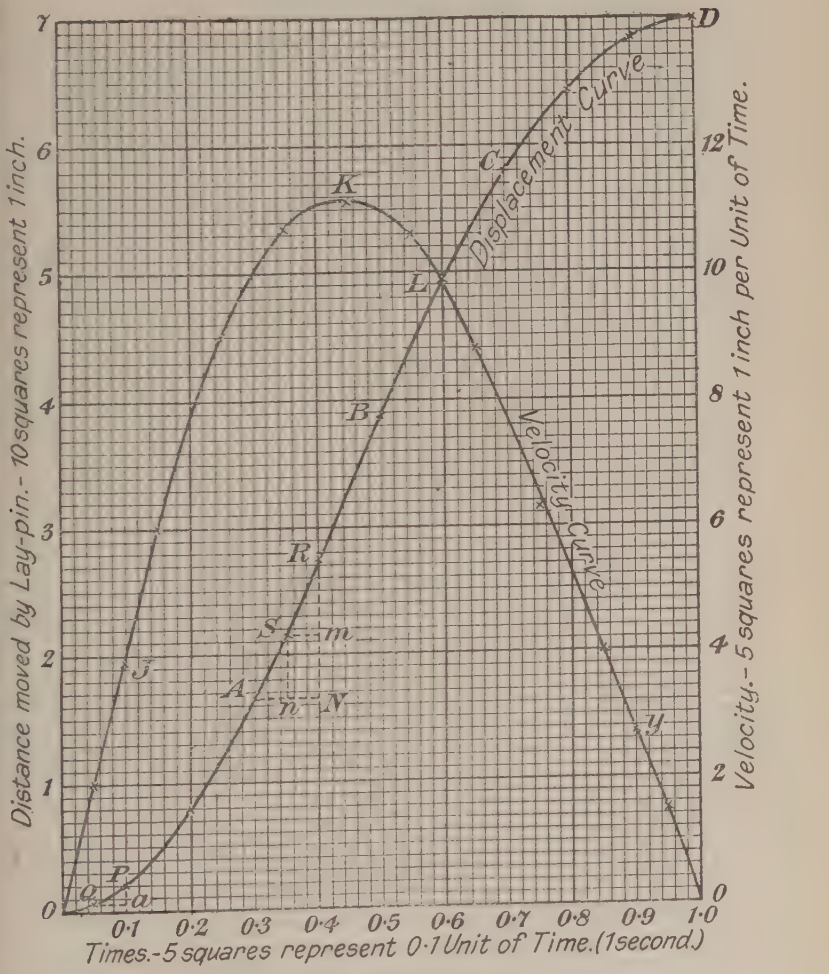


FIG. 173.

And thus again we assume that at the middle of this interval the velocity of the lay-pin = 10·7 inches per second $\left(\frac{ds}{dt}\right)$.

(3) The curve is steepest between R and L, becoming less steep as it passes through C to D.

(*c*) Now plot the *average velocities* $\frac{ds}{dt}$ (see Table of results, column 5) vertically at the *middle* of each interval of 0·1 second, and obtain a *Velocity-Time* Graph.

Examining this curve we notice :—

(1) When the crank-pin is at the inner dead centre the velocity of the lay-pin is zero.

(2) As the crank-pin moves from its inner dead centre to its outer dead centre the velocity of the lay-pin gradually increases till it attains a maximum of 11·1 inches per second at K and then gradually decreases till it again becomes zero when the crank-pin is at its outer dead centre.

(3) The maximum speed of the lay-pin is attained in 0·45 second, *i.e.*, in less than half the time for half a revolution of the crank.

(4) In the first tenth of a second whilst the crank turns through 18° the lay-pin has acquired a velocity of 3·9 inches per second (shown at J), but during the last tenth of a second, whilst the crank turns 18°, the velocity of the lay-pin decreases from 2·7 inches per sec. to zero (shown at Y).

Thus the lay-pin is travelling slower as the crank approaches or leaves the outer dead centre than it is as the crank approaches or leaves the inner dead centre.

(*d*) In Fig. 174 we have drawn the Displacement and Velocity Time Curves for *two complete revolutions of the crank.*

Examine the Displacement Curve and we notice :—

(1) The curve at 2·0 (indicating the time of the "beat-up") is sharper than at A (time of the "dwell" or shuttle crossing).

(2) DK (1·2 inches) and EL (1·2 inches) show the distances travelled by the lay-pin whilst crank revolves 54° on each side of the outer dead centre, that is in 0·6 second it travels 1·2 inches backwards from the cloth, comes to rest at the "dwell," and then travels 1·2 inches forwards towards the cloth.

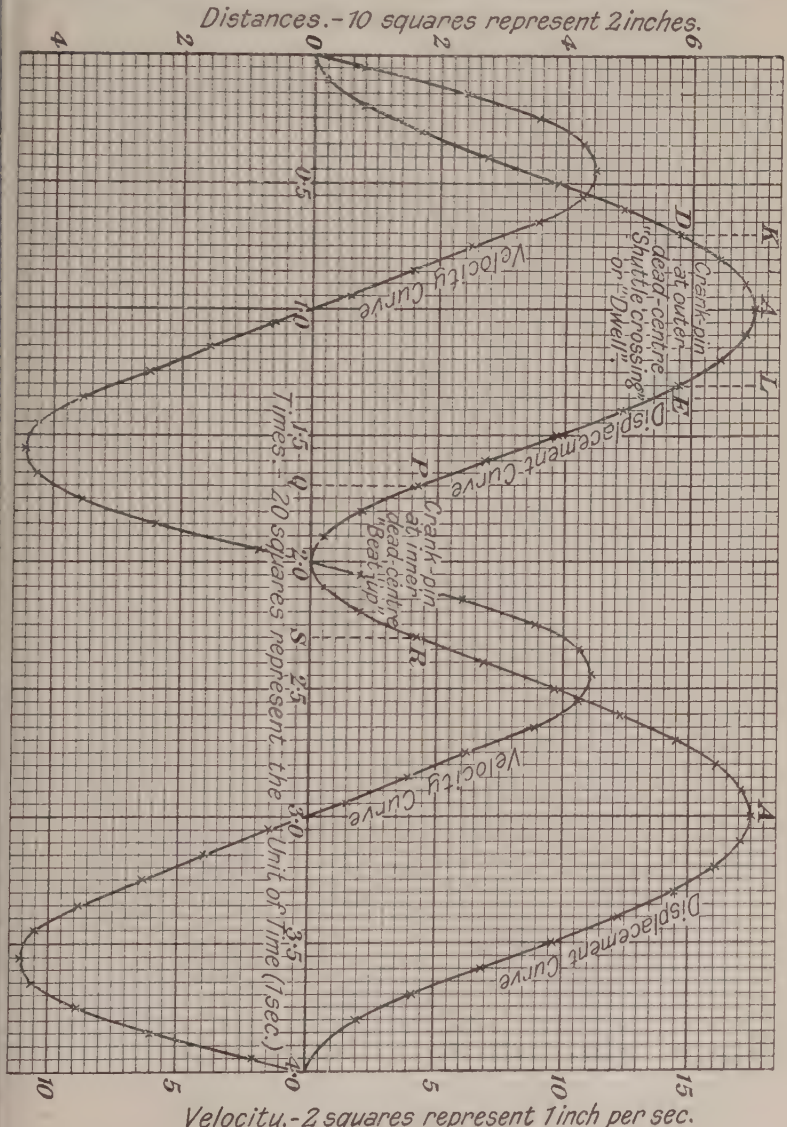
(3) PQ (1·7 inches) and RS (1·7 inches) show the distances travelled by the lay-pin whilst crank revolves 54° on each side of the inner dead centre, that is in 0·6 second it travels 1·7 inches forwards towards the cloth, comes to rest at the "beat-up," and then travels 1·7 inches backwards from the cloth.

Now the crank is revolving at a constant rate all the time, so that the speed of the lay-pin is slower as the crank-pin approaches and leaves the outer dead centre than when it approaches and leaves the inner dead centre. This allows time for the shuttle to cross, and the motion of the lay-sword being steadier, there is less danger of the shuttle leaving the shuttle-race.

The greater speed of the lay-pin at the "beat-up" serves to give greater force in the form of a smart blow to the reed as it comes in contact with the fell of the cloth.

Case II. Assume a *Long Crank* = 4 inches and a *Short Connecting Arm* 9 inches.

As in Case I, we start with the crank at the inner dead centre and let it revolve counter-clockwise. Assume the time for half a revolution of the crank to be the unit (1 second suppose).



The student should draw Fig. 175 as before and compare his measurements with those in the table below :—

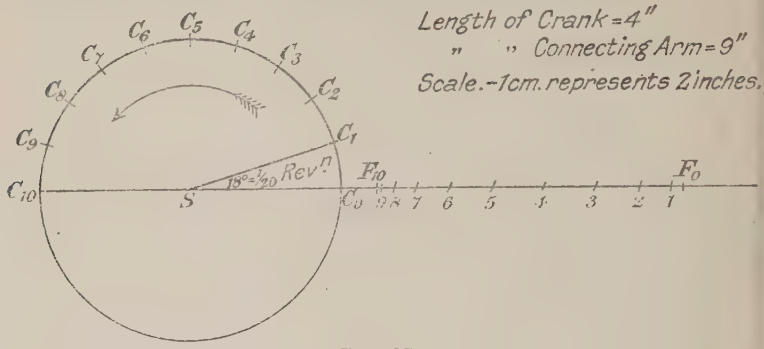


FIG. 175.

Distance travelled by "lay-pin" (F) $= s$ inches from the start.	Time taken $= t$ secs.	Distance travelled between each successive interval of 0·1 sec. ds.	Small increase in time dt.	$\frac{ds}{dt}$ = Average velocity during time dt in inches per sec. at *middle of each small interval of time.*
inches.	secs.	inches.	secs.	inches per second.
0	0	0·28	0·1	$\frac{0·28}{0·1} = 2·8$
0·28	0·1	0·77	0·1	$\frac{0·77}{0·1} = 7·7$
1·05	0·2	1·17	0·1	$\frac{1·17}{0·1} = 11·7$
2·22	0·3	1·36	0·1	$\frac{1·36}{0·1} = 13·6$
3·58	0·4	1·32	0·1	$\frac{1·32}{0·1} = 13·2$
4·9	0·5	1·15	0·1	$\frac{1·15}{0·1} = 11·5$
6·05	0·6	0·87	0·1	$\frac{0·87}{0·1} = 8·7$
6·92	0·7	0·60	0·1	$\frac{0·60}{0·1} = 6·0$
7·52	0·8	0·35	0·1	$\frac{0·35}{0·1} = 3·5$
7·87	0·9	0·13	0·1	$\frac{0·13}{0·1} = 1·3$
8·0	1·0			

In Fig. 176 we have the Displacement-Time and the Velocity-Time Curves for half a revolution of the crank; whilst in Fig. 177 these curves are shown for *two* complete revolutions.

(*a*) As in Case I, we notice :—

(1) The speed of the "lay-pin" increases, from zero when the crank-pin is at its inner dead centre, till it attains a maximum of 13·7 inches

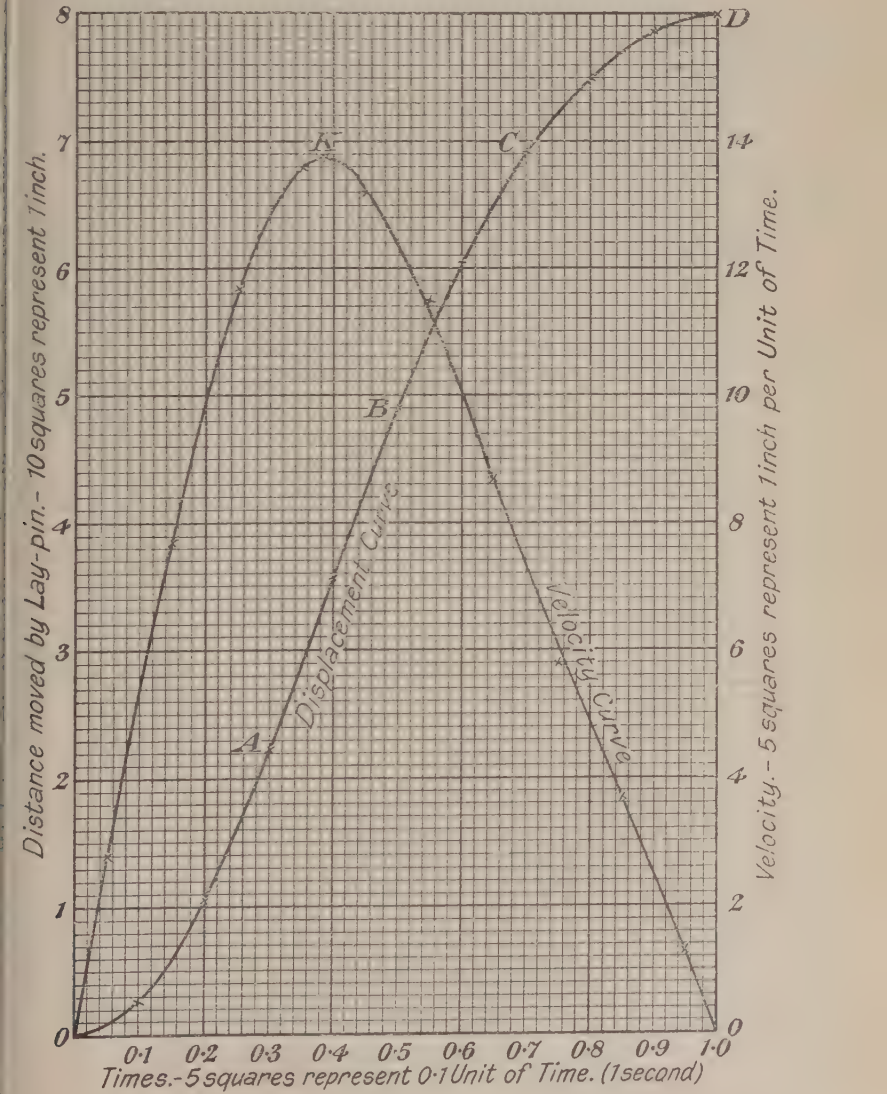
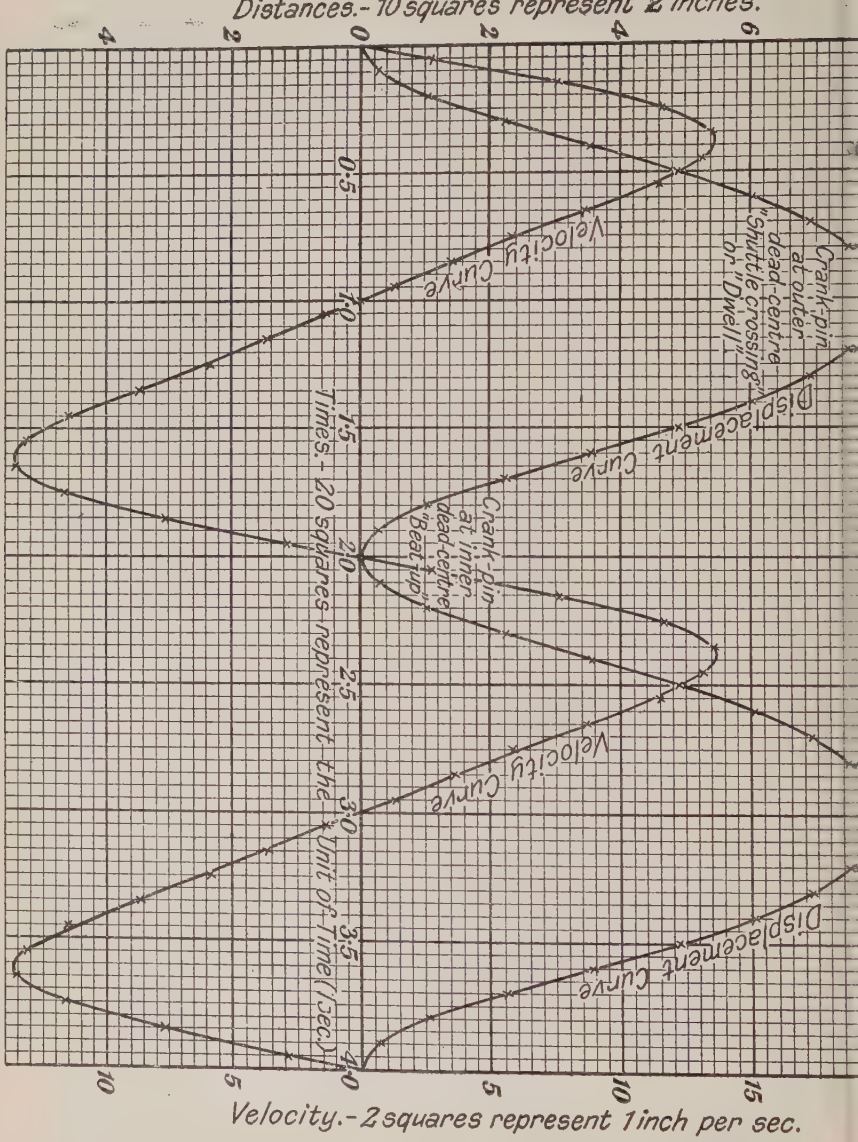


Fig. 176.

per second in 0·375 second, that is before the "lay-pin" has travelled half its distance from front to back of the loom [shown at *K* (Fig. 176)]. Its speed then diminishes, becoming zero again as the crank passes the outer dead centre.



(2) The speed of the "lay-pin" as the crank approaches and leaves the outer dead centre is less than its speed as the crank approaches and leaves the inner dead centre.

Then (*a*) the slower motion at *A* (Fig. 177) gives *time for the shuttle to cross*; (*b*) the quicker motion at *B* gives a *more distinct blow at the "beat-up."*

(*b*) Comparing Case I and Case II (see Figs. 174 and 177), we see that :—

(1) The maximum speed of the lay-pin is greater in Case II than in Case I.

(2) The motion is slower and steadier at the time of "dwell" in Case II than in Case I.

(3) The motion is quicker at the time of "beat-up" in Case II than in Case I.

Hence if we want to increase the eccentricity of the motion of the "going part" we must *lengthen the crank* and *shorten the connecting arm.*

Case II is thus suitable for broad looms, as this allows more time for the passage of the shuttle across the race. In narrow looms a small crank and long connecting arm are employed, only a slight dwell being necessary in such looms.

CHAPTER XIII.

Problems on the Construction of Woven Fabrics.

160.—Introduction. 161.—To find the Diameter of a Thread. 162.—The Diameters of Threads of the same Material vary inversely as the Square Roots of the Counts. 163.—For the same Material the Numbers of Diameters per inch vary directly as the Square Roots of the Counts. 164.—To find the Ends and Picks per inch for a given Weave. 165.—Changing the Yarns or the Setts. 166.—Changing from one Weave to another. 167.—Changing the Weight of a Cloth. 168.—Simultaneous Change of Weight and Weave. 169.—Twist or Turns per inch. 170.—The Bending of the Warp and Weft Threads—Angle due to Curvature.

160. *Introduction.*

The construction of an ideal cloth involves all the essential characteristics of any given fabric or texture.

The chief features to consider in the construction of a woven fabric are : the weave ; the diameter of the yarn ; the relative bending powers of the warp and weft. Other features which influence the structure are : the quality of the yarn used ; the elasticity, strength, and twist in the yarn.

Since we are only dealing with calculations in regard to cloth construction, we confine ourselves to the features which involve any calculation necessary to the building up or setting of cloths.

Thus we must be able :—

First. To find the diameter of the yarn to be used.

Second. To find the sett and picks for the design or weave.

Third. To find the ends and picks per inch with which to make a new cloth in the same weave as a known cloth but *with different counts of yarn* and yet preserve the same balance of structure.

Fourth. To find the ends and picks per inch in a new cloth made with the same counts of yarn but *altering the weave* and yet retaining the same balance of structure.

Fifth. To find the counts of yarn and the ends and picks per inch required to make a new cloth of similar structure to a given cloth, but *of a different weight.*

Sixth. To find the counts of yarn and ends and picks per inch of a new cloth with the same balance of structure as a given cloth, but *of a different weight and weave.*

Seventh. To find the *twist or turns per inch* suitable for a yarn of any count to be similar to a given yarn.

161. *To find the Diameter of a Thread.*

Method I. By actual measurement.

This consists in choosing a number of well-made threads of the yarn under consideration, carefully laying them side by side, and measuring the width across all of them. This measurement divided by the number of threads will give the average diameter of one thread.

Method II. By calculation from a rule which has been formulated from experience (an empirical rule).

The rule may be stated thus :—

"Extract the square root of the number of yards per lb. of the yarn, and from this result deduct 10% in the case of worsted yarns, 8% for cotton and linen, and 16% for woollen yarns. The reciprocal of this difference will be the diameter of the thread in inches."

[Note $\frac{1}{2}$ is the reciprocal of 2, $\frac{1}{8}$ is the reciprocal of 8. See § 141, Example 4.]

Thus to find the diameter of 1/30's worsted,

Number of yards per lb. $= 30 \times 560.$

$\sqrt{\text{yds. per lb.}} - 10\% \sqrt{\text{yds. per lb.}} = \frac{9}{10}\sqrt{\text{yds. per lb.}}$ (using fractions)

$= 0{\cdot}9\sqrt{\text{yds. per lb.}}$ (using decimals)

$= 0{\cdot}90\sqrt{16800}$

$= 116{\cdot}6$ (using logs.).

The reciprocal of $116{\cdot}6 = \frac{1}{116{\cdot}6}$.

$\therefore$ Diameter of 1/30's worsted $= \frac{1}{116{\cdot}6}$ inch,

or 116·6 threads may be placed side by side in a width of 1 inch without compression.

For cotton and linen yarns we deduct 8% or $\frac{8}{100}\sqrt{\text{yds. per lb.}}$ and as above we shall obtain $\frac{92}{100}\sqrt{\text{yds. per lb.}}$ or $0{\cdot}92\sqrt{\text{yds. per lb.}}$

For woollen yarns we deduct 16% or $\frac{16}{100}\sqrt{\text{yds. per lb.}}$ and obtain $\frac{84}{100}\sqrt{\text{yds. per lb.}}$ or $0{\cdot}84\sqrt{\text{yds. per lb.}}$

Then we may state the rule thus :—

$$\text{Diameter of a yarn} = \frac{1}{0{\cdot}90\sqrt{\text{yds. per lb.}}} \text{ inch for worsted}$$

$$= \frac{1}{0{\cdot}92\sqrt{\text{yds. per lb.}}} \text{ inch for cotton and linen}$$

$$= \frac{1}{0{\cdot}84\sqrt{\text{yds. per lb.}}} \text{ inch for woollen,}$$

and expressed in this form the student may conveniently use Log. Tables or the Slide Rule.

Method III. By calculation having first obtained the specific gravity of the yarn.

This is a scientific method, marred only by the fact that the specific gravity of yarn is difficult to obtain accurately.

As the result of numerous experiments it has been placed on record that the specific gravity of wool is 1·3 and that of cotton 1·5.

(*a*) Suppose we *calculate the diameter of* 20's *cotton* by this method assuming that the specific gravity of cotton = 1·5 and that the cotton thread is perfectly cylindrical.

Suppose the diameter is D inches.

Then

$$\text{Volume of a cotton hank} = \frac{\pi \times D^2 \times L}{4} \text{ when } L = \text{length of thread which is a cylinder}$$

$$= \frac{\pi D^2 \times 840 \times 36}{4} \text{ cubic inches}$$

$$= \frac{\pi \times D^2 \times 840 \times 36}{4 \times 1728} \text{ cubic feet.}$$

Now 1 cubic foot of water weighs 62·3 lbs.

$\therefore$ 1 cubic foot of cotton weighs $62{\cdot}3 \times 1{\cdot}5$ lbs.

Hence

$$\text{Weight of a cotton hank} = \frac{\pi D^2 \times 840 \times 36 \times 62{\cdot}3 \times 1{\cdot}5}{4 \times 1728} \text{ lbs.} \quad \text{(A)}$$

But the thread is 20's cotton.

$\therefore$ Weight of 1 hank $= \frac{1}{20}$ lb. (B)

Results (A) and (B) must be the same ; then

$$\frac{\pi \times D^2 \times 840 \times 36 \times 62{\cdot}3 \times 1{\cdot}5}{4 \times 1728} = \frac{1}{20}$$

$$\text{or} \quad D^2 = \frac{4 \times 1728}{3{\cdot}142 \times 840 \times 36 \times 62{\cdot}3 \times 1{\cdot}5 \times 20} \qquad \text{(C)}$$

$$\text{or} \quad D = \sqrt{\frac{4 \times 1728}{3{\cdot}142 \times 840 \times 36 \times 62{\cdot}3 \times 1{\cdot}5 \times 20}}$$

$$= \sqrt{\frac{1}{3{\cdot}142 \times 70 \times 62{\cdot}3 \times 1{\cdot}875}}$$

$$= \frac{1}{\sqrt{3{\cdot}142 \times 70 \times 62{\cdot}3 \times 1{\cdot}875}} = \frac{1}{160{\cdot}3} \text{ inch,}$$

or 160·3 threads may be laid side by side in a width of 1 inch.

Now this number is always greater than that in actual practice, since the cotton fibre contains air spaces and consequently the diameter obtained above is too small.

(*b*) Using Method II we have :—

$$\text{Diameter of 20's cotton} = \frac{1}{0{\cdot}92\sqrt{20 \times 840}} \text{ inch}$$

$$= \frac{1}{0{\cdot}92\sqrt{16800}} = \frac{1}{119{\cdot}2} \text{ inch,}$$

and the number of diameters or threads per inch = 119·2.

[*Note.*—Method II is the one generally used for calculating the diameters of threads.]

Example.

Calculate the diameter of a 30's cut Gala thread.

This is a woollen thread and in the Gala cut system the counts are equivalent to the number of hanks (200 yards) which weigh 1 lb.

∴ Number of yards per lb. = 30 × 200 = 6000.

$$\therefore \text{ Diameter of a 30's cut Gala thread} = \frac{1}{0{\cdot}84\sqrt{6000}} \text{ inch}$$

$$= \frac{1}{65{\cdot}07} \text{ inch.}$$

If $x = 0{\cdot}84\sqrt{6000}$

$\log x = \log 0{\cdot}84 + \frac{1}{2} \log 6000$

$= \bar{1}{\cdot}9243 + 1{\cdot}8891 = 1{\cdot}8134$

$x =$ antilog $1{\cdot}8134 = 65{\cdot}07$.

Hence Number of Diameters or threads per inch = 65 in practice.

162. "*The diameters of threads of the same material vary inversely as, or are inversely proportional to, the square roots of the counts.*"

This has been proved by one method in § 142, Chapter XI. The result may also be deduced from the last paragraph, § 161, Method III.

In calculating the diameter of the 20's cotton yarn, we obtained

$$\left.\begin{matrix} D^2 \\ (20\text{'s cotton}) \end{matrix}\right\} = \frac{4 \times 1728}{3{\cdot}142 \times 840 \times 36 \times 62{\cdot}3 \times 1{\cdot}5 \times 20} \quad (C)$$

If we were to follow the working there shown for a 30's cotton we should get

$$\left.\begin{matrix} D^2 \\ (30\text{'s cotton}) \end{matrix}\right\} = \frac{4 \times 1728}{3{\cdot}142 \times 840 \times 36 \times 62{\cdot}3 \times 1{\cdot}5 \times 30}.$$

Here we see that the fraction $\left\{\frac{4 \times 1728}{3{\cdot}142 \times 840 \times 36 \times 62{\cdot}3 \times 1{\cdot}5}\right\}$ occurs every time we are dealing with cotton threads.

Let us call its value K.

$$\therefore \quad \frac{D^2 \text{ (20's cotton)}}{D^2 \text{ (30's cotton)}} = \frac{K \times \frac{1}{20}}{K \times \frac{1}{30}}$$

$$= \frac{1}{20} \div \frac{1}{30} = \frac{30}{20}.$$

Taking square root of each ratio we have

$$\frac{\text{Diameter of 20's cotton}}{\text{Diameter of 30's cotton}} = \frac{\sqrt{30}}{\sqrt{20}},$$

i.e., the diameters are inversely proportional to the square roots of the counts.

163. "*For the same material, the number of diameters or threads per inch varies directly as, or is directly proportional to, the square roots of the counts.*"

This has been deduced from the last result in § 143, Chapter XI, and we will content ourselves here with stating the result arrived at :—

$$\frac{\text{Diameters per inch of 1st thread}}{\text{Diameters per inch of 2nd thread}} = \frac{\sqrt{\text{Counts of 1st thread}}}{\sqrt{\text{Counts of 2nd thread}}}.$$

This is sometimes expressed as :—

The "Sett" is directly proportional to $\sqrt{\text{Counts}}$.

EXAMPLE 1.

If the diameter of a 1/24's Yorkshire skeins is $\frac{1}{66{\cdot}0}$ inch, find the diameter of 1/30's Y.S.

If the diameter of 1/30's Y.S. $= D$ inches we have

$$\frac{D}{\frac{1}{66{\cdot}0}} = \frac{\sqrt{24}}{\sqrt{30}}.$$

Squaring both ratios. $$\frac{D^2}{(\frac{1}{66{\cdot}0})^2} = \frac{24}{30}$$

or $$D^2 = \frac{24}{30 \times 66{\cdot}0 \times 66{\cdot}0} = \frac{1}{5445}.$$

Taking square root. $$D = \frac{1}{\sqrt{5445}} = \frac{1}{73{\cdot}79} \text{ inch.}$$

EXAMPLE 2.

The number of diameters per inch of a 24's linen is given as 76. What are the linen counts when the number of diameters per inch is 170 ?

Suppose the required counts $= x$'s.

Since the diameters per inch are directly proportional to the square root of the counts,

$$\frac{76}{170} = \frac{\sqrt{24}}{\sqrt{x}}.$$

Squaring both ratios. $$\frac{76^2}{170^2} = \frac{24}{x}.$$

Inverting both ratios, $$\frac{x}{24} = \frac{170^2}{76^2}$$

or $$x = \frac{170^2 \times 24}{76^2} = 120.$$

$\therefore$ Required counts $= 120$'s linen.

164. *To find the Ends and Picks per inch for a given weave.*

We will try to arrive at some general rule by working examples from first principles.

EXAMPLE 1.

To find the ends and picks per inch in a plain weave made with 2/36 worsted (diameter $= \frac{1}{60}$ inch).

If the diameter of the yarn is not given we must first calculate it, using Method II, § 161.

In a plain weave each weft thread goes under a warp thread and over the next one and *vice versa*.

Thus in one repeat of the design A to B (Fig. 178) we have 2 ends of warp (1, 2) and 2 intersections of weft (a, b).

If we neglect curvature and compression, each warp end occupies $\frac{1}{90}$ inch and each thickness of weft also occupies $\frac{1}{90}$ inch.

The width of these four spaces occupied by (2 ends of warp + 2 intersections of weft) will be $\left(\frac{2}{90} + \frac{2}{90}\right)$ or $\frac{2+2}{90}$ inch, and in this width we must set 2 ends of warp.

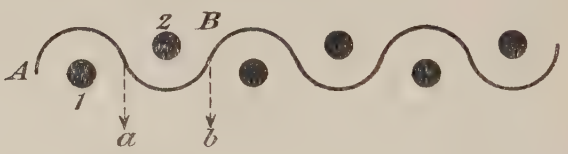


Fig. 178.

Hence in $\frac{2+2}{90}$ inch there are 2 ends of warp, and in 1 inch there will be $\frac{2}{\frac{2+2}{90}}$ ends of warp.

$$\therefore \text{ Sett required } = \frac{2}{\frac{2+2}{90}} \text{ ends per inch.}$$

$$= \frac{2 \times 90}{2+2} \qquad \text{(A) [Note this carefully]}$$

$$= 45 \text{ ends per inch.}$$

Since the cloth is woven on the "square," *i.e.*, the warp threads cross the weft threads in exactly the same fashion as the weft threads cross the warp threads, then

Number of picks per inch = 45.

Example 2.

To find the ends and picks per inch necessary to weave a $\frac{2}{2}$ twill using the same material, 2/36 worsted.

In this weave, each weft thread passes over two warp threads and under the next two and *vice versa*.

Thus in one repeat of the design A to B (Fig. 179) we have 4 ends of warp (1, 2, 3, 4) and 2 intersections of weft (a, b).

Supposing there is no curvature or compression of the threads, each end of warp and each thickness of weft occupies a width of $\frac{1}{90}''$.

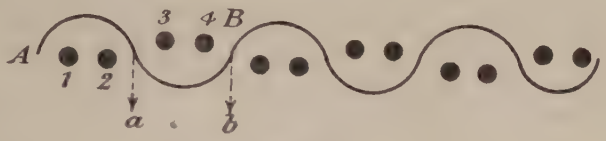


Fig. 179.

The width of the 6 spaces occupied by (4 ends of warp + 2 intersections of weft) will be $\left(\frac{4}{90} + \frac{2}{90}\right)$ or $\frac{4+2}{90}$ inch, and in this width we must have 4 ends of warp.

Hence in $\frac{4+2}{90}$ inch there are 4 ends of warp, or in 1 inch there are $\frac{4}{\frac{4+2}{90}}$ ends of warp.

$$\therefore \quad \text{Sett required} = \frac{4}{\frac{4+2}{90}} \text{ ends per inch}$$

$$= \frac{90 \times 4}{4+2} \qquad \text{(B) [Note this]}$$

$$= 60 \text{ ends per inch.}$$

Since it is a "square" cloth,

Number of picks per inch = 60.

Example 3.

To find the ends and picks per inch to weave a $\frac{1}{4}$ twill made with 30's skeins woollen (Yorks). Diameter = $\frac{1}{75}$ inch.

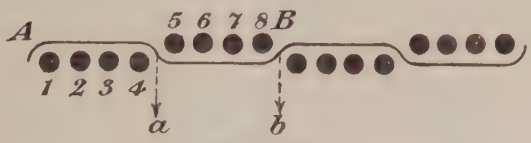


Fig. 180.

Here in one repeat of the design (Fig. 180) we have 8 ends of warp and 2 intersections of weft. Each end of warp and each thickness of weft occupies a width of $\frac{1}{75}$ inch.

The width of the 10 spaces occupied by (8 ends of warp + 2 intersections of weft) will be $\left(\frac{8}{75} + \frac{2}{75}\right)$ or $\frac{8+2}{75}$ inch, and in this width we must have 8 ends of warp.

Hence in $\frac{8+2}{75}$ inch there are 8 ends of warp, or in 1 inch there are $\frac{8}{\frac{8+2}{75}}$ ends of warp.

$$\therefore \quad \text{Sett required} = \frac{8}{\frac{8+2}{75}} \text{ ends per inch}$$

$$= \frac{75 \times 8}{8+2} \quad \text{,, ,, ,, (C) [Note this]}$$

$$= 60 \text{ ends per inch.}$$

Also there will be 60 picks per inch.

Now examining results (A), (B), and (C) where

$$\text{Sett required} = \frac{90 \times 2}{2+2} \quad \text{(A) Example 1,}$$

$$\text{,, ,,} = \frac{90 \times 4}{4+2} \quad \text{(B) Example 2,}$$

$$\text{,, ,,} = \frac{75 \times 8}{8+2} \quad \text{(C) Example 3,}$$

we see that (90, 90, and 75) are the numbers of diameters per inch in the given yarns; (2, 4, and 8) the other factors in the numerators are the numbers of warp ends in one repeat of the weave; whilst the denominators (2 + 2), (4 + 2), and (8 + 2) are the sums of the number of ends of warp and the number of intersections of weft in 1 repeat.

We thus arrive at what is known as "*Ashenhurst's Intersection Rule.*"

$$\text{Threads per inch} = \frac{\left\{\begin{matrix}\text{Diameter} \\ \text{reciprocal}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Ends in 1 repeat} \\ \text{of the design}\end{matrix}\right\}}{\left\{\begin{matrix}\text{Ends in 1 repeat} \\ \text{of the design}\end{matrix}\right\} + \left\{\begin{matrix}\text{Intersections in 1 re-} \\ \text{peat of the design}\end{matrix}\right\}}$$

This rule holds good for "square" cloths, *i.e.*, cloths in which the warp and weft are equal in thickness, the number of ends and picks are equal, and the bending powers of the warp and weft are equal.

For satin cloths it is necessary that the warp and weft should not be equal. If a warp-faced satin is required there should be more warp threads per inch than picks of weft, since the

result desired is a cloth in which the face is composed of warp threads with little or no bending power. In all satins one thread in each repeat goes to the back of the cloth, whilst the remaining threads close up and conceal the weft; it is therefore necessary that the sett should be fine. A good method is to use the diameter reciprocal as the number of threads per inch.

If a weft-faced satin is required the reverse of the above will be necessary.

Corkscrews, cords, and cloths of this nature which show a warp face require to be set as close as the diameter of the thread will permit. The bending takes place in the warp and the weft lies straight. The warp threads should be finer than the weft. If a weft face is required the conditions are reversed.

Example 4.

Warp and Weft Threads unequal in thickness.

Find the ends and picks per inch for a $\frac{4}{4}$ twill using 2/24's worsted warp (diameter = $\frac{1}{74}$ inch) and 2/16's worsted weft (diameter = $\frac{1}{60}$ inch).

(*a*) *To find the Ends per inch.*

In one repeat of the design we have 8 ends of warp and 2 intersections of weft.

The 8 ends of warp will occupy $\frac{8}{74}$ inch, the 2 intersections of weft will occupy $\frac{2}{60}$ inch, and the total width of these 10 spaces will be $(\frac{8}{74} + \frac{2}{60})$ inch.

In this width we place 8 ends of warp.

Hence in $(\frac{8}{74} + \frac{2}{60})$ inch there are 8 ends of warp, or in 1 inch there are $\dfrac{8}{\frac{8}{74} + \frac{2}{60}}$ ends of warp.

$$\therefore \quad \text{Sett required} = \frac{8}{\frac{8}{74} + \frac{2}{60}} = \frac{8}{\frac{4}{37} + \frac{1}{30}}$$

$$= \frac{8}{\frac{120 + 37}{1110}} = \frac{8 \times 1110}{157}$$

$$= 56{\cdot}6 \text{ ends per inch.}$$

(*b*) *To find the Picks per inch.*

In one repeat of the design we have 8 ends of weft and 2 intersections of warp.

The width of these 10 spaces will be $(\frac{8}{60} + \frac{2}{74})$ inch, and in this width there will be 8 picks.

Working step by step as in (*a*) we get the following result :—

$$\text{Number of Picks per inch} = \frac{8}{\frac{8}{60} + \frac{2}{74}}$$

$$= \frac{8}{\frac{4}{30} + \frac{1}{37}} = \frac{8}{\frac{148 + 30}{1110}}$$

$$= \frac{8 \times 1110}{178} = 49{\cdot}9.$$

Hence the cloth would be woven 57 ends and 50 picks per inch.

165. *Changing the Yarns or the Sett.*

In this case we have to find the ends and picks per inch with which to make a new cloth in the *same weave* as a known cloth, but with different counts of yarn and preserving similarity of structure or *vice versa*.

Example 1.

A cloth has 52 threads per inch and is made with 2/48's worsted. What number of ends would be required if 2/24's worsted were used, and retaining the same balance of structure ?

In §§ 143 and 163 we have established the fact that the " Sett " or " diameters per inch " is directly proportional to the square root of the counts.

Suppose required Sett $= x$ ends per inch, then

$$\frac{x}{52} = \frac{\sqrt{12}}{\sqrt{24}}.$$

Squaring both ratios, $\frac{x^2}{52^2} = \frac{12}{24}$

$$\text{or} \quad x^2 = \frac{12 \times 52 \times 52}{24} = 1352.$$

Taking square root of both sides,

$$x = \sqrt{1352},$$

$$\log x = \tfrac{1}{2} \log 1352 = 1{\cdot}5655,$$

$$\therefore \quad x = \text{antilog } 1{\cdot}5655 = 36{\cdot}77,$$

$$\therefore \quad \text{Required Sett} = 36{\cdot}8 \text{ ends per inch.}$$

Example 2.

A cloth is woven with 36-skein woollen (Yorks) in 18's reed 4's. It is required to reduce the fineness of Sett to 58 ends per inch. What count of yarn is required ?

As in Example 1 we have

$$\frac{\text{Given Sett}}{\text{Required Sett}} = \frac{\sqrt{\text{Given Counts}}}{\sqrt{\text{Required Counts}}}$$

If we suppose the required counts = x's Y.S., we have

$$\frac{72}{58} = \frac{\sqrt{36}}{\sqrt{x}}.$$

Squaring both sides, $\frac{72^2}{58^2} = \frac{36}{x}$.

Inverting both ratios, $\frac{x}{36} = \frac{58^2}{72^2}$.

Multiply by 36 and $x = \frac{58 \times 58 \times 36}{72 \times 72} = 23{\cdot}36$.

Then the yarn required would be 23's or 24's Y.S.

166. *Changing from one weave to another.*

Here we have to find the number of ends and picks per inch in a new cloth made by altering the weave of a given cloth, using the same counts of yarn and retaining the same balance of structure.

EXAMPLE 1.

A cloth is woven 60 ends and picks per inch in a $\frac{2}{2}$ twill. It is desired to change the weave to a $\frac{3}{3}$ twill. Find the sett and picks per inch using the same counts of thread.

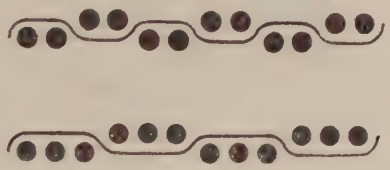


FIG. 181.

In a $\frac{2}{2}$ twill (Fig. 181) we have 4 ends of warp and 2 intersections of weft in 1 repeat of the design.

In a $\frac{3}{3}$ twill (Fig. 181) we have 6 ends of warp and 2 intersections in 1 repeat.

Now in changing the design we must compare the setting of equal numbers of warp threads beginning and ending with a repeat. This number will be a common multiple of 4 and 6 in this case, and the L.C.M. of 4 and 6 = 12.

Hence in a $\frac{2}{2}$ twill we have 12 warp ends and 6 intersections in 18 diameter spaces.

In a $\frac{3}{3}$ twill we have 12 warp ends and 4 intersections in 16 diameter spaces.

Now with 12 warp ends in 18 diameter spaces the Sett = 60 ends per inch.

With 12 warp ends in 1 diameter space the sett = 60×18 ends per inch.

With 12 warp ends in 16 diameter spaces the sett = $\frac{60 \times 18}{16}$ ends per inch.

$$\therefore \text{ Sett required } = \frac{60 \times 18}{16} = 67\tfrac{1}{2} \text{ ends per inch.}$$

Note.—The sett is closer since we have less intersections of weft for the same number of ends of warp.

Example 2.

A cloth is woven with 72 ends and picks per inch in a twill weave of which the first pick is given $\frac{3\ 1}{2\ 1}$. It is required to change the plan to the following $\frac{2\ 2\ 1}{4\ 3\ 1}$. Assuming that the same count of yarn is used, what number of ends and picks per inch will retain the same relative balance of structure ?

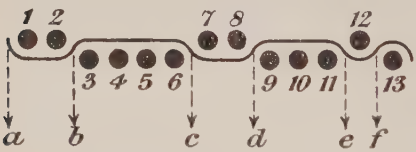


Fig. 182.

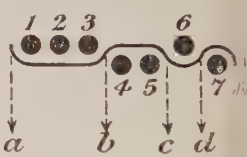


Fig. 183.

Method I.

In the required weave (Fig. 182) we have 13 warp ends and 6 intersections of weft in each repeat.

In the given weave (Fig. 183) we have 7 warp ends and 4 intersections of weft in each repeat.

We must compare the spacing of equal numbers of ends of warp for some complete number of repeats, and this we see must be a multiple of 13 and 7.

Now L.C.M. of 13 and 7 = 91.

Hence in the given weave we have 91 warp ends and 52 intersections in 143 diameter spaces.

In the required weave we have 91 warp ends and 42 intersections in 133 diameter spaces.

Now with 91 warp ends in 143 diameter spaces the Sett = 72 ends per inch.

With 91 warp ends in 1 diameter space the Sett = 72×143 ends per inch.

With 91 warp ends in 133 diameter spaces the Sett = $\frac{72 \times 143}{133}$ ends per inch.

$\therefore$ Sett required $= \frac{72 \times 143}{133} = 77{\cdot}4$ ends per inch.

Also picks per inch = 77·4.

EXAMPLE 2.

Method II.

Let us now work this example using the "Intersection Rule," § 164.

Suppose the number of diameters per inch of the yarn used = D. For the given weave $\frac{3}{2}\frac{1}{1}$ we have 7 warp ends and 4 intersections in each repeat, and the given Sett is 72 ends per inch.

Then by the Intersection Rule,

$$\frac{D \times 7}{7 + 4} = 72,$$

$$\text{or } D = \frac{72 \times 11}{7}. \qquad \text{(A)}$$

In the required weave $\frac{2}{4}\frac{2}{3}\frac{1}{1}$ we have 13 warp ends and 6 intersections in each repeat.

If the required Sett is x ends per inch,

By the Intersection Rule, $\frac{D \times 13}{13 + 6} = x$

$$\text{or } D = \frac{x \times 19}{13}. \qquad \text{(B)}$$

But the value of D is the same in results (A) and (B).

$$\therefore \quad \frac{x \times 19}{13} = \frac{72 \times 11}{7}.$$

Divide both sides by $\frac{19}{13}$ and

$$x = \frac{72 \times 11 \times 13}{7 \times 19}$$
$$= \frac{72 \times 143}{133}.$$

[*Note.*—This is the same fraction as in Method I.]

$\therefore$ we must have 77·4 ends and picks = 77·4.

EXAMPLE 3.

To change the weave, altering the counts, but keeping the ends and picks the same.

For a plain weave the warp and weft is 2/64's and the cloth contains 60 ends and picks per inch. Keeping the sett and picks the same, what counts of warp and weft would be necessary to retain the same relative structure of cloth if the weave is changed from plain to $\frac{2}{2}$ twill?

Suppose the number of diameters per inch of the 2/64's = D_1.

Suppose the number of diameters per inch of the required thread = D_2.

In the plain weave we have 2 ends of warp and 2 intersections for each repeat of pattern.

Hence by the Intersection Rule, § 164

$$\frac{D_1 \times 2}{2 + 2} = 60 \text{ ends per inch.} \qquad \text{(A)}$$

In a $\frac{2}{2}$ twill we have 4 ends of warp and 2 intersections in each repeat.

Hence $$\frac{D_2 \times 4}{4 + 2} = 60 \text{ ends per inch} \qquad \text{(B)}$$

since the sett is to be the same.

From (A) and (B),

$$\frac{D_1 \times 2}{4} = \frac{D_2 \times 4}{6}$$

or $$\frac{D_1}{2} = \frac{D_2 \times 2}{3}$$

or $$D_1 = \frac{D_2 \times 4}{3}.$$

Dividing by D_2, $$\frac{D_1}{D_2} = \frac{4}{3}. \qquad \text{(C)}$$

But the diameters per inch are directly proportional to the square roots of the counts § 163.

Then if counts of yarn required $= x$'s, we have

$$\frac{D_1}{D_2} = \frac{\sqrt{32}}{\sqrt{x}}.$$

From result (C), $\frac{D_1}{D_2} = \frac{4}{3}$.

$$\therefore \quad \frac{\sqrt{32}}{\sqrt{x}} = \frac{4}{3}.$$

Squaring both ratios, $\frac{32}{x} = \frac{16}{9}$.

Inverting both ratios, $\frac{x}{32} = \frac{9}{16}$

$$\text{or} \qquad x = \frac{9 \times 32}{16} = 18.$$

$\therefore$ Counts required $= 18$'s or 2/36's warp and weft.

We have established the Intersection Rule from first principles, § 164, and an intelligent use of this rule as shown in the foregoing examples will serve the student better than trying to remember formulæ like the following :—

To work Examples 1 *and* 2. Find L.C.M. of the two weaves, and

Required Sett =

$$\frac{(\text{Given Sett}) \times (\text{Ends in L.C.M.} + \text{Intersections in given cloth})}{(\text{Ends in L.C.M.} + \text{Intersections in required cloth})}.$$

To work Example 3.

Required Counts =

$$\frac{\left(\text{Given Counts}\right) \times \left\{\left(\begin{matrix}\text{Ends in}\\ \text{L.C.M.}\end{matrix}\right) + \left(\begin{matrix}\text{Intersections of}\\ \text{required weave}\end{matrix}\right)\right\}^2}{(\text{Ends in L.C.M.} + \text{Intersections of given weave})^2}.$$

167. *Changing the Weight of a Cloth.*

(*a*) If we keep the sett and picks per inch constant, then the weight of a cloth varies inversely as the count of the yarn used, for 1 hank of 36's weighs $\frac{1}{36}$ lb., and 1 hank of 72's weighs $\frac{1}{72}$ lb. This is worked out in § 138 (*a*), page 280.

Expressed symbolically,

$$\text{Weight } (W) = \frac{K}{C \text{ (counts)}} \quad \text{where } K \text{ is constant.}$$

(*b*) If we keep the counts of the yarn the same we may alter the weight by altering the sett and picks per inch.

It is easy to see that if we double the ends and picks per inch then we double the weight, *i.e.*, the weight varies directly as the sett when the counts are the same, or

$$W = aS \text{ (sett) where } a \text{ is a constant, § 137.}$$

(*c*) When both the sett and the counts vary we have

$$W = \frac{K}{C} \times aS \quad [\S\ 139, \text{page } 281]$$

$$= \frac{KaS}{C} \quad \text{where } Ka \text{ will be constant.} \qquad \text{I}$$

(*d*) Now the sett varies directly as the square root of the counts (§ 163), or

$$S = b\sqrt{C} \qquad \text{II}$$

Substitute this value for S in I, and

$$W = \frac{Kab\ \sqrt{C}}{C} \qquad [\text{Since} \quad \sqrt{16} = 4$$

$$\sqrt{16} \times \sqrt{16} = 4 \times 4 = 16.$$

$$= \frac{Kab}{\sqrt{C}} \qquad \therefore \frac{16}{\sqrt{16}} = \sqrt{16}.$$

$$= \frac{\text{Some Constant}}{\sqrt{C}} \qquad \text{Similarly} \quad \frac{C}{\sqrt{C}} = \sqrt{C}.]$$

or $W \times \sqrt{C} =$ a constant quantity.

Comparing this with results in § 138, page 280, we see that "the weight varies inversely as, or is inversely proportional to the square root of the counts."

(*e*) In this paragraph we are to find the counts of yarn, ends, and picks per inch to make a new cloth similar in structure to a given cloth but of a different weight; *both cloths to be made in the same weave.*

Example 1.

A cloth is made as follows: $\frac{2}{2}$ twill; 40 ends and picks per inch; warp and weft 20's skeins woollen (Yorks). It is required to make another cloth of the same weave structure but $\frac{1}{4}$ heavier. Find the counts and sett required.

Both the counts and the sett will vary, for to increase the weight we shall require a thicker yarn, but to preserve the balance, the threads and picks will not be as close.

(i) As we have tried to show above, the weight varies inversely as the $\sqrt{\text{Counts}}$.

If the required counts = x's Y. skeins,

$$\frac{\text{Given Weight}}{\text{Required Weight}} = \frac{\sqrt{\text{Required Counts}}}{\sqrt{\text{Given Counts}}}$$

or
$$\frac{4}{5} = \frac{\sqrt{x}}{\sqrt{20}}. \qquad \text{(A)}$$

Squaring both ratios, $\frac{x}{20} = \frac{16}{25}$,

$$\text{or} \quad x = \frac{16 \times 20}{25} = 12{\cdot}8.$$

$\therefore$ Counts required = 12·8's Y. skeins.

(ii) *To change the Sett.*

The sett varies directly as the square root of the counts. Hence, if required sett = y ends per inch,

$$\frac{y}{40} = \frac{\sqrt{12{\cdot}8}}{\sqrt{20}} = \frac{4}{5} \quad \left| \begin{array}{l} \text{From (A) above,} \\ \frac{\sqrt{x}}{\sqrt{20}} = 12{\cdot}8 = \frac{4}{5} \end{array} \right.$$

$$\therefore \quad y = \frac{4 \times 40}{5} = 32.$$

$\therefore$ The cloth must be woven with 32 ends and picks per inch using a 12·8's Y.S.

(iii) *Proof of this example.*

Suppose the warp is 64 yards long, and 64 inches wide in the loom.

(*a*) Then in the given cloth,

$$\text{Weight of warp and also of weft} = \frac{64 \times 40 \times 64}{20 \times 256} = 32 \text{ lbs.}$$

(*b*) In the cloth required, if we use 12·8's yarn and 32 ends and picks as calculated,

$$\text{Weight of warp} = \frac{64 \times 32 \times 64}{12{\cdot}8 \times 256} = \frac{512}{12{\cdot}8} = 40 \text{ lbs.},$$

which is $\frac{1}{4}$ heavier.

Let us try and *check the Sett.*

(*c*) Diameter of 20's Y.S. $= \frac{1}{0{\cdot}84\sqrt{20 \times 256}} = \frac{1}{60{\cdot}12} = \frac{1}{60}$ inch.

By the Intersection Rule,

$$\text{Ends per inch} = \frac{60 \times 4}{4 + 2} = 40.$$

(*d*) Diameter of 12·8's Y.S.

$$= \frac{1}{0{\cdot}84\sqrt{12{\cdot}8 \times 256}} = \frac{1}{48{\cdot}08} = \frac{1}{48} \text{ inch,}$$

and $$\text{Ends per inch} = \frac{48 \times 4}{4 + 2} = 32.$$

EXAMPLE 2.

It is required to make an 8 oz. cloth similar to a 10 oz. cloth made of 2/16's worsted, 40 threads and picks per inch. How many ends and picks and what counts of warp and weft are required for the new cloth ?

(i) *To find the Counts.*

The weight is inversely proportional to the square root of the counts.

If x's worsted be the counts of the yarn required,

$$\frac{\text{Given weight}}{\text{Required Weight}} = \frac{\sqrt{\text{Required Counts}}}{\sqrt{\text{Given Counts}}}$$

$$\text{or} \qquad \frac{10}{8} = \frac{\sqrt{x}}{\sqrt{8}}. \qquad \text{(A)}$$

Squaring both ratios, $$\frac{x}{8} = \frac{100}{64}$$

$$\text{or} \qquad x = \frac{100 \times 8}{64} = 12{\cdot}5.$$

Then counts of warp and weft = 2/25's worsted.

(ii) *To find the Sett.*

The sett varies directly as the square root of the counts. Hence if y ends per inch is the required sett,

$$\frac{y}{40} = \frac{\sqrt{12{\cdot}5}}{\sqrt{8}} = \frac{10}{8} \text{ from (A) above.}$$

$$\therefore \qquad y = \frac{10 \times 40}{8} = 50.$$

Then the new cloth must be woven 50 ends and picks per inch with 2/25's worsted warp and weft.

168. *Simultaneous change of Weight and Weave.*

Here we have to find the counts of yarn and ends and picks per inch to make a cloth with the same balance of structure as a given cloth, but of different weight and weave.

Example 1.

A worsted fabric is made of 2/36's worsted with 45 ends and picks per inch plain weave. It is required to change to $\frac{2}{2}$ twill and increase the weight $\frac{1}{2}$. Find the counts and sett.

This example we shall work in steps, referring the student to the various preceding paragraphs for the detailed methods employed.

I. We keep the weave the same (*i.e.*, plain) and find the counts and ends per inch *increasing the weight in the ratio* of 3 : 2.

(*a*) *To find the Counts.*

If required counts $= x$'s worsted,

$$\frac{2}{3} = \frac{\sqrt{x}}{\sqrt{18}} \quad \text{[see Examples, § 167]}$$

$$\text{or} \quad \frac{x}{18} = \frac{4}{9}.$$

$$\therefore \quad x = \frac{4 \times 18}{9} = 8\text{'s worsted.} \qquad (\alpha)$$

(*b*) *To find the Sett.*

If sett required $= y$ ends per inch,

$$\frac{y}{45} = \frac{\sqrt{8}}{\sqrt{18}} = \frac{2}{3}$$

$$\text{or} \quad y = \frac{2 \times 45}{3} = 30 \text{ ends and picks per inch.} \qquad (\beta)$$

II. Now change the weave from plain to $\frac{2}{2}$ twill, and find the new ends per inch. [See Examples, § 166.]

If diameter of yarn used (8's worsted) $= \frac{1}{D}$ inch, and given sett = 30 ends per inch—plain weave, then

$$\frac{D \times 2}{2 + 2} = 30,$$

$$\text{or} \quad D = \frac{4 \times 30}{2}. \qquad \text{(A)}$$

If the required sett $= y'$ in the $\frac{2}{2}$ twill,

$$\frac{D \times 4}{4 + 2} = y'$$

$$\text{or} \quad D = \frac{6 \times y'}{4}, \qquad \text{(B)}$$

But D is the same for both weaves.

$$\therefore \quad \frac{6 \times y'}{4} = \frac{4 \times 30}{2}$$

$$\text{or} \quad y' = \frac{4 \times 30 \times 4}{2 \times 6} = 40.$$

Then the sett = 40 threads per inch. (γ)

III. Now setting 40 threads per inch (γ) in place of 30 ends per inch (β) and using 8's worsted will give a cloth heavier than required in the ratio of 40 : 30 or 4 : 3.

We now find the counts and ends per inch reducing the weight in the ratio of 4 : 3. [See Examples, § 167.]

(*c*) *To find the Counts.*

If required Counts = x',

$$\frac{4}{3} = \frac{\sqrt{x'}}{\sqrt{8}}$$

Squaring both sides, $\frac{x'}{8} = \frac{16}{9}$

$$\text{or} \quad x' = \frac{16 \times 8}{9} = 14\tfrac{2}{9}\text{'s worsted.}$$

(*d*) *To find the Sett.*

If sett required = y'' ends per inch,

$$\frac{y''}{40} = \frac{\sqrt{14\frac{2}{9}}}{\sqrt{8}} = \frac{4}{3}$$

$$\text{or} \quad y'' = \frac{4 \times 40}{3} = 53\tfrac{1}{3} \text{ ends per inch,}$$

so that finally the cloth required must be woven $53\frac{1}{3}$ ends and picks per inch with a $14\frac{2}{9}$'s worsted yarn.

Problems of this type can always be worked using steps I, II, and III above.

169. *Twist or Turns per inch.*

We require to find the twist or turns per inch for a yarn in any count to be similar to a given yarn.

Now since the thicker the yarn the fewer the turns per inch, then the *twist varies inversely as the diameter of the yarn, i.e.*, if T_1 and T_2 are the numbers of turns per inch of threads with diameters D_1 and D_2 respectively,

$$\frac{T_1}{T_2} = \frac{D_2}{D_1}. \qquad \text{(i)} \quad [\S\ 138\ (a), \text{p. } 280.]$$

But the *diameter varies inversely as the square root of the counts.* [§ 162, p. 390.]

So that if C_1 and C_2 are the corresponding counts of the above-mentioned threads, we have

$$\frac{D_2}{D_1} = \frac{\sqrt{C_1}}{\sqrt{C_2}}. \quad \text{(ii)}$$

Then from (i) and (ii) we have

$$\frac{T_1}{T_2} = \frac{\sqrt{C_1}}{\sqrt{C_2}},$$

or *the number of turns per inch varies directly as the square root of the counts.* [§ 137, p. 278.]

Example 1.

Given a 2 30's worsted with 15 turns per inch, how many turns per inch will be required to make a similar yarn in 2/40's worsted ?

If number of turns per inch required $= x$, we have

$$\frac{x}{15} = \frac{\sqrt{20}}{\sqrt{15}}.$$

Squaring both ratios, $\frac{x^2}{15^2} = \frac{20}{15}$

or $$x^2 = \frac{20 \times 15 \times 15}{15} = 300.$$

Taking square root of both sides.

$$x = \sqrt{300} = 17{\cdot}3.$$

Then the yarn required has 17·3 turns per inch.

170. *The Bending of the Warp and Weft threads—Angle due to Curvature.*

In establishing the Intersection Rule for the setting of the warp threads in § 164, we supposed that the warp and weft threads lay perfectly straight in the cloth. Actually this is not the case. If we draw a thread of warp or weft from a piece of cloth we find that it is wavy ; that the weft has been bent round the warp and the warp round the weft in some degree.

Suppose the diagram (Fig. 184) represents the cross section

of a piece of cloth—plain weave—and suppose that there is equal bending of the warp and weft.

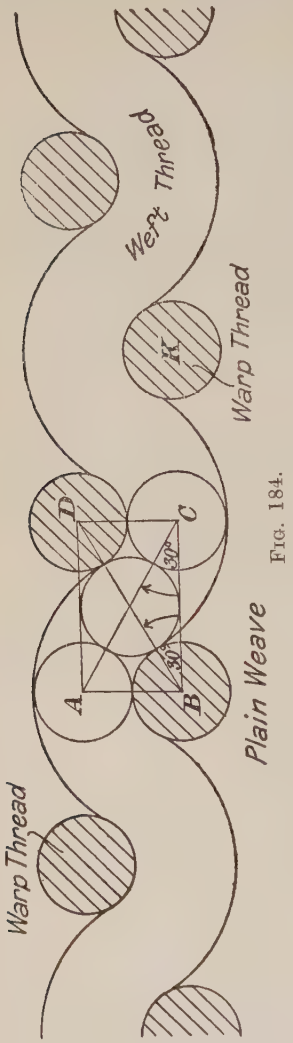


Fig. 184.

If A and C mark the centres of the weft thread when at its highest and lowest positions (over and under the warp threads) respectively, then the angle ACB is called *the Angle due to Curvature* of the weft. The angle DBC will measure the curvature of the warp. This angle due to curvature will vary with different weaves and thicknesses of warp and weft, but in this book we are only concerned with the calculations involved if the angle due to curvature is known.

Example 1.

(*a*) Suppose we have a plain cloth made with 2/32's worsted (diameter $= \frac{1}{8\cdot5}$ inch). Suppose angle due to curvature, which is the same in both warp and weft, is 30°. Find the possible ends and picks per inch.

In the figure (Fig. 184) Angle $ACB = 30°$ and Angle $DBC = 30°$.

Since the diameters of warp and weft threads are the same ($\frac{1}{8\cdot5}$ inch) BD represents $\frac{2}{8\cdot5}$ inch (1 weft diameter + 1 warp diameter).

If there were no bending we should say that one warp thread together with one intersection of weft would occupy $\frac{2}{8\cdot5}$ inch measured across the cloth.

But the diagram shows that the distance occupied by them across the cloth is represented by BC.

$$\text{Now} \quad \frac{BC}{BD} = \text{Cos } DBC \qquad [\S\ 120, \text{p. } 225.]$$

$$= \text{Cos } 30° = 0{\cdot}8660.$$

$$\therefore \quad BC = 0{\cdot}8660 \times BD$$

$$= 0{\cdot}8660 \times \tfrac{2}{8\cdot5} = 1{\cdot}732 \times \tfrac{1}{8\cdot5} \text{ inch.}$$

In a plain weave one repeat of the pattern (from B to K, Fig. 184) contains 2 warp threads and 2 intersections of the weft thread. Then one repeat occupies $(2 \times 1{\cdot}732 \times \frac{1}{85})$ inch across the cloth. Hence in "setting" the warp,

In $(2 \times 1{\cdot}732 \times \frac{1}{85})$ inch we must have 2 warp threads.

In 1 inch we must have $\dfrac{2}{2 \times 1{\cdot}732 \times \frac{1}{85}}$ warp threads.

$$\therefore \quad \text{Ends per inch} = \frac{2 \times 85}{2 \times 1{\cdot}732} = 49{\cdot}07.$$

Since the warp bends equally with the weft, picks per inch $= 49{\cdot}07$.

If we find the sett by the Intersection Rule we have

$$\text{Ends per inch} = \frac{85 \times 2}{2 + 2} = \frac{85 \times 2}{4} = 42{\cdot}5.$$

Example 2.

Let us find the ends and picks per inch in a $\frac{2}{2}$ twill made with 2/36 worsted (diameter $= \frac{1}{90}$ inch), and let us suppose the angle due to curvature is again 30°.

Considering the diagram (Fig. 185) we have distance SR measured across the cloth is occupied by one repeat of the design.

Now $SR = SB + BC + CQ + QR$,

and of these SB represents 1 diameter $= \frac{1}{90}''$,

CQ ,, 1 ,, $= \frac{1}{90}''$.

Since $D\hat{B}C = 30°$ and $\hat{P}RQ = 30°$,

$$\frac{BC}{BD} = \text{Cos } 30° = 0{\cdot}8660 \qquad \text{(i)}$$

and

$$\frac{QR}{PR} = \text{Cos } 30° = 0{\cdot}8660. \qquad \text{(ii)}$$

Now BD and PR each represent 2 diameters $= \frac{2}{90}$ inch.

Hence from (i) $BC = 0{\cdot}8660 \times BD$

$= 0{\cdot}8660 \times \frac{2}{90}$ inch

$= 1{\cdot}732 \times \frac{1}{90}$ inch.

Similarly $QR = 1{\cdot}732 \times \frac{1}{90}$ inch.

Then $SR = SB + BC + CQ + QR$

$$= \left\{ \frac{1}{90} + \left(\frac{1{\cdot}732}{90}\right) + \frac{1}{90} + \frac{1{\cdot}732}{90} \right\} \text{ inch}$$

$$= \frac{(2 \times 1{\cdot}732) + 2}{90} \text{ inch},$$

and in this distance SR we must place 4 warp threads, *i.e.*,

In $\frac{(2 \times 1{\cdot}732) + 2}{90}$ inch we must have 4 warp threads,

or in 1 inch we must have $\frac{4}{\frac{(2 \times 1{\cdot}732) + 2}{90}}$ warp threads.

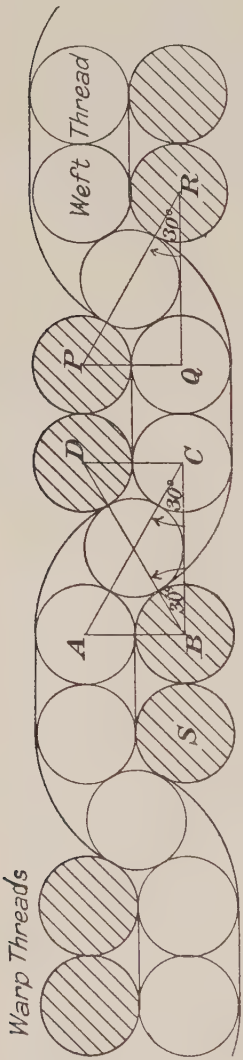


FIG. 185.

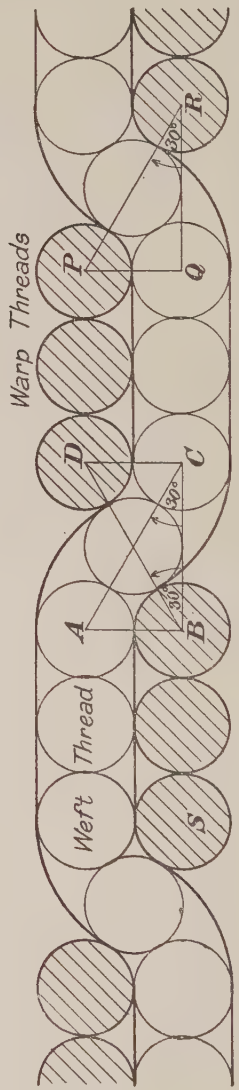


FIG. 186.

$$\therefore \text{ Ends per inch and also picks per inch} = \frac{4 \times 90}{(2 \times 1{\cdot}732) + 2}$$

$$= \frac{4 \times 90}{5{\cdot}464}$$

$$= 65{\cdot}89.$$

If we find the sett by the Intersection Rule we have

$$\text{Ends per inch} = \frac{90 \times 4}{4 + 2} = \frac{90 \times 4}{6} = 60,$$

so that allowing for bending we may use a closer setting.

Example 3.

To find the ends and picks per inch in a $\frac{2}{3}$ twill woven with a 2/24's worsted (diameter = $\frac{1}{74}$ inch). Suppose equal bending of warp and weft and that the angle due to curvature is 30°.

Considering the diagram (Fig. 186), SR represents the distance measured across the cloth occupied by one repeat of the design (*i.e.*, 6 warp ends and 2 intersections of weft).

Angle DBC = Angle $PRQ = 30°$.

$BD = PR$ = Two diameters = $\frac{2}{74}$ inch.

Then $\frac{BC}{BD} = \text{Cos } 30° = 0{\cdot}8660,$

or $BC = 0{\cdot}8660 \times BD$

$= 0{\cdot}8660 \times \frac{2}{74}$ inch.

$= (1{\cdot}732 \times \frac{1}{74})$ inch.

Similarly $QR = (1{\cdot}732 \times \frac{1}{74})$ inch.

Also $SB = 2$ diameters $= \frac{2}{74}$ inch.

$CQ = 2$,, $= \frac{2}{74}$ inch.

Then $SR = SB + BC + CQ + QR$

$$= \frac{2}{74} + \frac{1{\cdot}732}{74} + \frac{2}{74} + \frac{1{\cdot}732}{74}$$

$$= \frac{(2 \times 1{\cdot}732) + 4}{74} \text{ inch,}$$

and in this distance SR there are 6 ends of warp.

Hence in $\frac{(2 \times 1{\cdot}732) + 4}{74}$ inch we must have 6 ends of warp.

In 1 inch we must have $\frac{6}{(2 \times 1{\cdot}732) + 4}$ ends of warp.

$$\therefore \quad \text{Ends per inch} = \frac{6 \times 74}{(2 \times 1{\cdot}732) + 4} = \frac{444}{7{\cdot}464} = 59{\cdot}5.$$

By the Intersection Rule,

$$\text{Ends per inch} = \frac{74 \times 6}{6 + 2} = \frac{74 \times 6}{8} = 55{\cdot}5.$$

Note.—In these three examples we have kept the angle of curvature the same (30°) and altered the weave.

In each case the allowance for curvature gives a greater number of ends per inch than that obtained by the Intersection Rule; but the difference between the two decreases as the proportion between the ends of warp and intersections increases thus:

	Ends per Inch.		Increase of A over B.	Percentage increase.
	Allowing for curvature (30°).	By Intersection Rule.		
	A.	B.		
Example 1 (plain)	49·07	42·5	6·57	$\frac{6{\cdot}57 \times 100}{42{\cdot}5} = 15{\cdot}5\%$
,, 2 ($\frac{2}{2}$ twill)	65·89	60	5·89	$\frac{5{\cdot}89 \times 100}{60} = 9{\cdot}8\%$
,, 3 ($\frac{3}{3}$ twill)	59·5	55·5	4	$\frac{4 \times 100}{55{\cdot}5} = 7{\cdot}2\%$

(*b*) We will next consider the effect of keeping the weave the same whilst increasing the angle due to curvature.

Example 4.

As in Example 1, we will consider a plain weave made with 2/32's worsted (diameter = $\frac{1}{8{\cdot}5}$ inch). Suppose the angle due to curvature is 45°..

Referring to Fig. 184, Example 1, we have

$$\text{Angle } DBC = 45°.$$

BD as before represents $\frac{2}{8{\cdot}5}$ inch (1 weft diameter and 1 warp diameter).

BC represents the distance across the cloth occupied by one warp thread and one intersection of weft.

Now $$\frac{BC}{BD} = \text{Cos } 45° = 0{\cdot}707.$$

$$\therefore \quad BC = 0{\cdot}707 \times BD$$

$$= 0{\cdot}707 \times \frac{2}{85} = \frac{1{\cdot}414}{85} \text{ inch.}$$

Then one repeat of the design (B to K) occupies $\frac{2 \times 1{\cdot}414}{85}$ inch and in this distance BK there are 2 ends of warp.

Hence in $\frac{2 \times 1{\cdot}414}{85}$ inch we must have 2 warp threads,

or in 1 inch we must have $\frac{2}{\frac{2 \times 1{\cdot}414}{85}}$ warp threads.

$$\therefore \quad \text{Ends per inch} = \frac{2 \times 85}{2 \times 1{\cdot}414} = 60{\cdot}12. \qquad \text{(A)}$$

By the Intersection Rule,

$$\text{Ends per inch} = \frac{85 \times 2}{2 + 2} = \frac{85 \times 2}{4} = 42{\cdot}5. \qquad \text{(B)}$$

Increase of (A) over (B) $= 60{\cdot}12 - 42{\cdot}5 = 17{\cdot}62.$

Percentage increase $= \frac{17{\cdot}62 \times 100}{42{\cdot}5} = 41{\cdot}5\%.$

Comparing Examples 1 and 4 we see that, with the same weave, as the angle increases the cloth may contain relatively more threads of warp or weft per inch, and inversely an increase in the number of warp or weft threads causes a greater angle due to curvature in the respective set of threads in which the increase takes place.

It is to be hoped the student will now be able, with the aid of a diagram and his mathematical tables, to solve problems which occur in his lectures and his practical work.

USEFUL CONSTANTS AND MATHEMATICAL TABLES.

USEFUL CONSTANTS.

(Many of the more fundamental data given here should become, by repeated use, part of the mental equipment of technical students. They are given in this place for convenience of reference.)

1 Inch = 25·40 millimetres. 1 mm. = ·03937 inch.
1 Gallon = ·1604 cubic foot = 10 lb. of water at 62° F.
1 Knot = 6080 feet per hour = 1 Nautical mile per hour.
Weight of 1 lb. in London = 445,000 dynes.
One pound avoirdupois = 7000 grains = 453·6 grammes.
1 Cubic foot of water weighs 62·3 lb.
1 Cubic foot of air at 0° C. and 1 atmosphere, weighs ·0807 lb.
1 Cubic foot of Hydrogen at 0° C. and 1 atmosphere, weighs ·00559 lb.
1 Foot-pound = $1{\cdot}3562 \times 10^7$ ergs.
1 Horse-power-hour = 33000 × 60 foot-pounds.
1 Electrical unit = 1000 watt-hours = 1·34 horse-power-hours.
Joule's Equivalent to suit Regnault's H, is { 774 ft.-lb. = 1 Fah. unit. / 1393 ft.-lb. = 1 Cent. ,,
1 Horse-power = 33000 foot-pounds per minute = 746 watts.
Volts × ampères = watts.
1 Atmosphere = 14·7 lb. per square inch = 2116 lb. per square foot = 760 mm. of mercury = 10^6 dynes per sq. cm. nearly.
A column of water 2·3 feet high corresponds to a pressure of 1 lb. per sq. inch.
Absolute temp., $t = \theta°$ C. + 273° or $\theta°$ F. + 459·4°.
Regnault's H = 606·5 + ·305 $\theta°$ C. = 1082 + ·305 $\theta°$ F.
$p\, u^{1.0646} = 479.$

$$\log_{10} p = 6{\cdot}1007 - \frac{B}{t} - \frac{C}{t^2},$$

where $\log_{10} B = 3{\cdot}1812$, $\log_{10} C = 5{\cdot}0882$.
p is in pounds per sq. inch, t is absolute temperature Centigrade.
u is the volume in cubic feet per pound of steam.
$\pi = 3{\cdot}1416$.
One radian = 57·30 degrees.
To convert common into Napierian logarithms, multiply by 2·3026.
The base of the Napierian logarithms is $e = 2{\cdot}7183$.
The value of g at London = 32·182 feet per sec. per sec.

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	9	13	17	21	26	30	34	38
											4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	12	15	19	23	27	31	35
											4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	11	14	18	21	25	28	32
											3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	7	10	13	16	20	23	26	30
											3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	28
											3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	9	11	14	17	20	23	26
											3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	14	16	19	22	24
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17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3	5	8	10	13	15	18	20	23
											2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
											2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
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20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

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MATHEMATICAL TABLES

Logarithms.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

MATHEMATICAL TABLES

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1427	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

MATHEMATICAL TABLES

Angle. Degrees.	Radians.	Chord.	Sine.	Tangent.	Co tangent.	Cosine.			
0°	0	0	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine	Co-tangent	Tangent	Sine	Chord	Radians	Degrees
								Angle	

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PRINTED IN GREAT BRITAIN BY R. CLAY AND SONS, LTD.,
BRUNSWICK STREET, STAMFORD STREET, S.E. 1, AND BUNGAY, SUFFOLK.